Fundamentals of open (quantum) systems

Iacopo Carusotto

Pitaevskii BEC Center, INO-CNR and Dipartimento di Fisica, Università di Trento
I-38123 Trento, Italy

Non-Markovian pumping

ACADÉMIE
DES SCIENCES
INSTITUT DE FRANCE

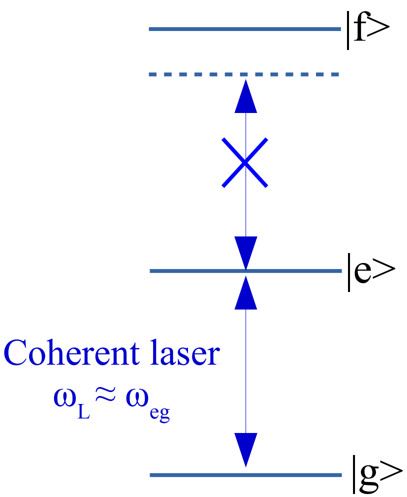
Comptes Rendus. Physique
2025, Vol. 26, p. 533-569
https://doi.org/10.5802/crphys.258

Intervention in a conference / *Intervention en colloque*

How to exploit driving and dissipation to stabilize and manipulate quantum many-body states

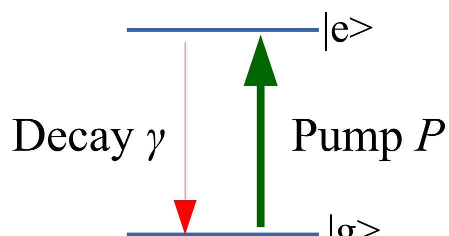
Different pumping schemes in a nutshell

Coherent pump

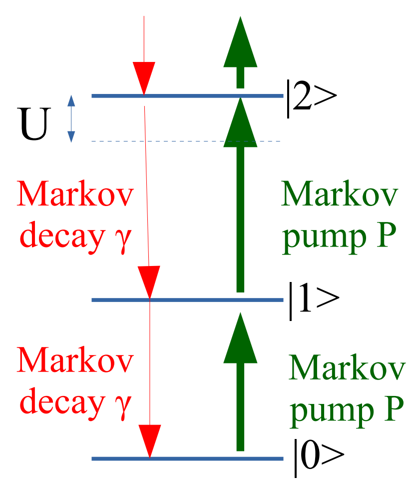


- Coherent laser drives absorption and stimulated emission
- Restricted to resonant transition, no population into |f>
- |e> population saturates to ½ under CW strong pumping
- Population inversion requires e.g. pulsed excitation

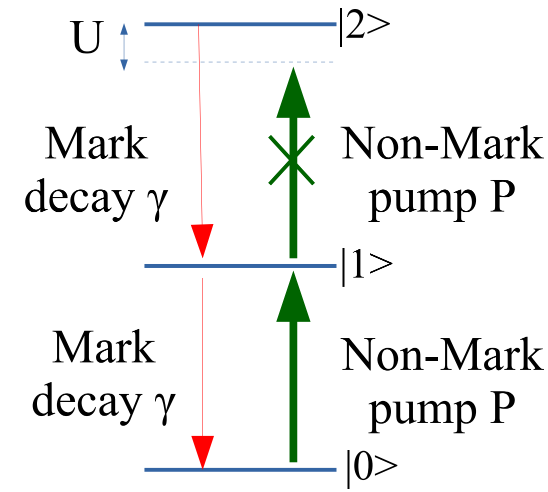
Standard (Markovian) incoherent pump



- For $P >> \gamma$ atom pumped in |e>
- Markovian pump energy-insensitive, keeps exciting to higher states



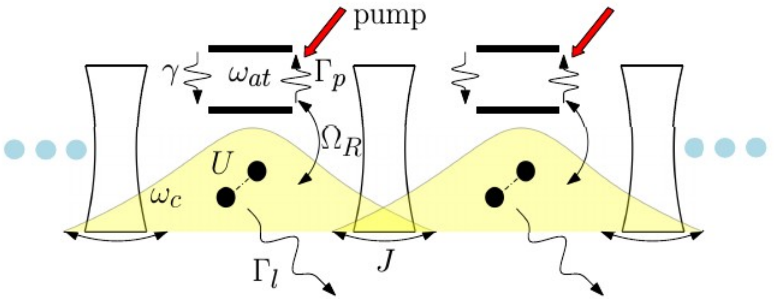
Frequency-dependent (i.e. non-Markovian) incoherent pump



- Transition to |2> detuned → forbidden by freq-depend.
- For $P >> \gamma$ population accumulates into state |1>
- Naturally obtained via frequency-dependent gain, e.g. population-inverted emitters

Many-cavity array: many-body states stabilized

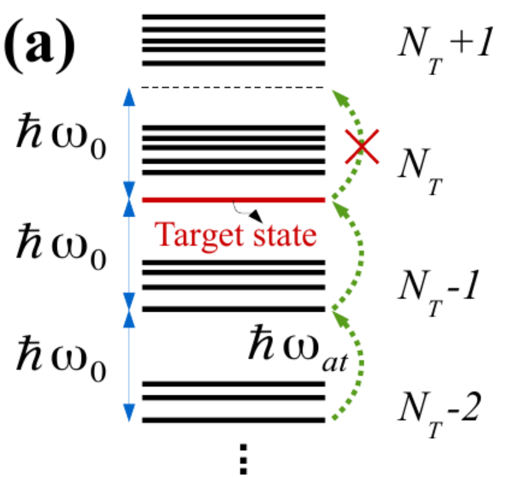
- Fabricate array of cavities with strong interactions
- Frequency-dependent incoherent pumping,
  e.g. inverted emitters in each cavity



Non-Markovian pump:

- Inverted emitters → Lorentzian emission line around ω_{at}
- Photon injection only active if many-body transition
  is near resonance, otherwise losses dominate
- For P >> γ photons injected until band is full (MI) or
  many-body gap develops (FQH)
- Many-body gap blocks excitation to higher states and larger N

→ desired correlated state gets stabilized !



General idea:
Kapit, Hafezi, Simon, PRX 2014
Lebreuilly et al. CRAS (2016)

Umucalilar-IC, PRA 2017
Lebreuilly, Biella et al., PRA 2017

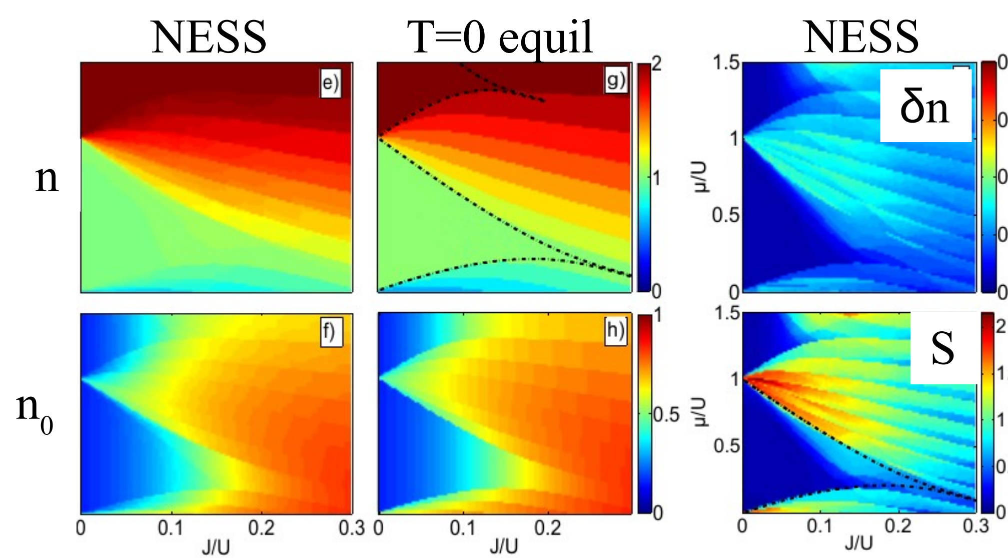
Mott insulator of light

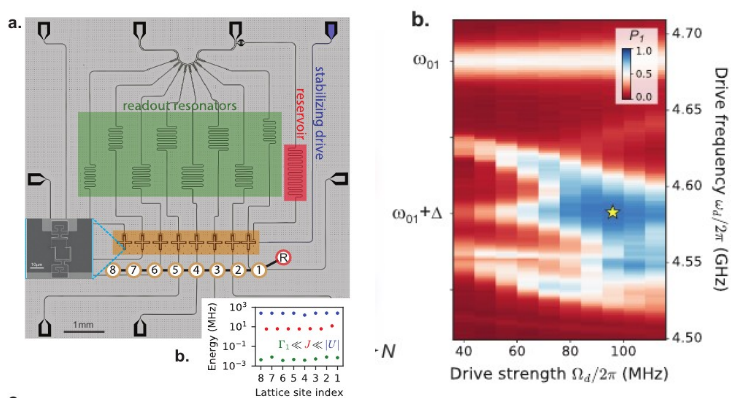
- One-dimensional array, strong nonlinearity U >> J
- Most naive non-Markovian master equation:
  frequency-dependent emission → rescaled jump operators

$$\bar{\mathcal{L}}_{\rm em}(\rho_{\rm ph}) = \frac{\Gamma_{\rm em}}{2}\sum_{i=1}^{k}\left[2\bar{a}_i^\dagger \rho_{\rm ph}\bar{a}_i - \bar{a}_i\bar{a}_i^\dagger\rho_{\rm ph} - \rho_{\rm ph}\bar{a}_i\bar{a}_i^\dagger\right]$$

$$\langle f'|\bar{a}_i^\dagger|f\rangle = \frac{\Gamma_{\rm pump}/2}{\sqrt{(\omega_{\rm at}-\omega_{f',f})^2+(\Gamma_{\rm pump}/2)^2}}\langle f'|a_i^\dagger|f\rangle$$

- driven-dissipative steady state stabilizes
  strongly correlated Mott-insulator state
- resembles low-T equilibrium
- (in principle) no restriction to small N_{ph}
  only requirement → many-body energy gap





First expt: Ma *et al.* Nature 2019

Theoretical calculations
Lebreuilly, Biella et al., 1704.01106 & 1704.08978
Related work in Kapit, Hafezi, Simon, PRX 2014

First experimental observation of a Mott insulator state of impenetrable photons

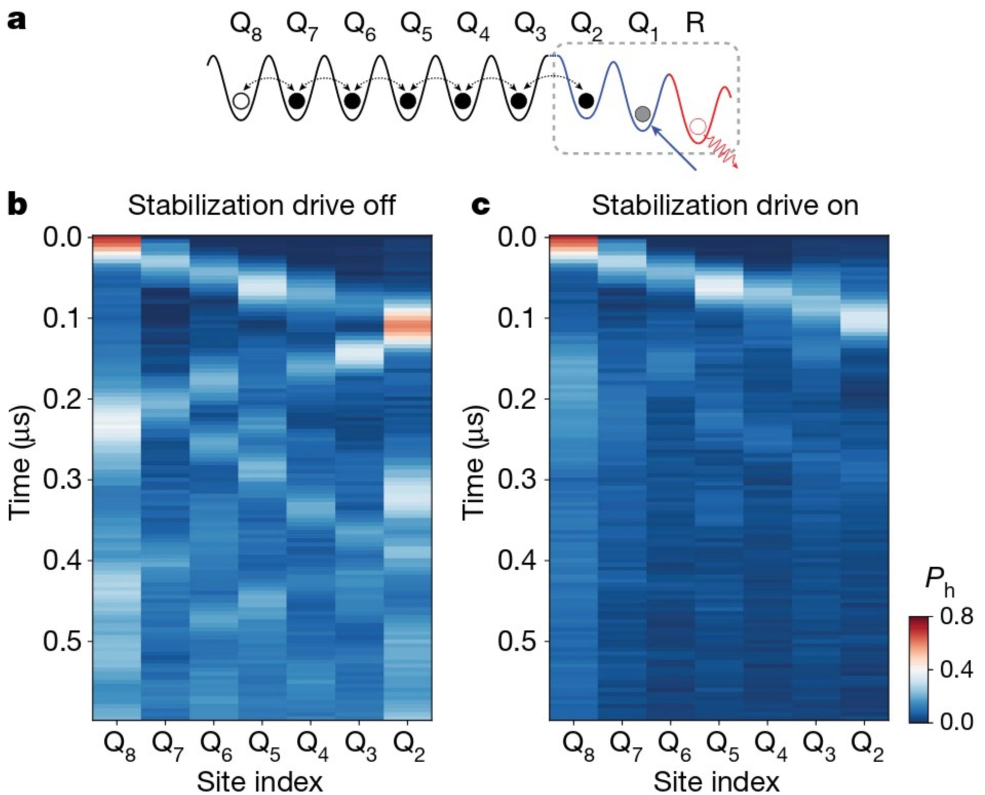
Mott insulator of light: dynamics

Pioneering expt: start with hole in Q_8

- No stabilizer: holes oscillates back and forth
- Stabilizer: hole quickly filled

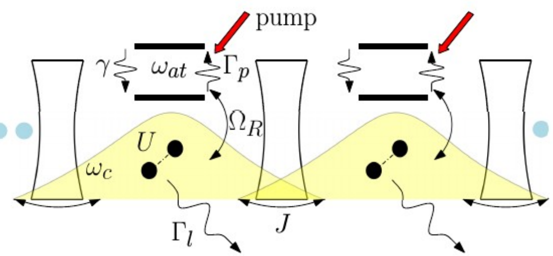
Suggests experimental access to
full dispersion of collective excitations

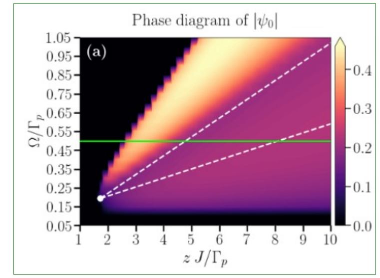
Question: what effect of driving & dissipation?



First expt: Ma *et al.* Nature 2019

Collective modes around Mott-superfluid phase transition



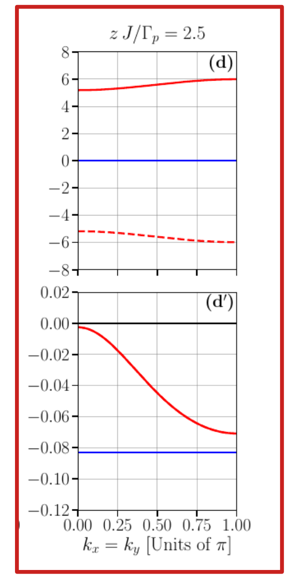


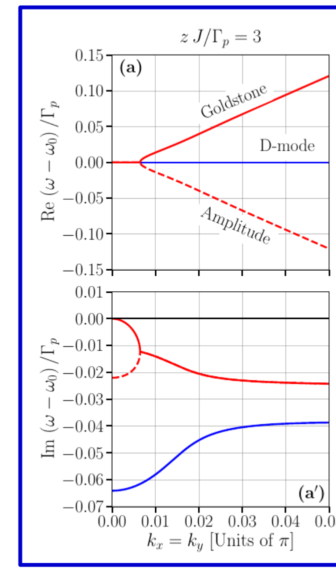
Exact description of non-Markovianity of emitter

→ explicit inclusion of two-level emitters:

- Markovian incoherent pump Γ_P
- Coupling to cavity mode Ω_R → emission irreversible via Γ’s
- Frequency-dependent emission of linewidth Γ_P

Biella, Lebreully et al., 1704.01106 & 1704.08978

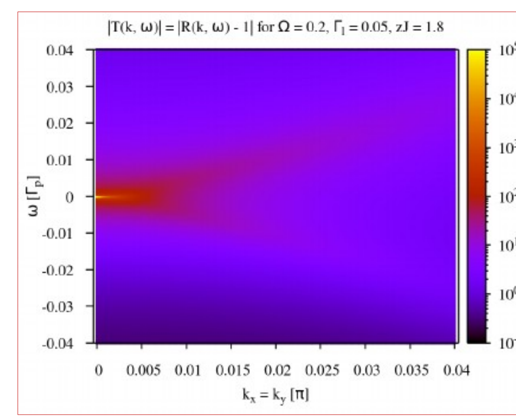




Superfluid-insulator non-equilibrium phase transition

Interesting behaviour of collective excitation modes across transition:

- Linearized Gutzwiller approach; visible in transmission/reflection/FWM
- Gap closes in Mott insulating phase approaching critical point
- Diffusive Goldstone mode in superfluid
- Similar physics as in polariton BECs
  (Wouters, Szymanska/Keeling, Diehl, expt: Bramati)



F. Caleffi, M. Capone, IC, *Collective excitations of a strongly-correlated non-equilibrium photon fluid across the Mott/superfluid phase transition*, PRL 2023; Fabio Caleffi, PhD thesis @ SISSA (2022)

What about large FQH fluids?

Coherent pump:

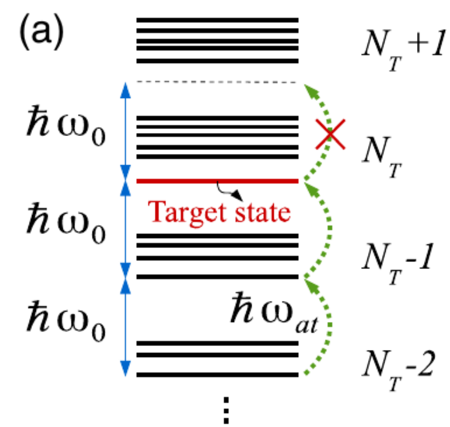
- Able to selectively generate few-body states
- Limited by (exponentially) decreasing matrix element for larger systems

Frequency-dependent incoherent pump:

- Interactions → many-body gap Δ
- Edge excitations not gapped. Hard-wall confinement gives small δ
- Non-Markovianity blocks excitation to higher states

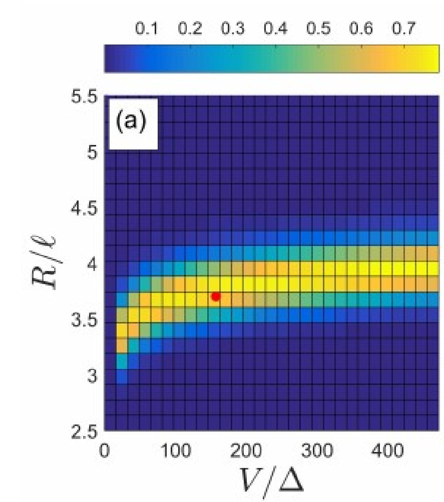
Ab initio calculations only possible for small systems:

- Large overlap with Laughlin states
- Residual excitations localized mostly on edge



Open question:

- What are ultimate limitations of this pumping method?
- Non-eq features in collective excitation modes?
- How to extend ideas to atomic clouds?



- R. O. Umucalilar, IC, *Generation and spectroscopic signatures of a fractional quantum Hall liquid of photons in an incoherently pumped optical cavity*, PRA 2017.
- R. O Umucalilar, J. Simon, IC, *Autonomous stabilization of photonic Laughlin states through angular momentum potentials*, PRA 2022
- Interesting subtleties: Kurilovich et al., *Stabilizing the Laughlin state of light: dynamics of hole fractionalization*, arXiv:2111.01157

Non-Equilibrium condensation

(aka lasing in a spatially-extended system)

Non-equilibrium Bose–Einstein condensation in photonic systems

Jacqueline Bloch[1], Iacopo Carusotto[2] and Michiel Wouters[3]

Review article on Nat. Rev. Phys. (2022)

2006 - Non-equilibrium Bose-Einstein condensation

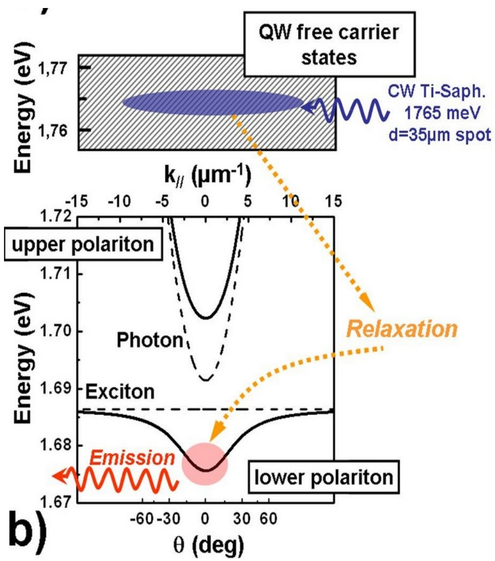
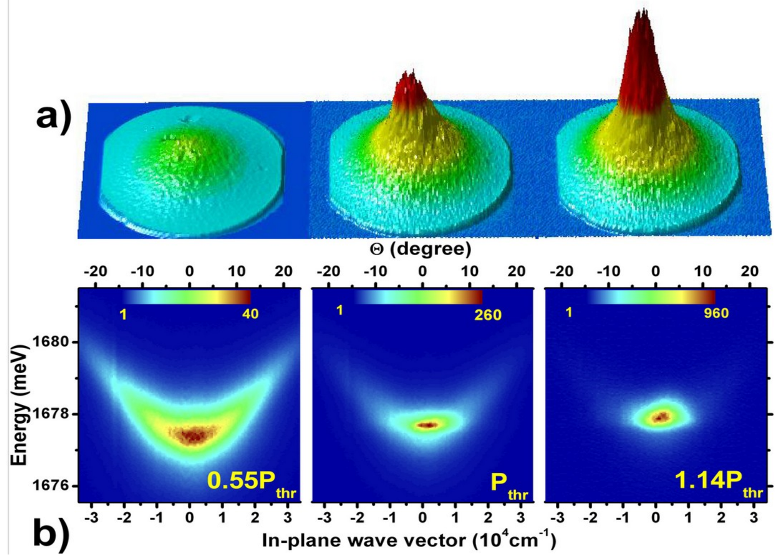
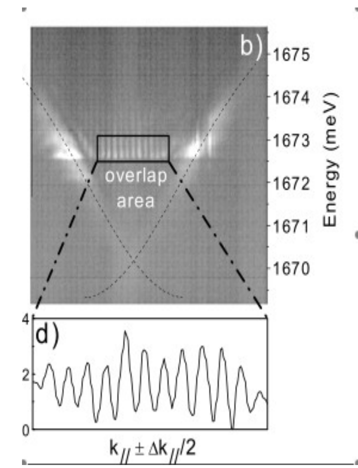


Figure from Kasprzak et al., Nature 2006



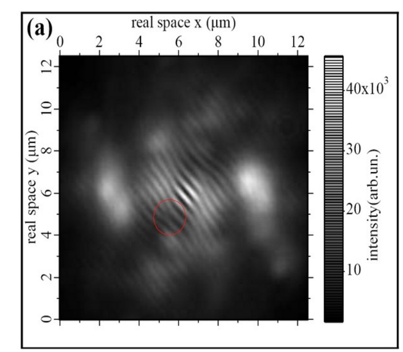
Momentum distribution

Kasprzak et al., Nature **443**, 409 (2006)



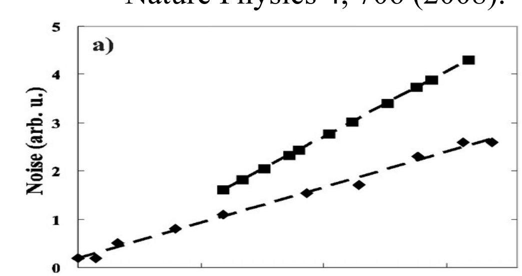
Interference

Richard et al., PRL **94**, 187401 (2005)



Quantized vortices

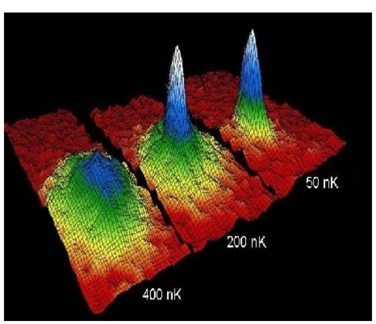
K. Lagoudakis *et al.*
Nature Physics 4, 706 (2008).



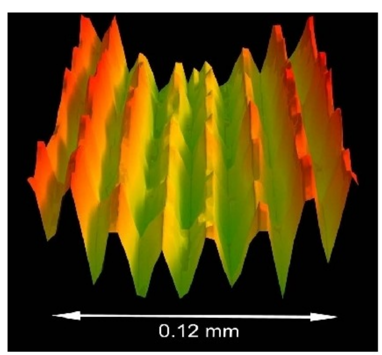
Suppressed fluctuations

A. Baas et al., PRL **96**, 176401 (2006)

Many features very similar to atomic BEC



The first atomic BEC
M. H. Anderson et al.
Science **269**, 198 (1995)



Interference pattern of two expanding atomic BECs
M. R. Andrews, Science **275**, 637 (1995)

Quantum/thermal fluctuations: quasi-BEC in low-D

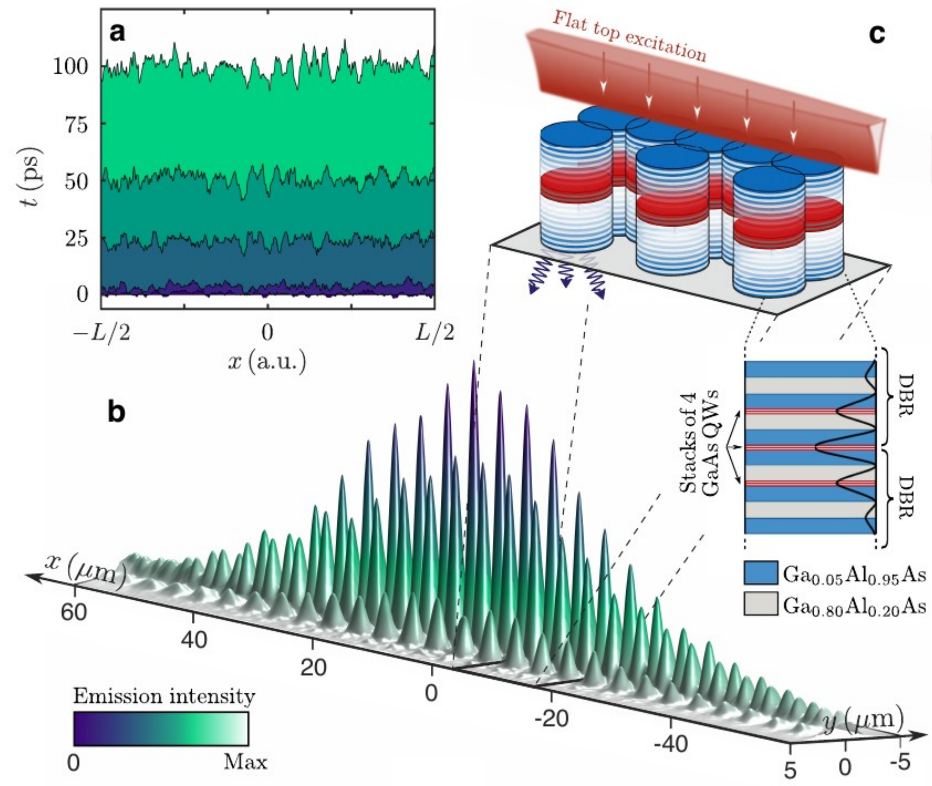
Quasi-condensation features:

Hohenberg-Mermin-Wagner theorem:

- at equilibrium, no BEC in d<3

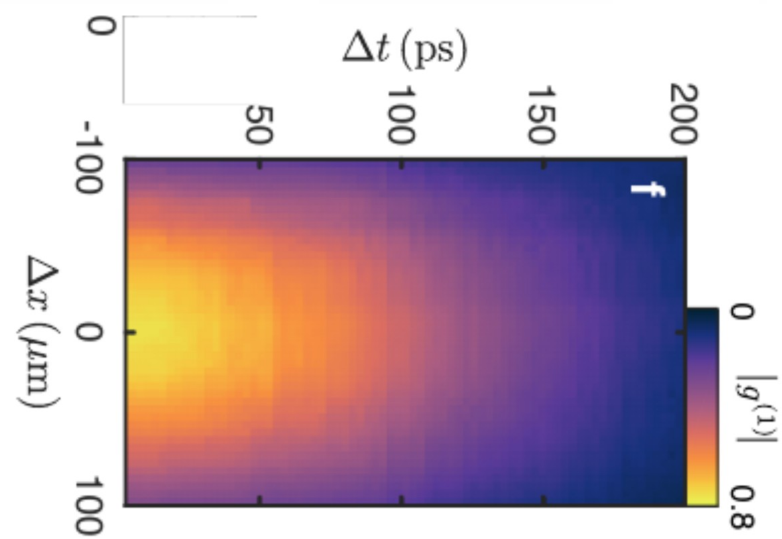
Non-equilibrium condensation or lasing:

- no BEC in 1D: exponential decay of $g^{(1)}(x)$
  (Graham-Haken, 1970; Wouters-IC, PRB 2006)
- debate in 2D: KPZ nonlin. destroy BKT phase?
  (Dagvadorj et al., PRX 2015; Altman et al., PRX 2015;
  Zamora et al., PRX 2017; …)



Experiment @ C2N Palaiseau:

- 1D lattice with array of semiconductor micropillars
- measure space-time coherence function $g^{(1)}(\Delta x,\Delta t)$
- Coherence only in limited space/time,
  then smoothly decays → quasi-BEC effect



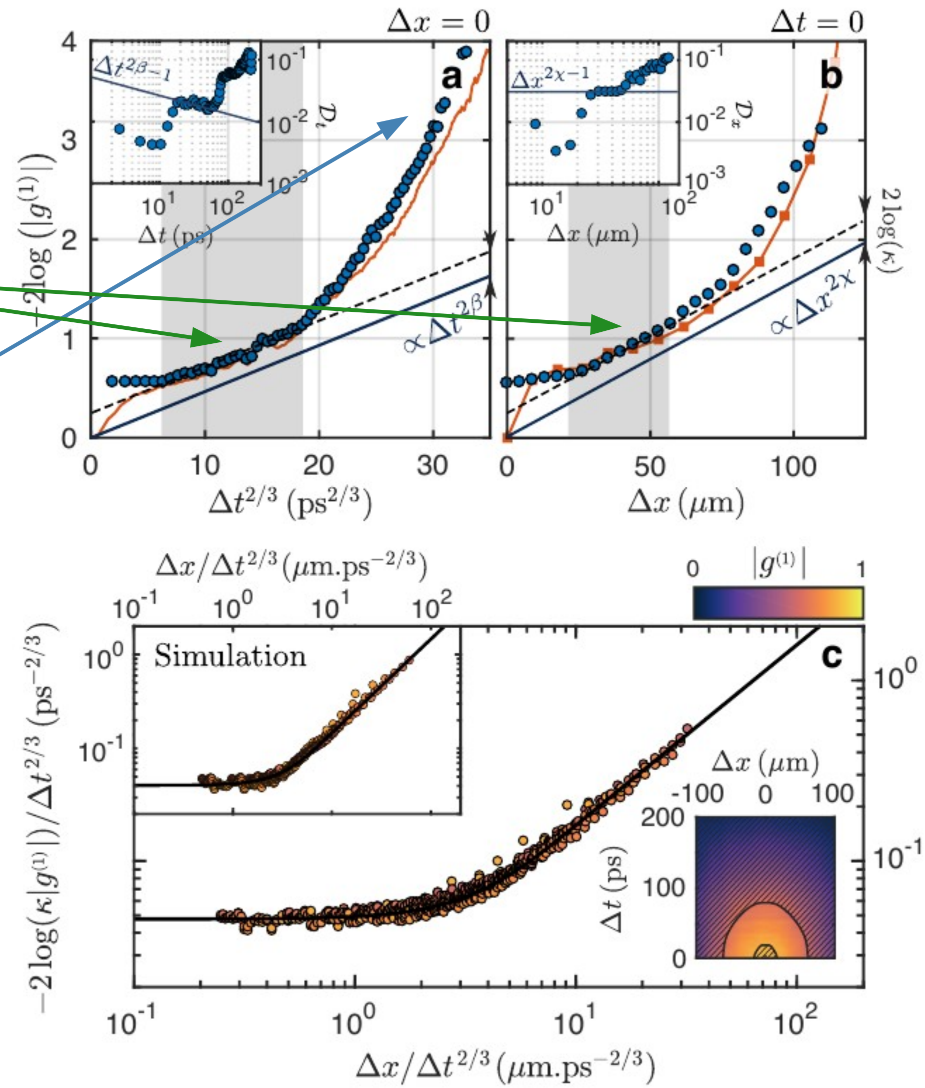
Fontaine, Squizzato, Baboux, Amelio, Lemaître, Morassi, Sagnes, Le Gratiet, Harouri, Wouters, IC, Amo, Richard, Minguzzi, Canet, Ravets, Bloch, *Observation of KPZ universal scaling in a 1D polariton condensate*, Nature 2022

KPZ universality features

Power laws in the decay of $g^{(1)}(\Delta x,\Delta t)$

KPZ exponent

Schawlow-Townes-like decay
(+ Henry factor)



Collapse of cuts of $g^{(1)}(\Delta x,\Delta t)$
on Kardar-Parisi-Zhang universal curve

→ Quasi-BEC dynamics involves non-equilibrium effects &
interactions between Bogoliubov excitations

Fontaine et al., *Observation of KPZ universal scaling in a 1D polariton condensate*, Nature 2022
Recent expts in 2D: Widmann et al. Science 2026 and Fontaine et al. (to appear)

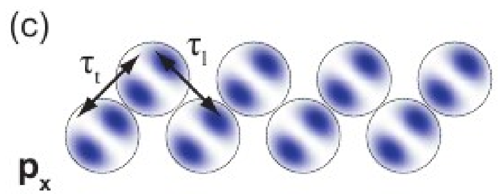
Topological lasing and non-Hermitian lattices

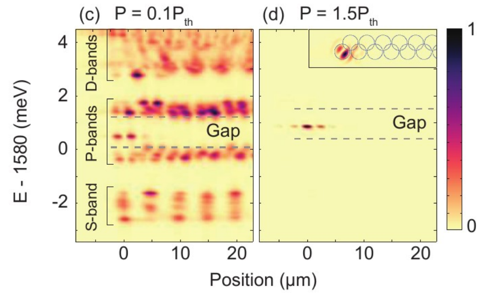
(global motion as a additional source of dissipation)

2017 - Topological lasing

a.k.a. non-equilibrium BEC in a topological edge state

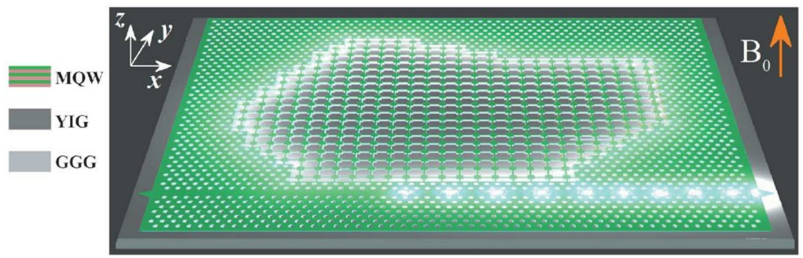
In other words: what happens if gain added to topological photonics model ?

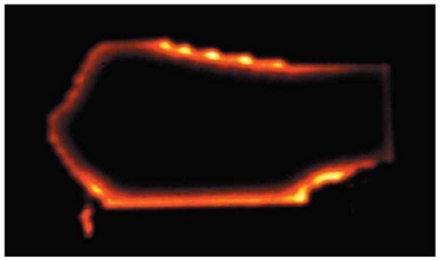


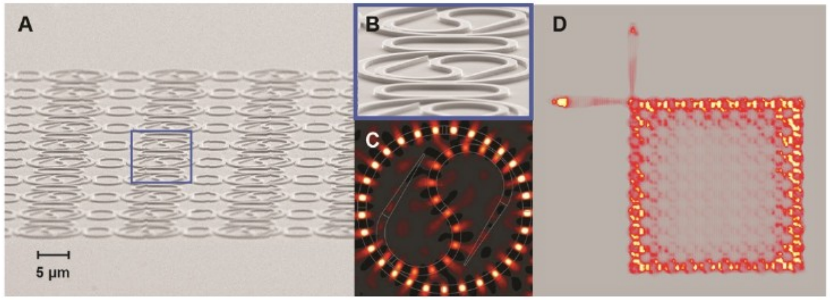


St. Jean, et al., Nat. Phot. ‘17
System: 1D SSH array of micropillar cavities for exciton-polaritons under incoherent pump

Bahari et al., Science 2017
System: 2D photonic crystal slab, amplification by QWs, magnetic field to break T





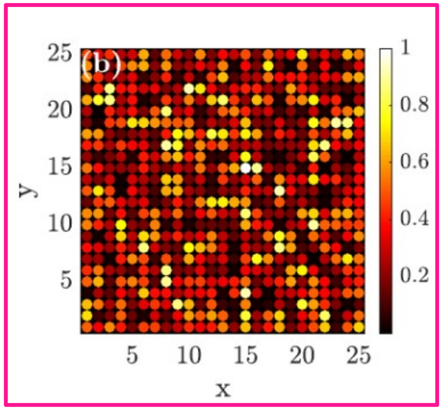


Bandres et al., Science 2018
System: array of Si-based ring resonators with optically pumped III-V amplifier layer.
Tai-Ji shape to break inversion symmetry

Early theoretical work by Conti & Pilozzi, Solnyshkov, Nalitov & Malpuech...
Other expts: Khajavikhan’s group, PRL 2018; Klembt et al., Nature 2018; Dikopoltsev et al. Science 2021,…

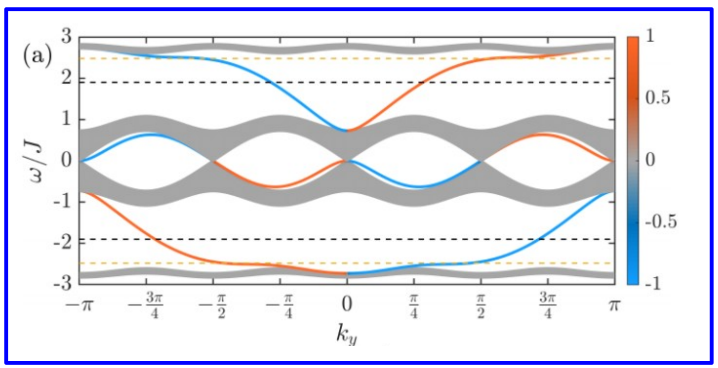
What new physics in these systems? What technological applications?

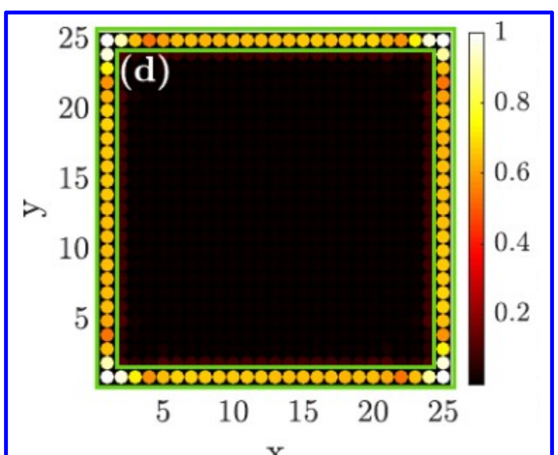
Topological lasing in 2D models: why interesting?



Topologically trivial system:

- pumping many cavities gives complicate many-mode emission because of unavoidable fabrication disorder
- hard to preserve coherence and fully exploit gain
- serious technological problem for high-power semiconductor laser applications





Topological system:

- Non-trivial band topology
  → chiral edge states, unidirectionally propagating around (finite) system
- 2D Topolaser operation into edge mode stabilized when edge only is pumped
- Chiral motion → phase locks many sites

First proposal of topological photonics: Haldane & Raghu, 2006.
First expt: Wang et al., Nature 2009 (Soljacic's group @ MIT)

Figures from M. Seclì's Msc thesis @ UniTN, 2017 and Seclì *et al.*, Phys. Rev. Research 2019
See also: Harari et al., Science 2018.

Theoretical model

Linear, single-particle physics → Harper-Hofstadter model

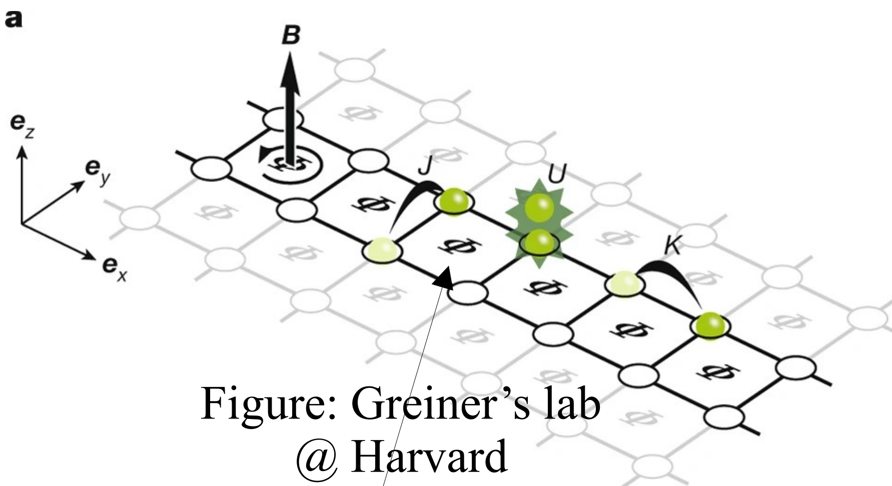
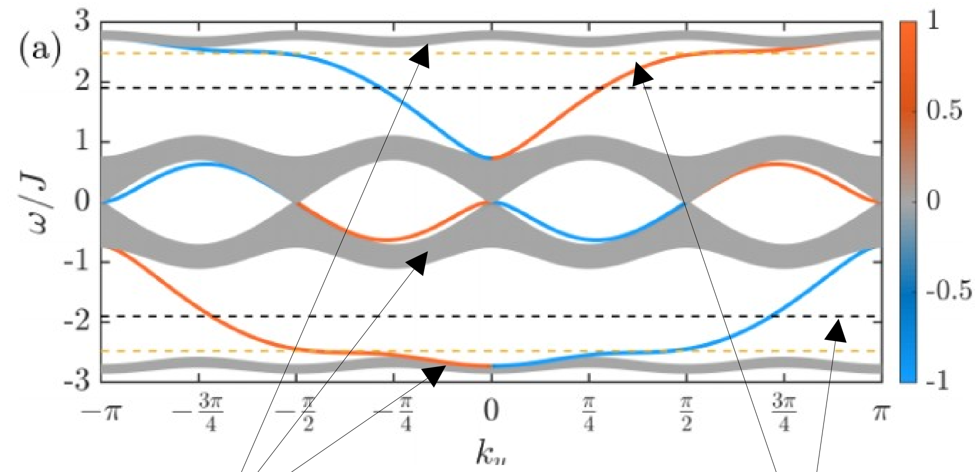


Figure: Greiner's lab @ Harvard



Topological bands (non-zero Chern)

Chiral edge states

Magnetic field encoded in hopping phase

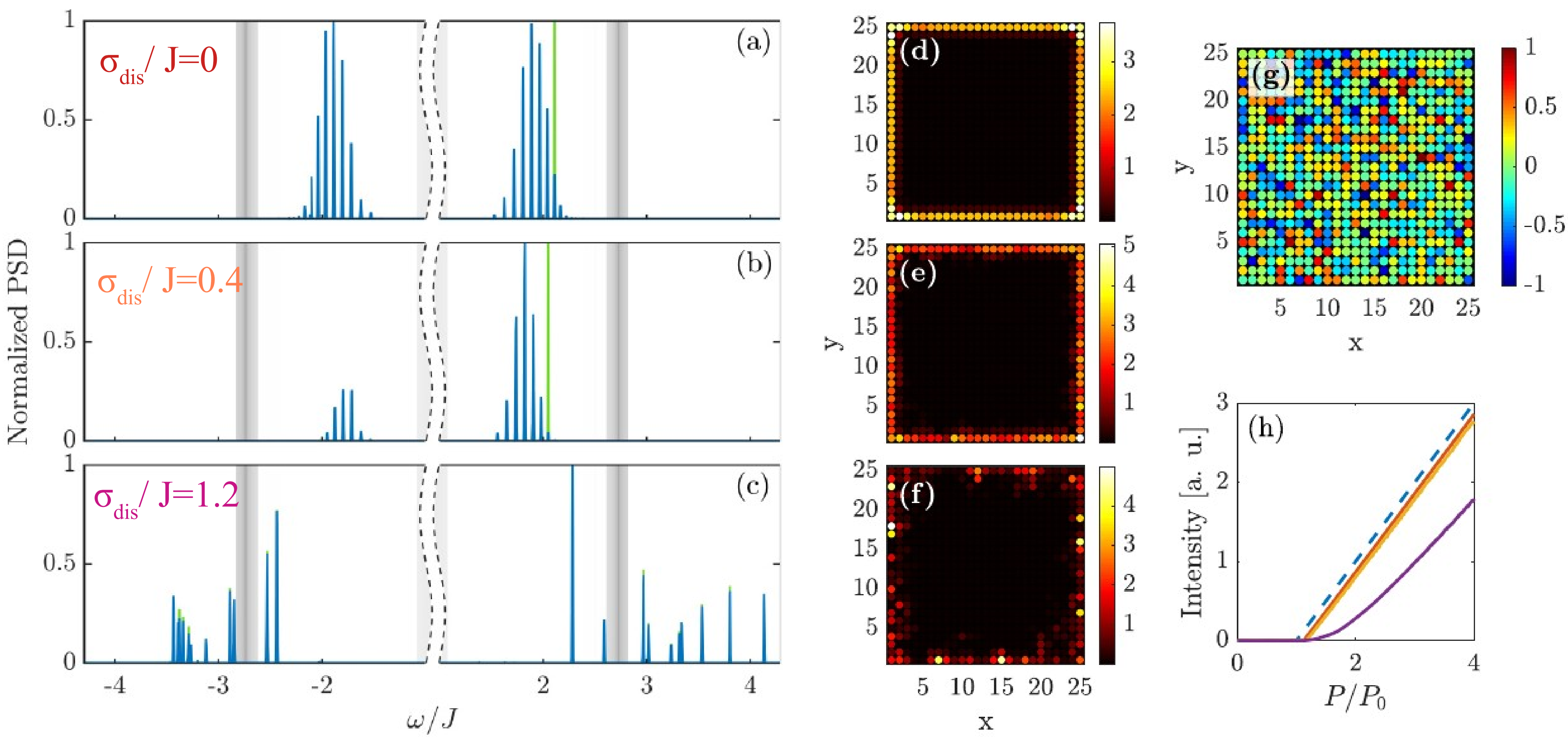
Driven-dissipative Harper-Hofstadter-Bose-Hubbard model:

$$i\partial_t \psi_{x,y} = -J[\psi_{x,y+1} + \psi_{x,y-1} + e^{-2\pi i \alpha y}\psi_{x-1,y} + e^{2\pi i \alpha y}\psi_{x+1,y}]$$

$$+\frac{i}{2}\left[\frac{P\delta_{y,1}}{1+n_{x,y}/n_S} - \gamma\right]\psi_{x,y} + \sqrt{2D_{x,y}}\xi_{x,y}$$

Usual driving and dissipation

Topological lasing in 2D models: immune to disorder



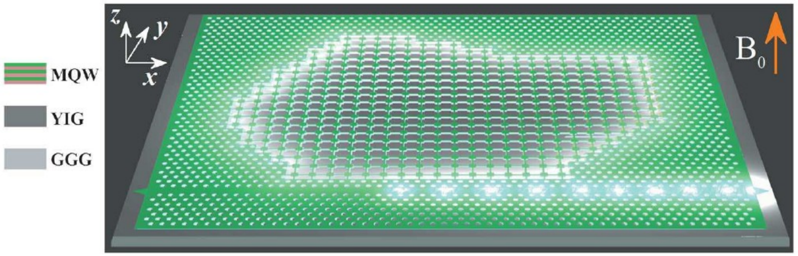
- Chiral propagation immune to disorder
- Efficient single mode lasing with high slope efficiency
- But lasing mode randomly selected according to periodic boundary conditions – like in any ring laser

Figures from M. Seclì's Msc thesis @ UniTN, 2017 and Seclì *et al.*, Phys. Rev. Research 2019
See also: Harari et al., Science 2018.

Coherence of topolaser emission (I)

Important fundamental & applied questions:

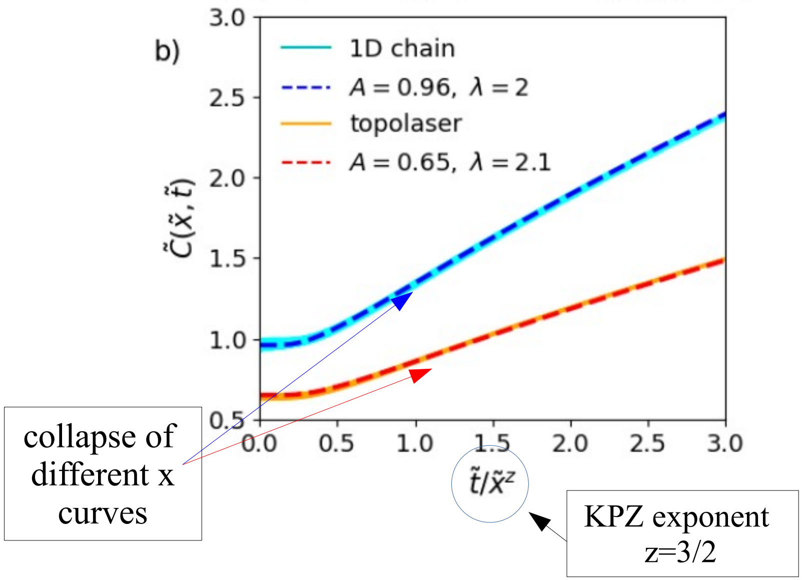
- What are ultimate limitations of coherence?
- How robust is coherence to disorder?
- What advantage over standard lasers?



Laser operation in spatially extended system:

- Linearized theory not enough, crucial role of nonlinearities
- Kardar-Parisi-Zhang model of non-equilibrium stat mech (Canet/Minguzzi, Altman/Diehl, Gladilin/Wouters)
- spatio-temporal scaling properties of phase-coherence

$$\tilde{C}(\tilde{t},\tilde{x}^z) \equiv -2(\phi^*)^{-2}\tilde{x}^{-2\chi}\log g^{(1)}_{CM}(\tilde{x},\tilde{t})$$



Topological laser:

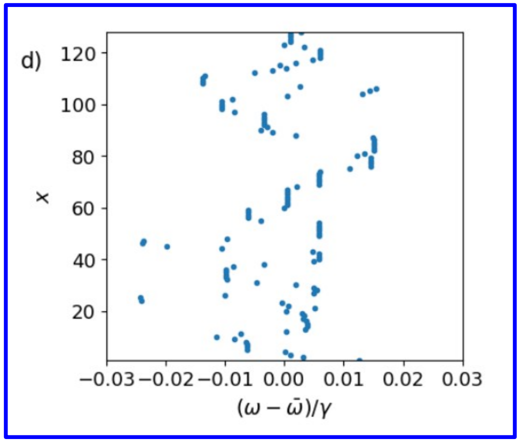
- One-dimensional edge state gives effective 1D dynamics
- KPZ spatio-temporal scaling of $g^{(1)}(x,t)$
- Most promising for experiments → periodic boundary conditions

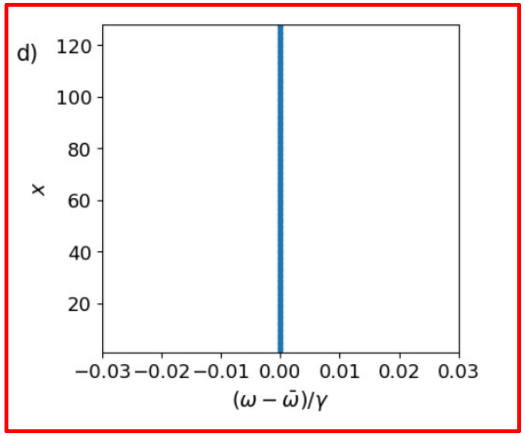
Closely related to polariton quasi-BECs @ C2N

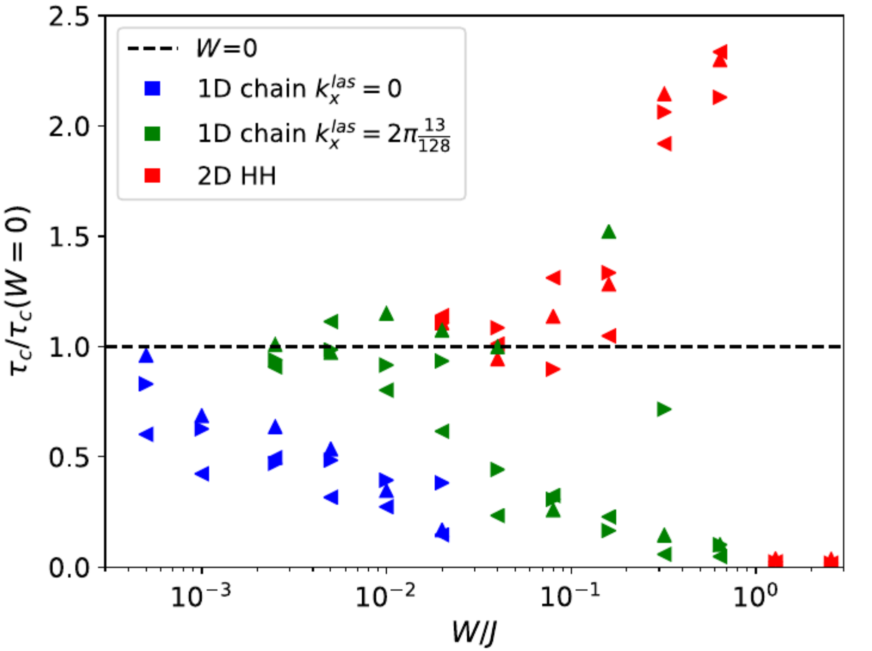
Q. Fontaine et al., Nature 2022

I. Amelio and IC, PRX 10, 041060 (2020)

Coherence of topolaser emission (II)







In the presence of static disorder:

- Non-Topological: weak disorder suppresses temporal coherence (mode fragmentation, multimode emission, localization, etc.)
- Topological: robust spatio-temporal coherence, chiral propagation travels through/around defects without backscattering.

Technologically important in (semiconductor) laser technology:

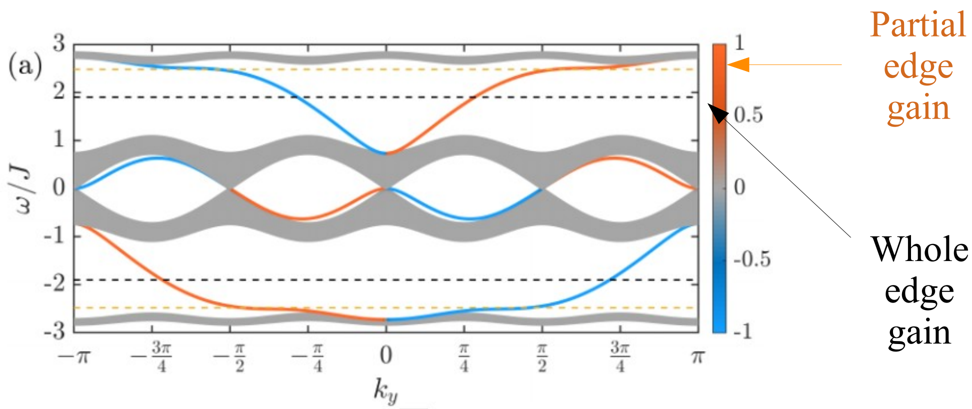
Allows to phase lock many individual lasers → strong intensity and high coherence

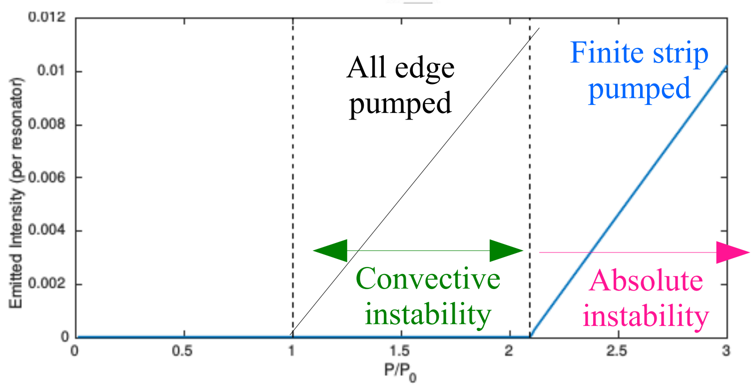
Next steps: extend theory to Class-B lasers. Control instabilities and maintain single-mode emission

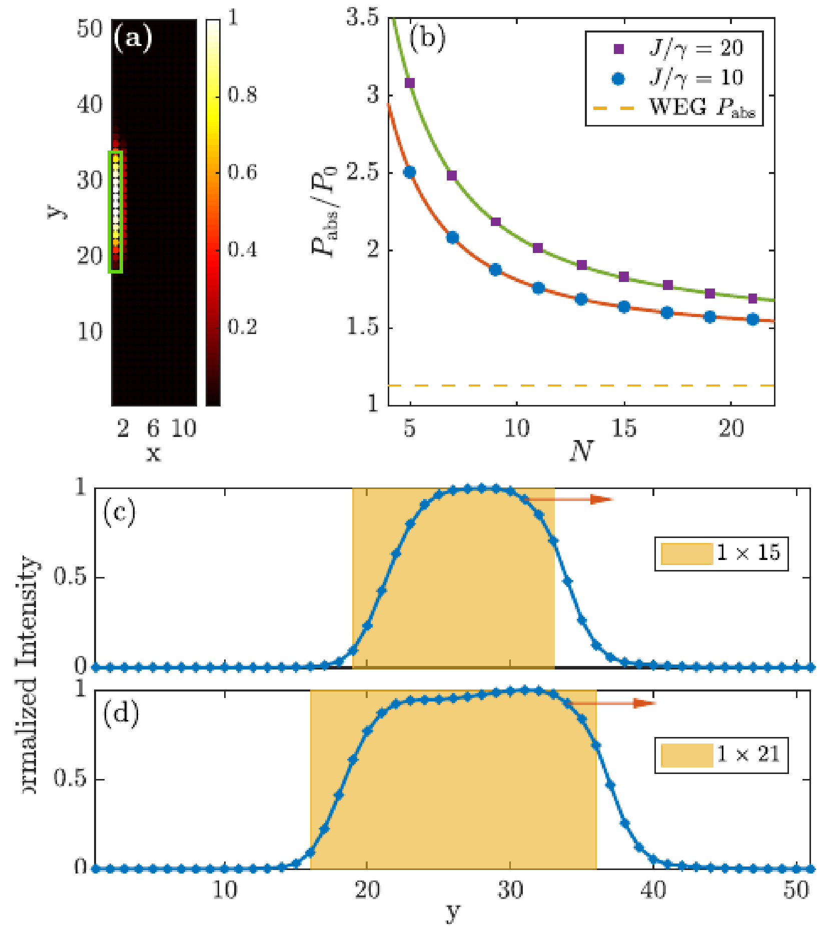
Fundamental question → effect of convective/absolute instability on coherence properties

I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 10, 041060 (2020)

Finite pumped region → additional decay channels







Gain localized on finite strip of sites on edge:

Convective vs. absolute instability of hydrodynamics

→ different threshold of edge-mode lasing

→ different lasing frequency
(competition between v_g and l_{pen})

Topological effects visible in lasing threshold for high number of pumped sites

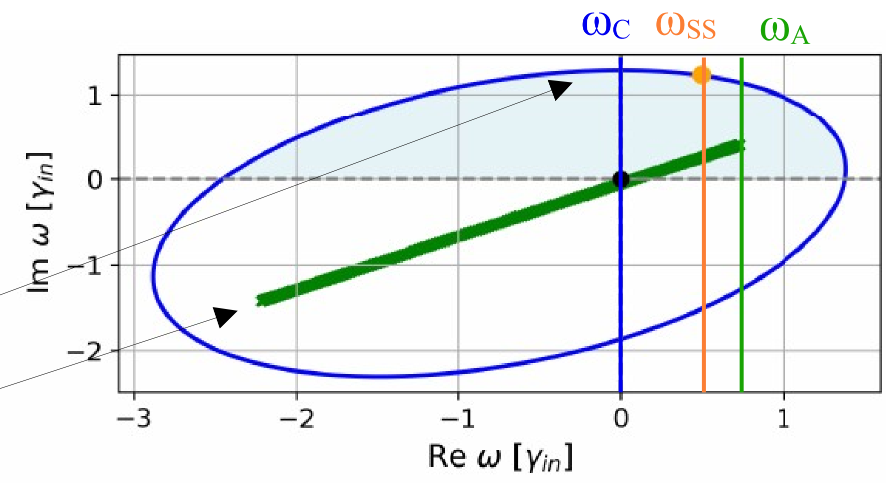
Figures from M. Seclì's Msc thesis @ UniTN, 2017 and Seclì et al., Phys. Rev. Research 1, 033148 (2019)

Lasing in finite Hatano-Nelson chain

Hatano-Nelson chain

$$H = \sum_{j=1}^{N-1} \left[t_R c_{j+1}^\dagger c_j + t_L c_j^\dagger c_{j+1} \right]$$

- t_L and t_R complex numbers, $t_R \neq t_L^*$
- paradigmatic model of non-Hermitian and directional propagation.
  - complex dispersion of infinite lattice
  - complex eigenenergies of finite chain



If only infinite lattice unstable

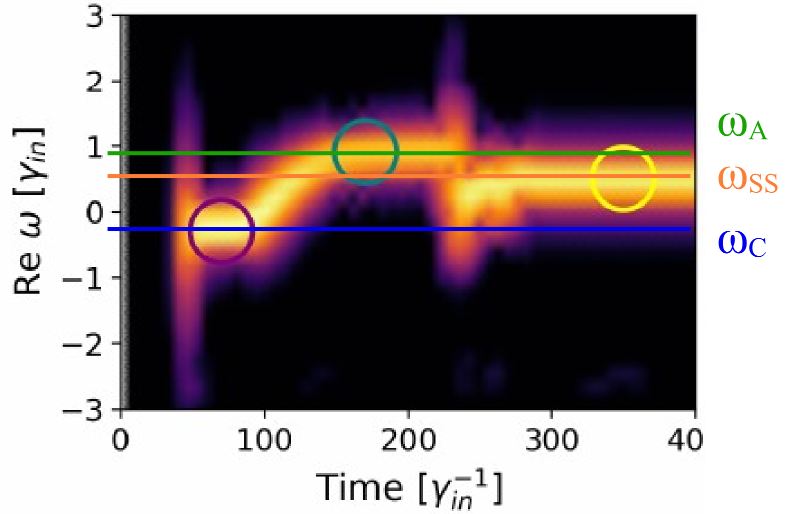
→ convective instability

If also finite chain unstable

→ absolute instability to lasing state

Three-stage switch-on dynamics:

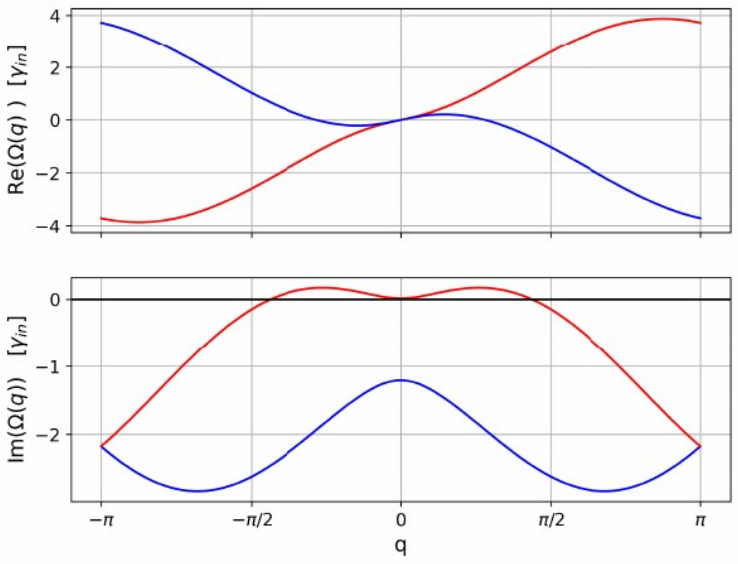
- convective instability mostly amplifies around ω_C
- convected out of lattice, replaced by absolutely unstable ω_A
- steady state ω_{SS} determined by saturated gain/losses



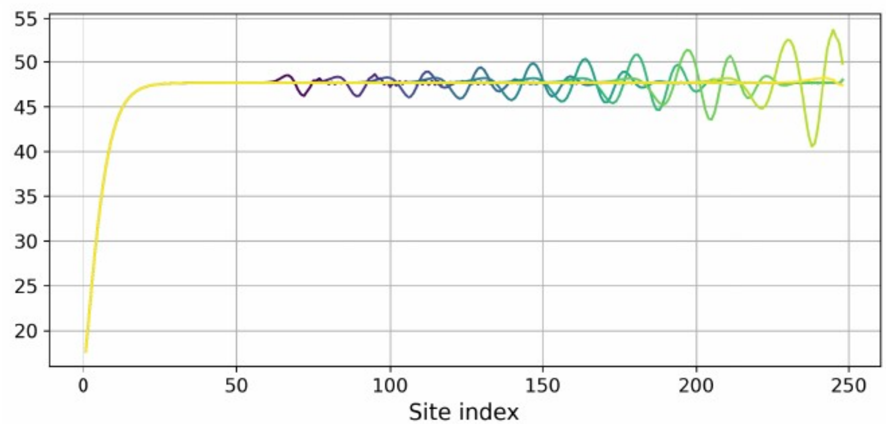
Figures from E. Stazzu's MSc thesis @ UniTN, in preparation (2026)

Noise in finite Hatano-Nelson lasers

- Bogoliubov modes around steady-state
  → no absolute instability
- But may be convectively unstable



- Any perturbation gets amplified
  while it drifts through the chain
- Leads to extra quantum noise
  beyond Schawlow-Townes



Figures from E. Stazzu's MSc thesis @ UniTN, in preparation (2026)

Conclusions & perspectives

General overview of the physics of open quantum systems

- Methods and phenomenology of simple systems nowadays quite well understood
- Many challenges open at the intersection with many-body physics
  - 2^{nd} order phase transitions → lasing transition is preminent example:
    KPZ physics well established in 1d. What about 2d, 3d, 4d ?
    What about supersolids with 2x broken symmetries?
    What collective excitations in different regimes?
  - 1^{st} order phase transitions → dynamics closely related to false vacuum decay.
    New features due to non-equilibrium? Applications to detectors?
    [Selvakumaran et al., arXiv 2026]
  - Driven-dissipative stabilization of strongly correlated states of matter
    also for quantum computing applications

If you wish to know more...

Quantum fluids of light

Iacopo Carusotto*
INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy

Cristiano Ciuti†
Laboratoire Matériaux et Phénomènes Quantiques, Université Paris Diderot-Paris 7 et CNRS,

I. Carusotto, C. Ciuti, Rev. Mod. Phys. **85**, 299 (2013)

REVIEWS OF MODERN PHYSICS, VOLUME 91,

Topological photonics

Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC, RMP **91**, 015006 (2019)

Quantum Fluids of Light
in Encyclopedia of cond-mat physics
arXiv:2211.10980

Non-equilibrium Bose–Einstein condensation in photonic systems

Jacqueline Bloch[1], Iacopo Carusotto[2] and Michiel Wouters[3]

Review article on Nat. Rev. Phys. (2022)

ACADÉMIE DES SCIENCES
INSTITUT DE FRANCE
1666

Comptes Rendus. Physique
2025, Vol. 26, p. 533-569
https://doi.org/10.5802/crphys.258

Intervention in a conference / *Intervention en colloque*

How to exploit driving and dissipation to stabilize and manipulate quantum many-body states

nature physics
FOCUS | REVIEW ARTICLE
https://doi.org/10.1038/s41567-020-0815-y

Check for updates

Photonic materials in circuit quantum electrodynamics

Iacopo Carusotto[1], Andrew A. Houck[2], Alicia J. Kollár[3,4], Pedram Roushan[5], David I. Schuster[6,7] and Jonathan Simon[6,7]

Review article on Nature Physics (2020)

We acknowledge generous financial support from:

PROVINCIA AUTONOMA DI TRENTO

Q@TN

DFG Deutsche Forschungsgemeinschaft

FOR 5688

European Commission

Horizon 2020
European Union funding
for Research & Innovation

QST

Italiadomani

Funded by the European Union
NextGenerationEU