Theoretical studies of thermal vortices in a 2D Bose gas

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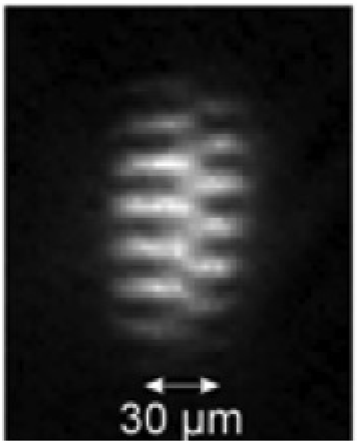
Motivation

Recent observations of thermal vortices in 2D Bose gases at finite T



Interference fringes after expansion of two 2D clouds

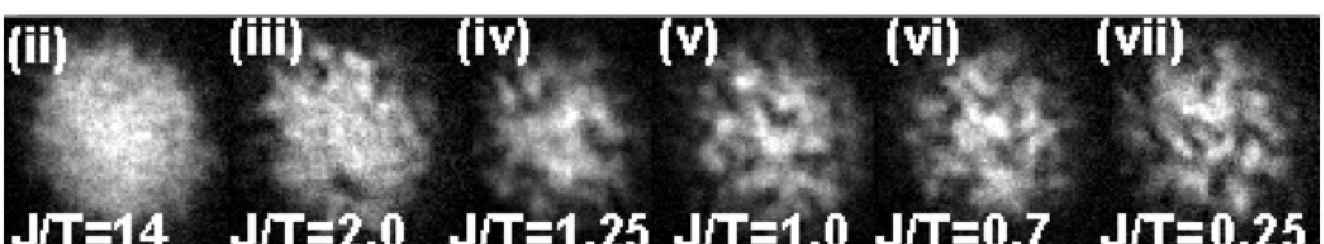
Hadzibabic, Kruger, Cheneau, Battelier, Dalibard, Nature (2006)

Theory questions

- Calculate density and correlation of vortex positions
- Extract from expts some info on the state of the system
- Relation to BKT phase transition ?
- Other interesting observables ?

Our point of view: **semiclassical field theory**

- results quantitatively accurate for $k_B T > \mu$
- based on C-number wavefunctions : can locate vortices
- no need for UV cut-off as in classical theories



Array of Josephson junctions in an optical lattice. Expansion: phase defects → vortices

V. Schweikhard, S. Tung, E. A. Cornell, 2007

The physical system

$$H = \sum_{\boldsymbol{k}} \frac{\hbar^2 k^2}{2\mathrm{m}} a^\dagger_{\boldsymbol{k}} a_{\boldsymbol{k}} + \frac{g_0}{2} \int dr \; \Psi^\dagger(\boldsymbol{r}) \Psi^\dagger(\boldsymbol{r}) \Psi(\boldsymbol{r}) \Psi(\boldsymbol{r})$$

Bose field Hamiltonian on a 2D lattice:

- parabolic free particle dispersion
- interactions: discrete-δ potential of strength $g_0 = \sqrt{8\pi} \frac{\hbar^2}{m} \frac{a_{3D}}{a_z}$
- models atoms in effectively 2D geometry
- homogeneous system, periodic boundary conditions

The semiclassical field method

Density matrix ρ written in terms of Glauber-P distribution: $\rho = \int d\psi \; P[\psi] \, |coh:\psi\rangle\langle coh:\psi|$

Imaginary-time evolution: pseudo-Fokker-Planck eq. for $P[\psi]$, $\tau = 0 \rightarrow \beta = \frac{1}{k_B T}$

$$\frac{\partial}{\partial \tau} P[\psi] = -E[\psi] \, P[\psi] + -\partial_\psi (F[\psi] \, P[\psi]) + + \frac{g_0}{4 \, dV} \partial_\psi^2 (\psi^2 \, P[\psi])$$

Classical field: first term

Semi-classical: first two terms

Exact: all three terms

Classical field → $P[\psi] \sim \exp(-E[\psi] / k_B T)$

Cut-off needed to avoid blackbody catastrophe

Exact — major difficulty →

Non-positive diffusion

$$D = -\frac{g_0}{4 \, dV} \begin{pmatrix} 0 & \psi^2 \\ \psi^{*2} & 0 \end{pmatrix}$$

No mapping on stoch. problem

Other methods necessary

IC, Y. Castin, J. Dalibard, PRA 2000

IC, Y. Castin, J. Phys B 2001

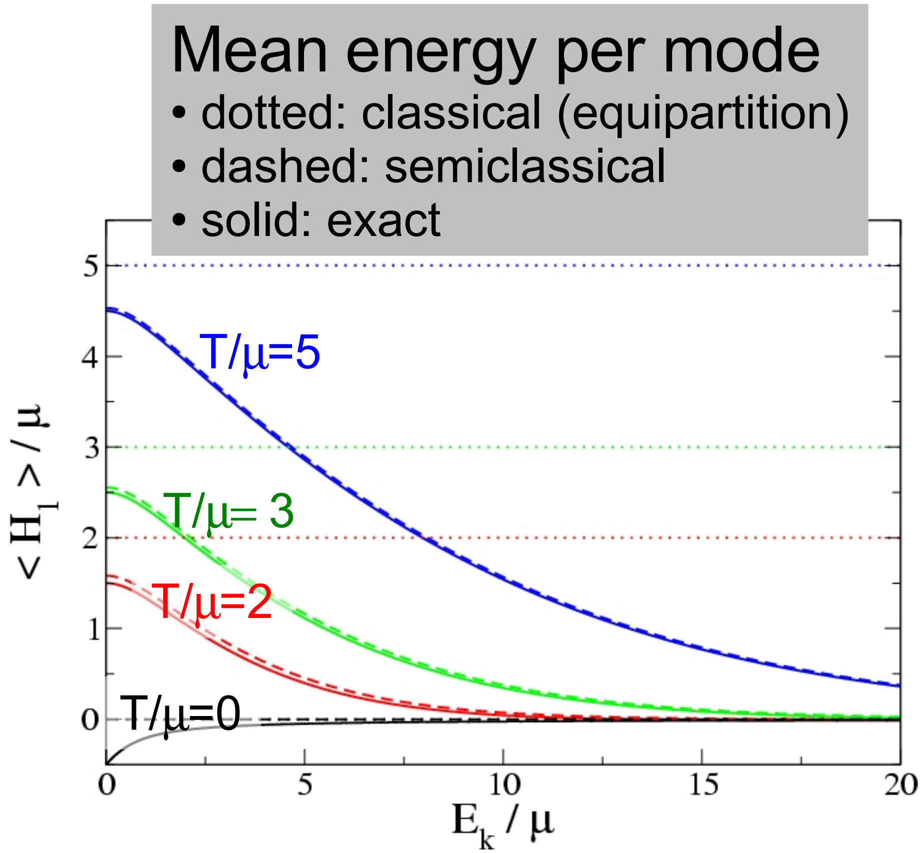
Semi-classical →

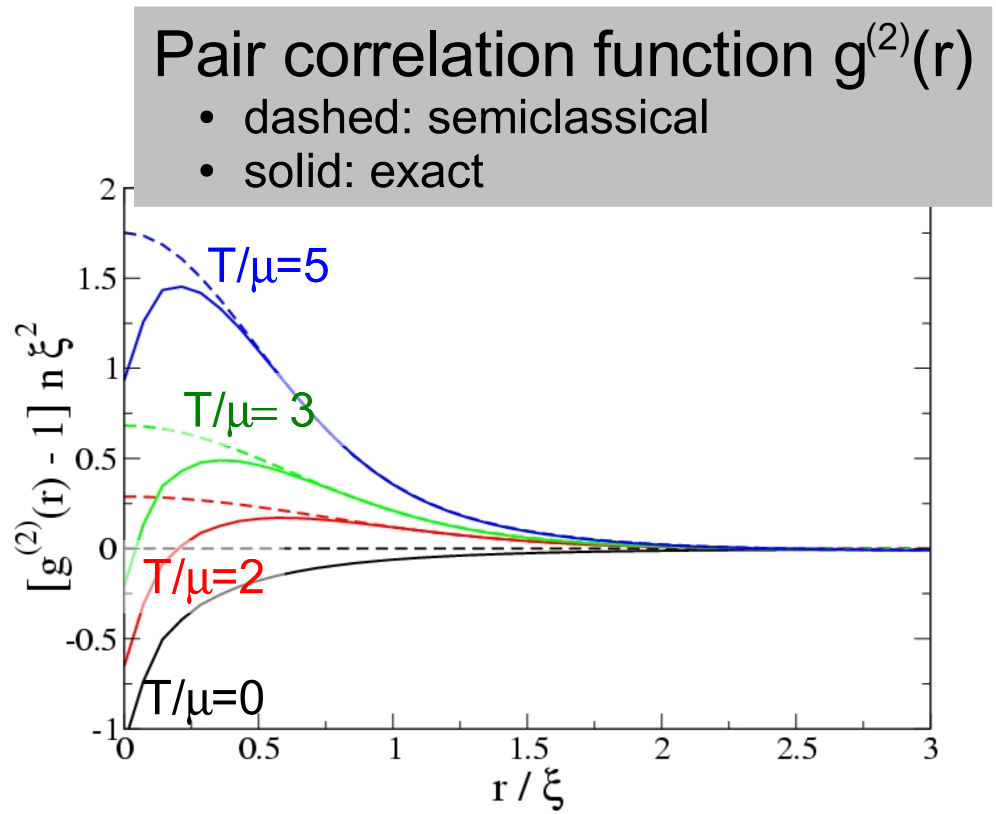
Mapping onto stochastic problem:

- random sampling of initial wavefunction
- weight: classical energy $E[\psi] = \int \psi^* (h_0 - \mu) \psi + \frac{g_0}{2} |\psi|^4$
- drift: imaginary-time GPE $F[\psi] = -\frac{1}{2} (h_0 - \mu + g_0 |\psi|^2) \, \psi$
- pseudo-diffusion negligible if: high temperature $k_B T > \mu$, weak interaction $g_0 \ll 1$
- similar method in Canonical ensemble

Validation of the semiclassical field method

Simplifiied 2D Bogoliubov Hamiltonian $H=\sum_k\left(\frac{\hbar^2 k^2}{2\,m}+\mu\right)a_k^\dagger a_k+\frac{\mu}{2}\left(a_k^\dagger a_{-k}^\dagger+a_k a_{-k}\right)$

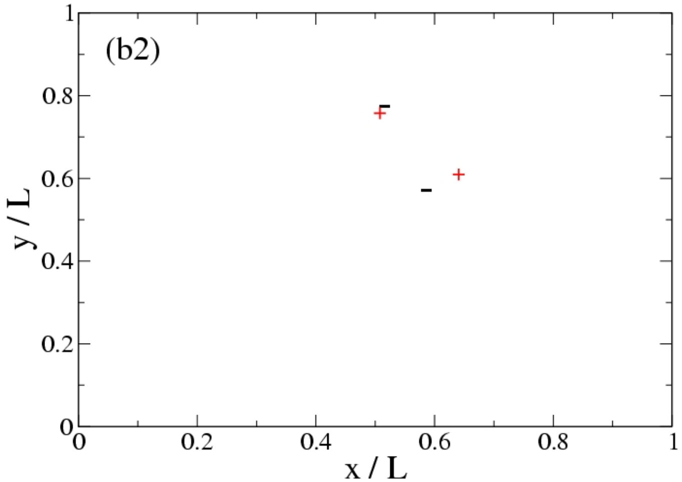


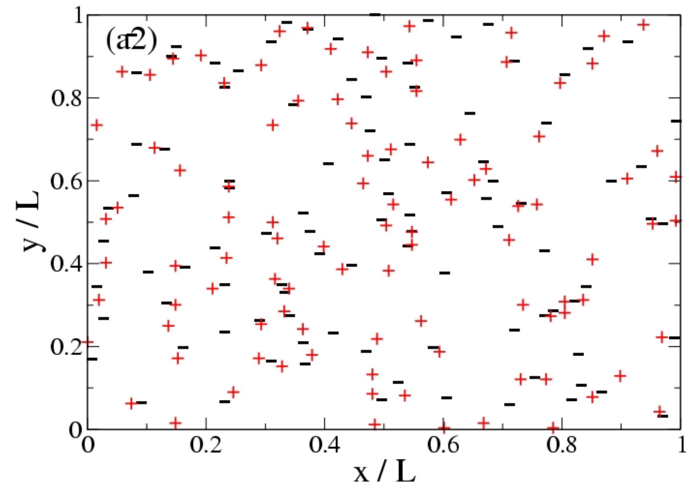


- Semiclassical method quantitatively excellent as long as $T \gg \mu$
- Only fails in short-distance effects due to quantum fluctuations (two-body scatt. function)
- No UV divergences in observables

Semiclassical MC results for the 2D Bose gas

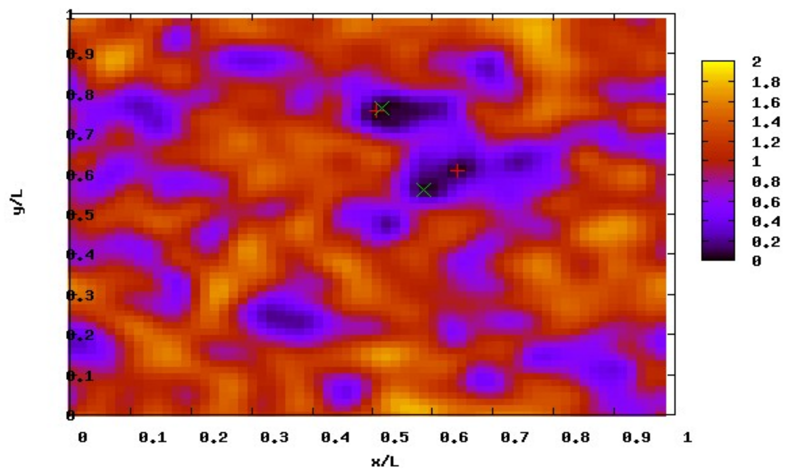
Canonical ensemble, N=1000 atoms

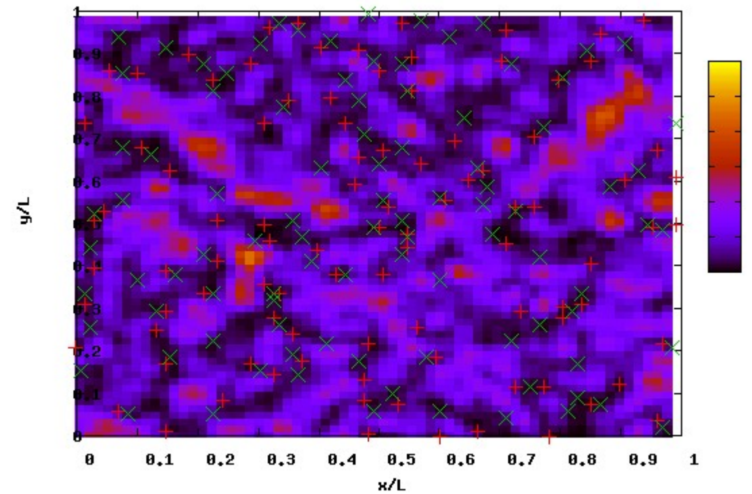




Low T ←

→ **High T**





Simulated snapshots of *in situ* density and vortex locations

How to understand them? What physics do they teach us?

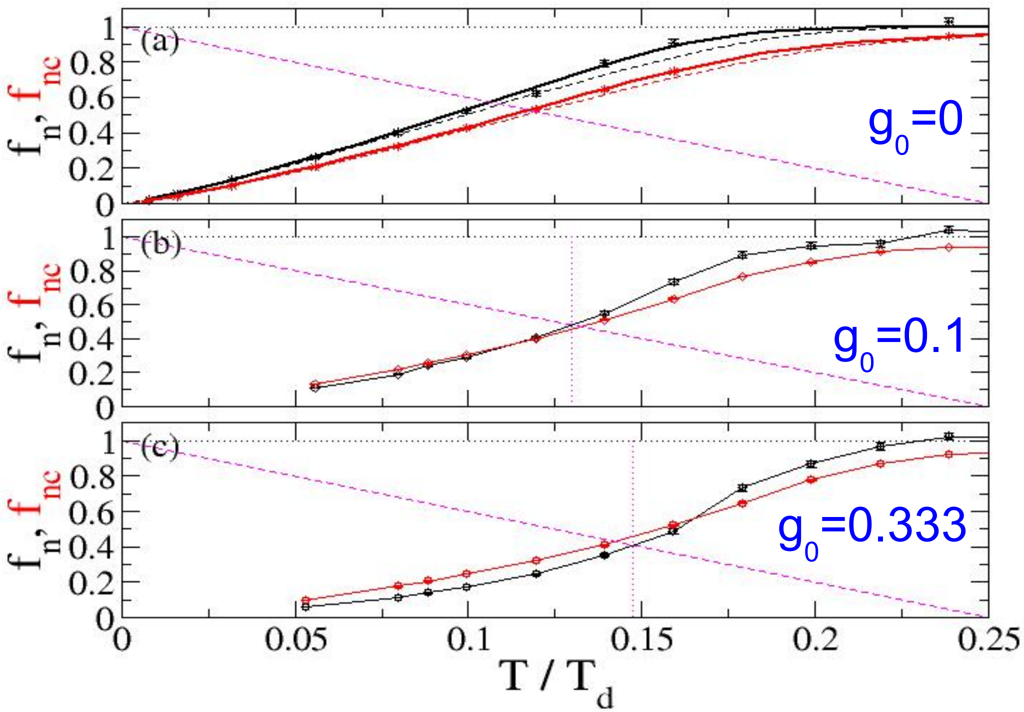
1- Normal and Non-Condensed fractions

Normal fraction:

$$f_n = \frac{\langle P_x^2 \rangle}{N\, m\, k_B T}$$

Non-condensed fraction:

$$f_{nc} = 1 - N_0 / N$$



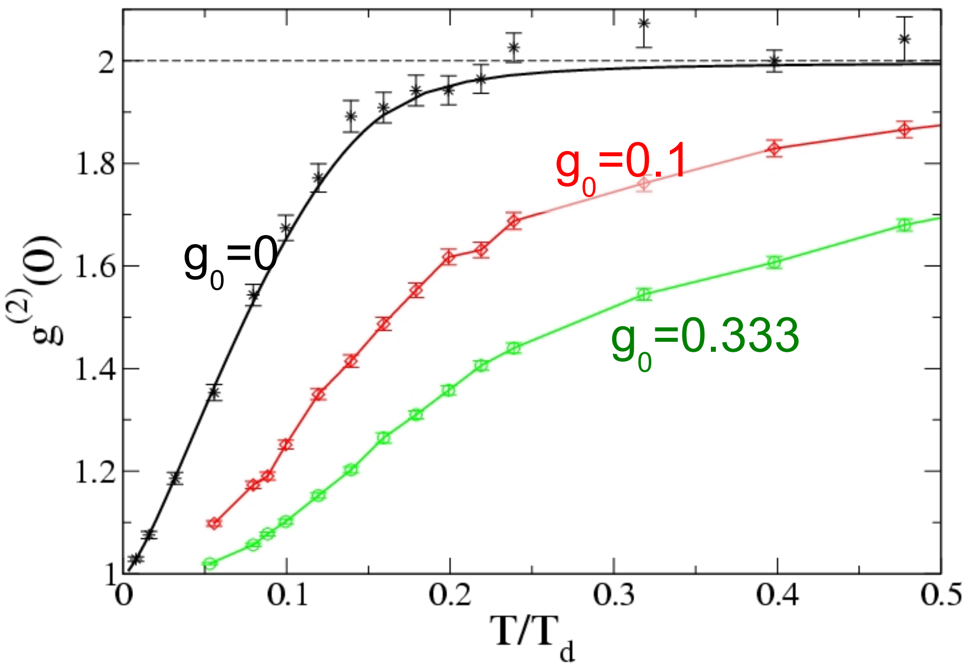
finite size

- Bose-Einstein condensation
- universal jump in superfluid fraction smeared out

- T_{BKT} (roughly) estimated from jump condition $\quad T_{BKT} = \frac{\pi n}{2m} f_s = \frac{1}{4} T_{deg} f_s$
- Reasonable agreement with theory $\quad n_{BKT} = \frac{mT}{2\pi} \log \frac{\xi}{m g_0}, \quad \xi \approx 380$

2- Density fluctuations

$$g^{(2)}(x,x') = \frac{\langle n(x)\, n(x') \rangle}{\langle n(x) \rangle \langle n(x') \rangle}$$



Ideal thermal gas:

- HB-T effect $g^{(2)}(x=x')=2$

Density fluctuations reduced by:

- Bose-Einstein condensation effect (ideal gas g_0=0, canonical ensemble)
- repulsive interactions (g_0>0) (quasi-condensation phenomenon)

density fluctuations generally significant around T_{BKT}:

can play important role in superfluidity breaking !!

3- Current-Current correlations

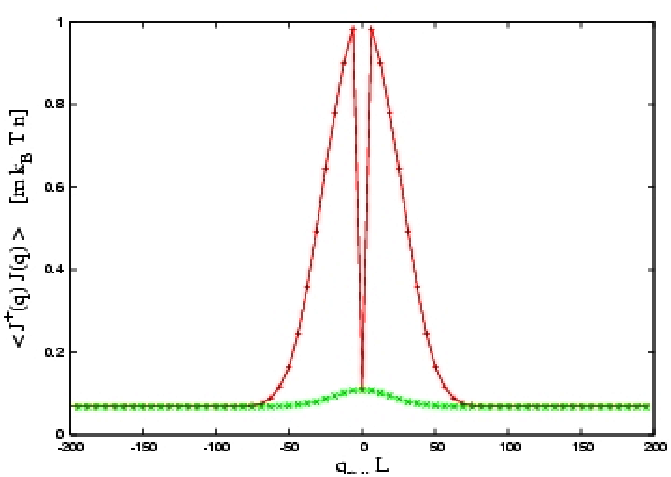
Finite temperature fluctuating, zero-mean mass current

Current-current correlations: $$C_{ij}(x-x')=\langle J_i(x) J_j(x')\rangle$$

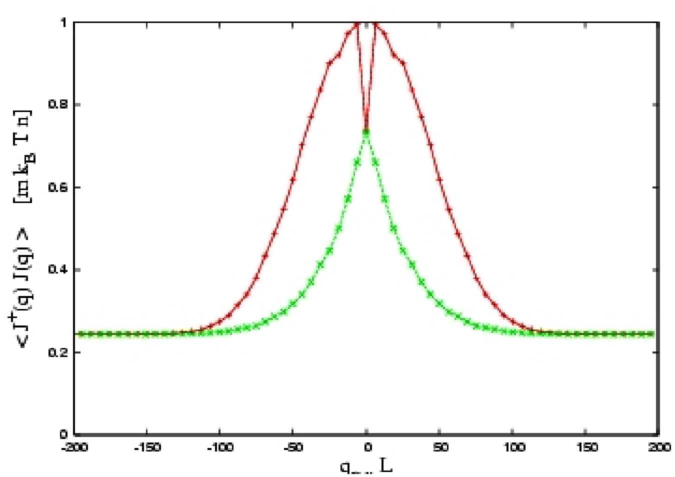
Fourier space

- longitudinal $C_L(\, q \to 0\,) \to m\, n\, k_B T$ (f-sum rule)
- transverse $C_T(\, q \to 0\,) \to m\, n\, k_B T * f_n$ (resp. to transv. gauge field)

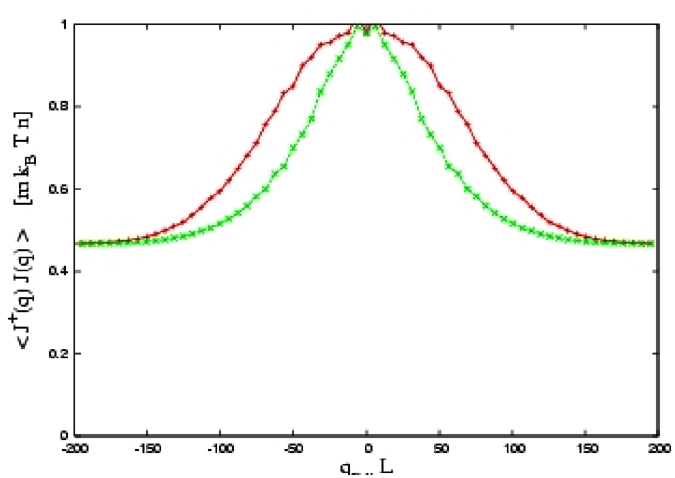
correlations related to response functions via fluctuation-dissipation theorem



$T/T_{deg} = 0.055$



$T/T_{deg} = 0.16$



$T/T_{deg} = 0.33$

4- Density of thermal vortices

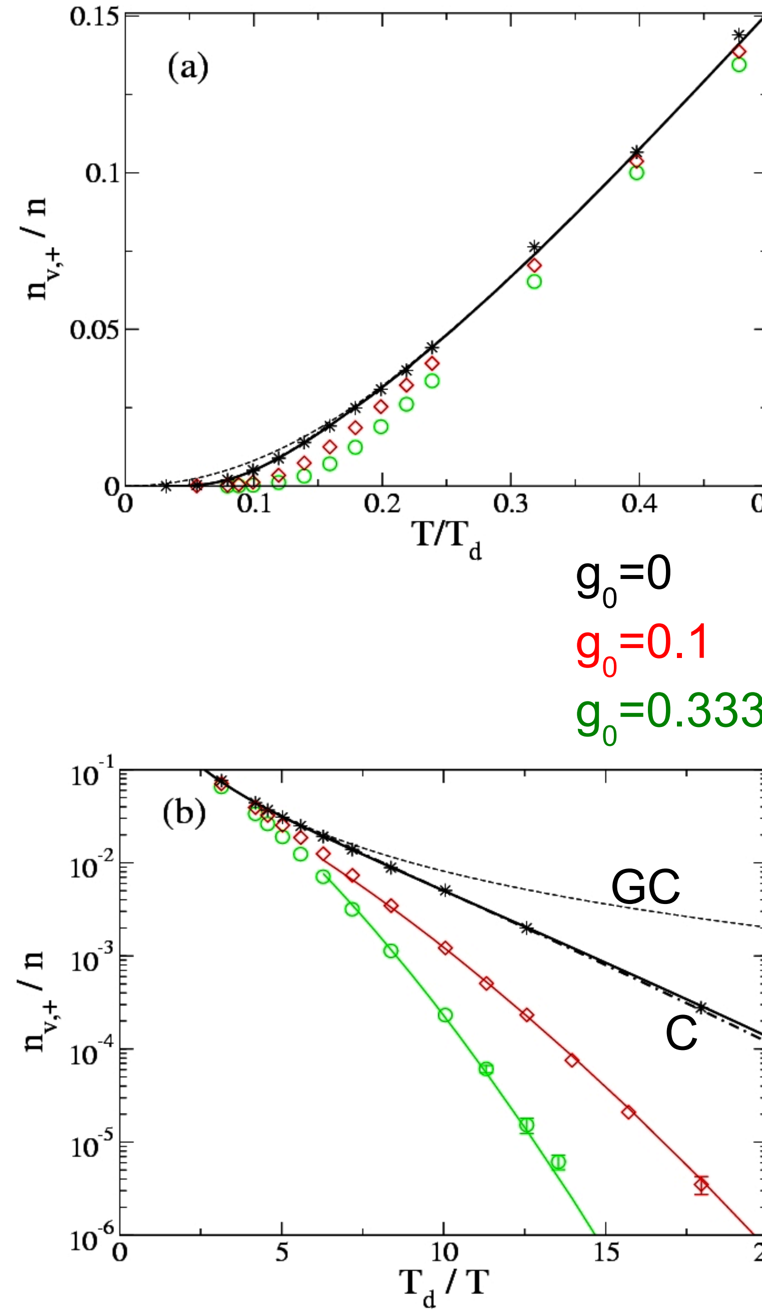
Canonical (C) ensemble:

- ideal and interacting: similar physics
- high T: linear increase $n_v \sim T/T_{deg}$
- low T: activation law $n_v \sim \exp(-\Delta / k_B T)$

Ideal gas, Grand-Canonical (GC):

- low T : quadratic $n_v \sim (T/T_{deg})^2$
- high T: linear increase $n_v \sim T/T_{deg}$

(old result: Berry, Halperin)



Vortex density: physical discussion (I)

Vortex density: probability of having a node $\psi(r)=0$ in classical field

Ideal gas, Grand-Canonical (GC):

- strong density fluctuations $g^{(2)}(r=0)=2$ ⟶ $n_{v,+}^{GC} \simeq n \cdot \begin{cases} T/T_{deg} & \text{for } T \gg T_{deg} \\ (T/T_{deg})^2 & \text{for } T \ll T_{deg} \end{cases}$

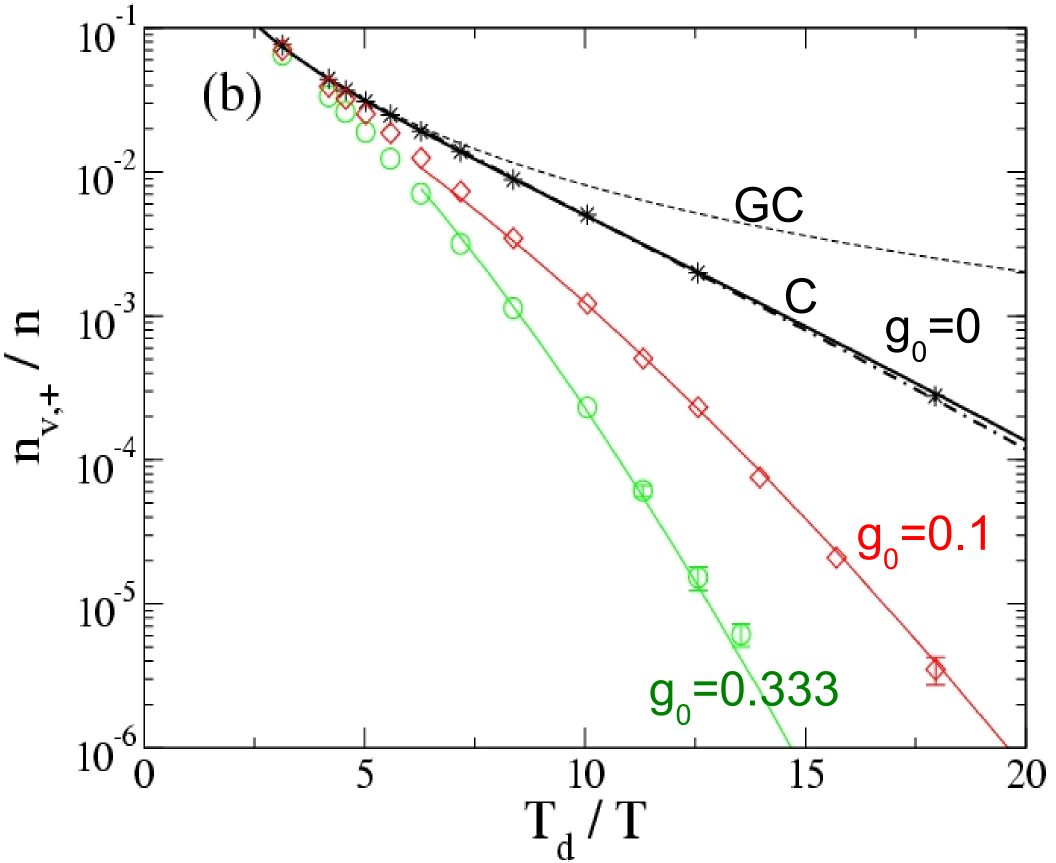
Ideal gas, Canonical (C):

- condensate present
- density fluctuation suppressed by BEC
- activation law:

$$n_{v,+} = n_{v,+}^{GC} \cdot \exp[-N_0 / \delta N]$$

- Condensate depleted in larger system:

GC result valid down to lower T



Vortex density: physical discussion (II)

SC energy functional: $U[\psi]=\sum_{k} |\alpha_k|^2\, k_B T\, (e^{\beta E_k}-1)+\frac{g_0}{2}\int d\mathbf{r}\, |\psi(\mathbf{r})|^4$

- $U[\psi]$ depends on T to include Bose statistics of high energy modes

  Classical GP energy would give $\Delta \to 0$ for UV cut-off $\to \infty$

- Activation law $n_v(T) = C(T)\cdot \exp(-\Delta/k_B T)$

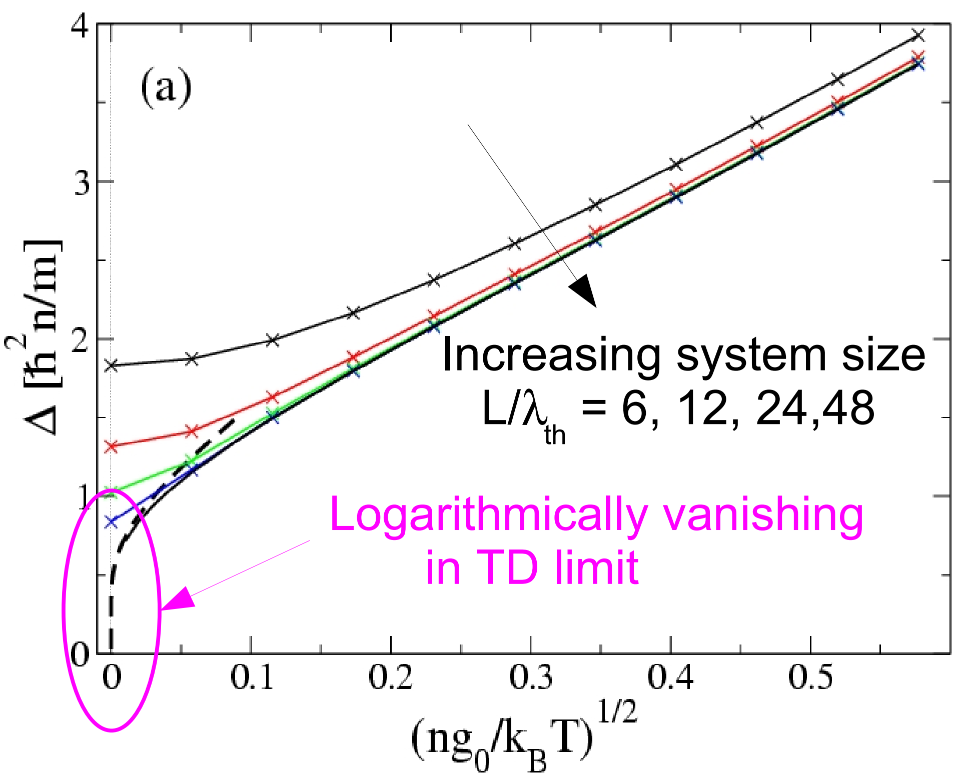
- Activation energy $\Delta(T)=\min_{node} U[\psi]-\min_{no\ node} U[\psi]$

Ideal gas $g_0=0$:

$$\Delta \simeq \frac{\pi\hbar^2 n}{m\ \log(L/\lambda_{th})} \overset{L\to\infty}{\rightarrow} 0$$

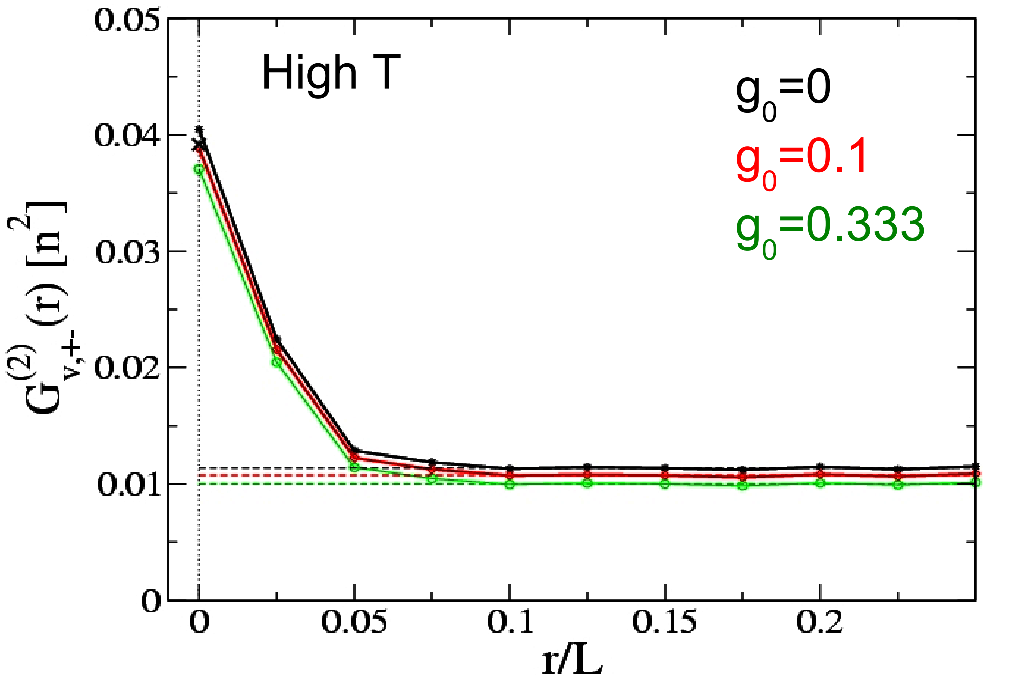
Interacting gas $g_0>0$:

$$\Delta \leq \frac{2\pi\hbar^2 n}{m}\frac{1-2ng_0/k_B T}{\log[k_B T/2ng_0]} \overset{g_0\to 0}{\rightarrow} k_B T_{BKT}$$

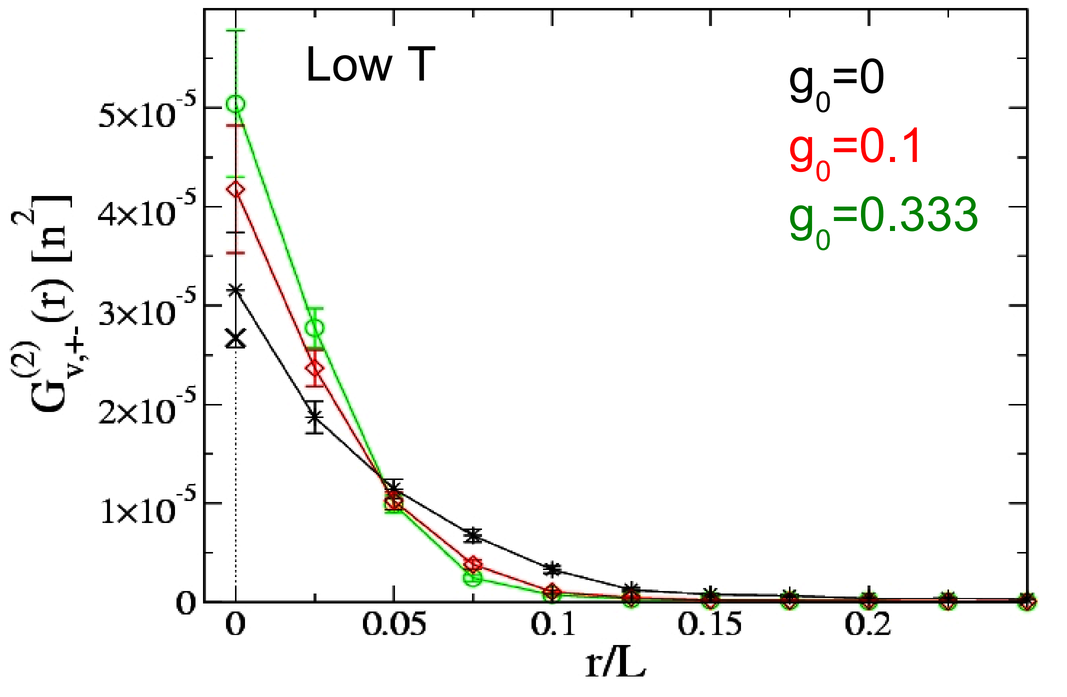


5- Vortex-Vortex correlations

$$G^{(2)}_{v,+-}(r) = \langle n_{v,+}(r)\, n_{v,-}(0) \rangle$$

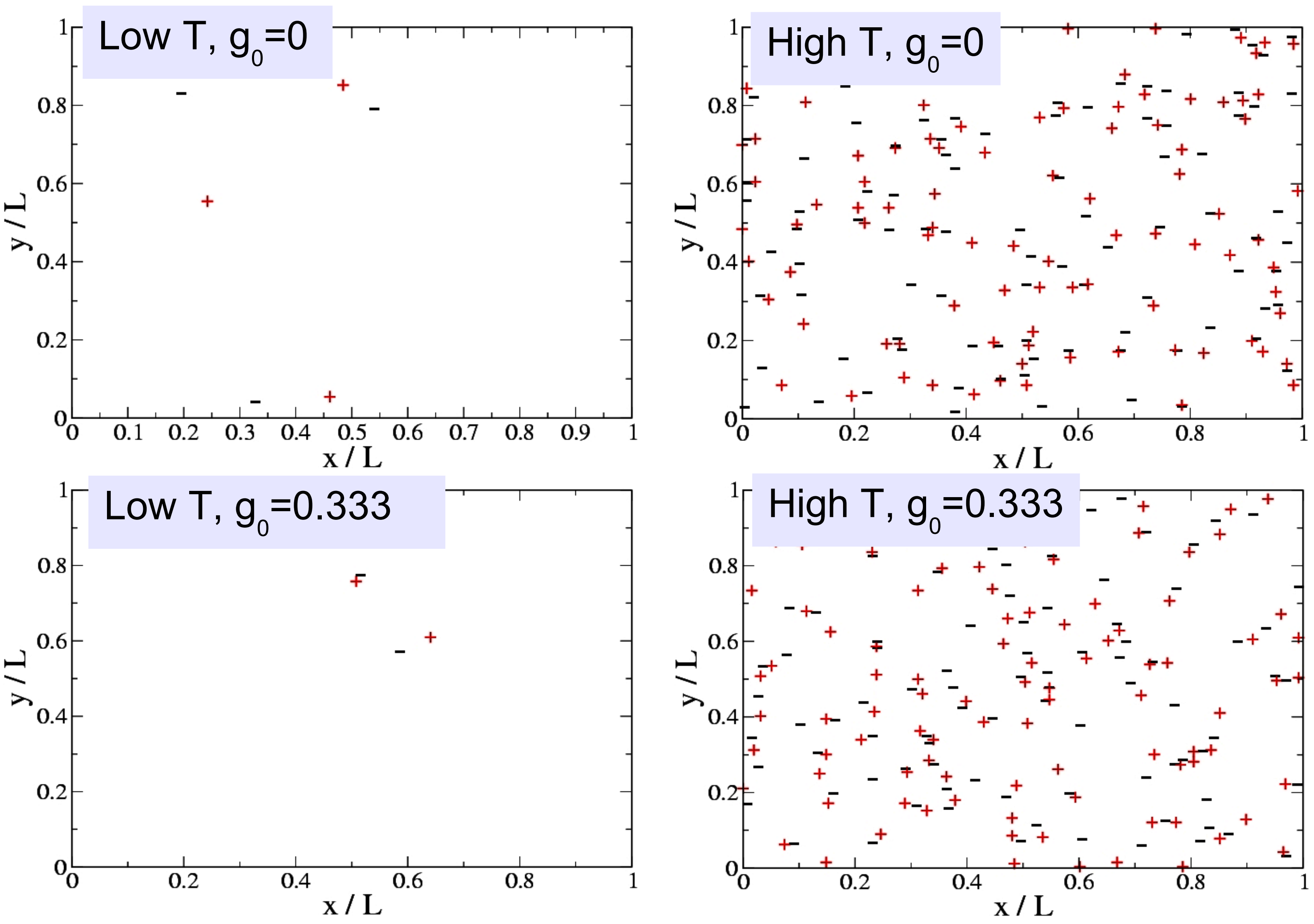


- High T (still degenerate):
  - peak at r = 0: +/- attraction
  - not much dependence on g_0
  - similar to GC.



- Low T (activation regime):
  - longer correlations for $g_0=0$

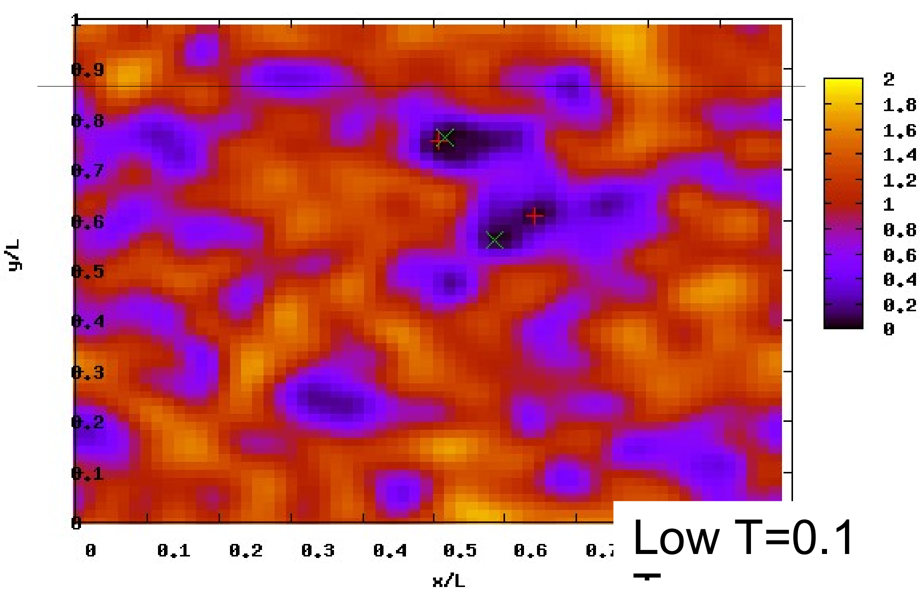
Some snapshots of vortex locations

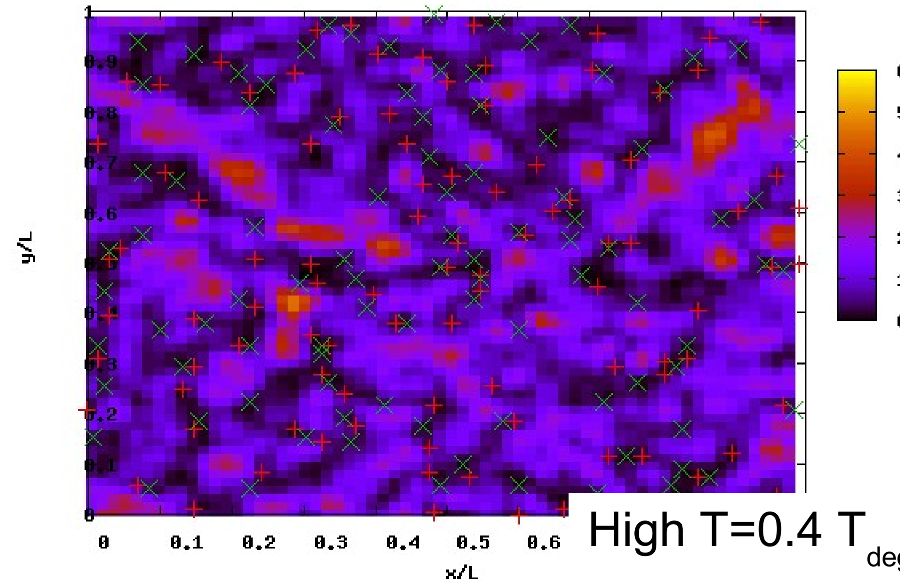


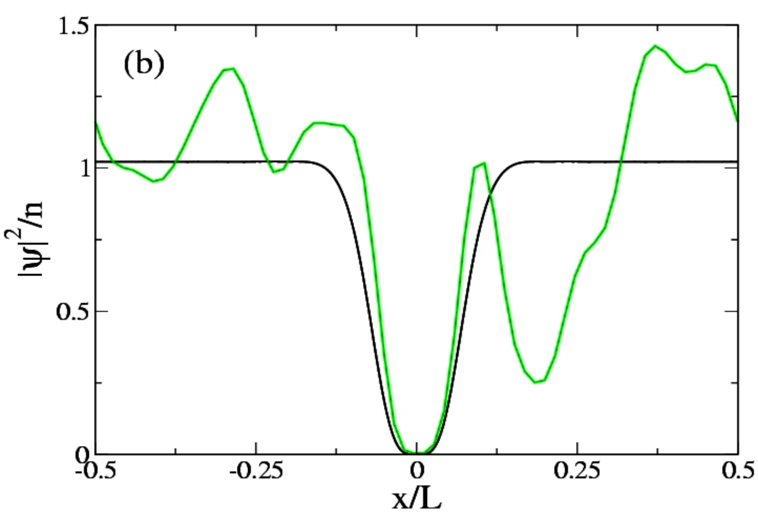
- Selected realizations with thermally activated bound pairs
- Wider pairs in ideal gas
- Vortex proliferation
- Hard to distinguish free vortices from bound pairs and clusters

How to experimentally observe all these features?

Look for minima in density snapshot







Hard to identify vortices:

- smooth transition from density fluctuation to vortex pair.
- both coexist in a 2D gas at finite T
- around T_{BKT}: density fluctuations are already large
- Easier in arrays of Josephson junctions (JILA):

  density fluctuations frozen

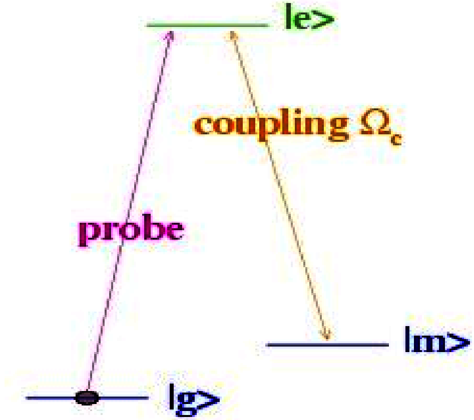
Alternative strategy: looking at mass current

Optical probe

- Strong sensitivity to Doppler shift $\omega \rightarrow \omega - k\,v$, i.e.
  strong dispersion of dielectric constant $\omega \frac{d\epsilon}{d\omega} \simeq \frac{c}{v_g} \gg 1$
- Significant refraction index, but weak absorption

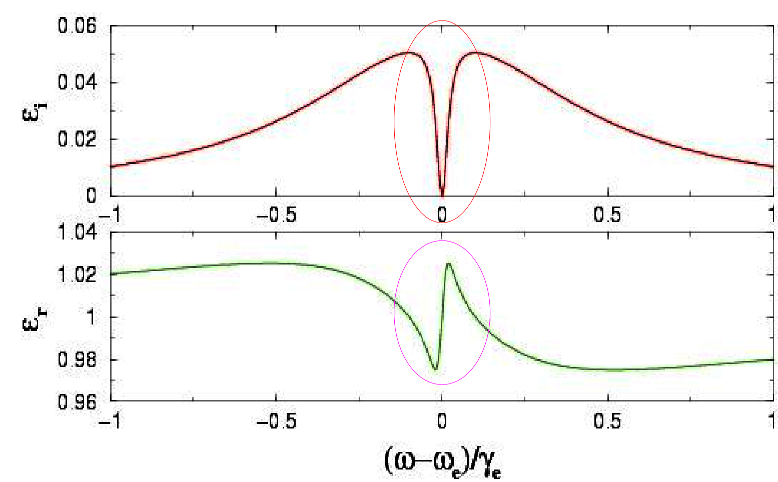
Electromagnetically Induced Transparency effect:

- 3-level atom in Λ configuration
- black line in absorption (Alzetta *et al.*, 1977)
- very small v_g, down to m/s regime (Hau 2000)



Refraction index experienced by probe:

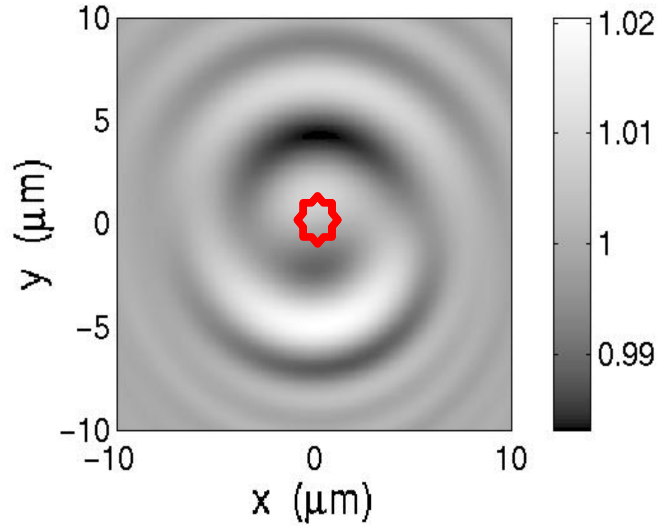
- depends on local current: $\epsilon_p \simeq 1 + \alpha\, \vec{J} \cdot (\vec{k}_c - \vec{k}_p)$
- accessible: phase-contrast imaging, diffraction
- non-destructive, *in situ* measurement



Simulated images for single (bent) vortex in 3D BEC

Top view

k_c along x, k_p along z

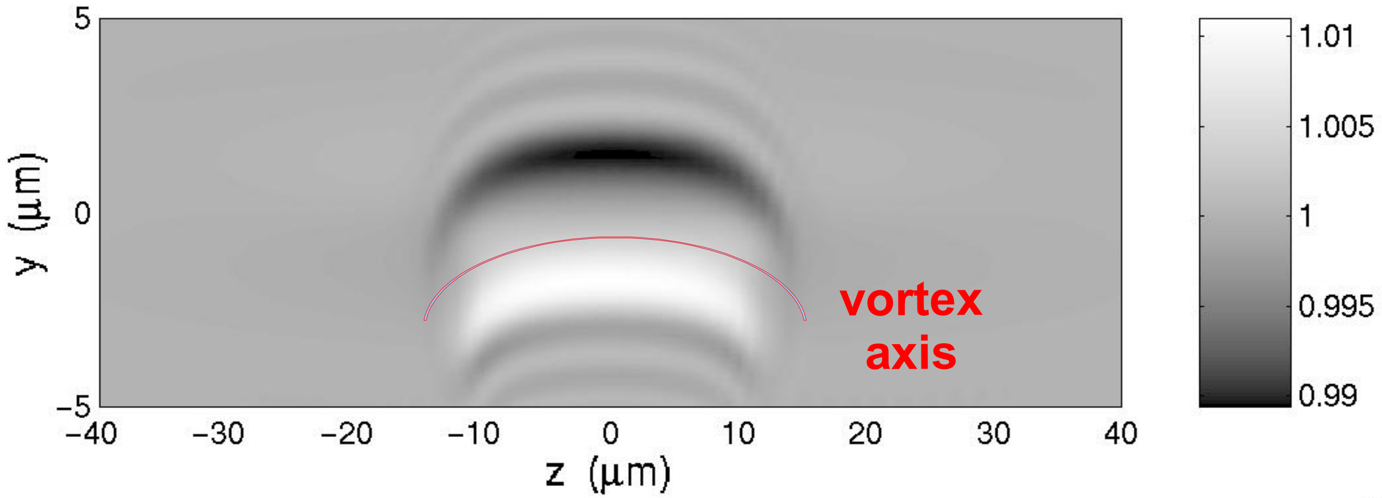


Images for v_g = 1 m/s

Side view

k_c along z, k_p along x

$$\Delta\phi \simeq \pi \frac{\hbar\,|\boldsymbol{k}_p|}{m\,v_g}$$

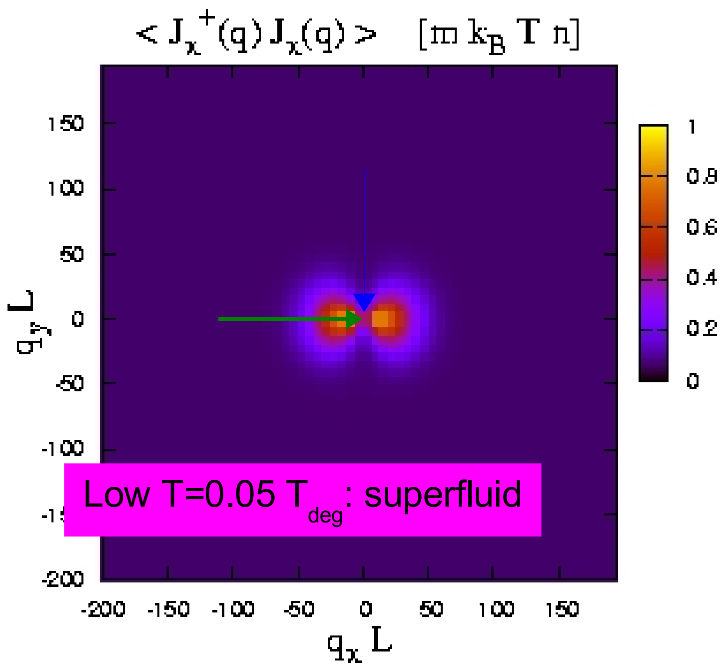


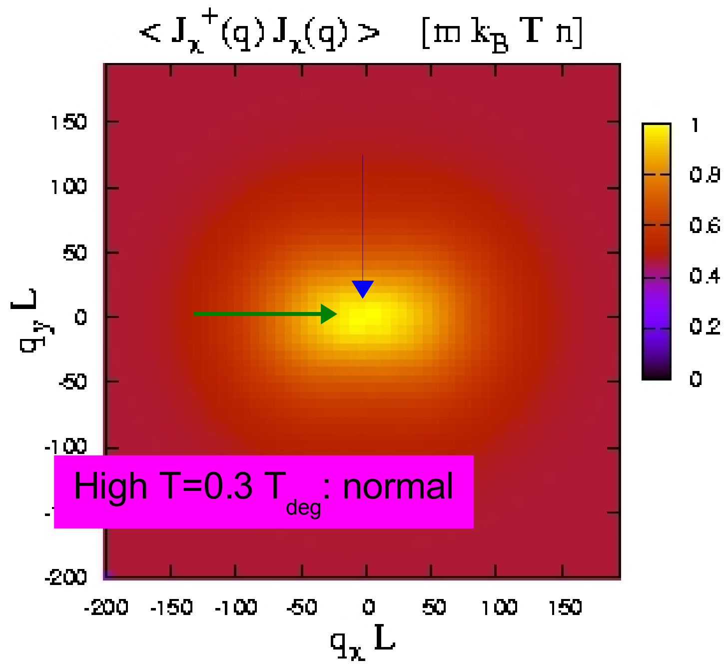
M. Artoni and IC, *In situ velocity imaging of ultracold atoms using slow light*, PRA **67**, 11602R (2003)
similar ideas: Leonhardt and Piwnicki, PRL **84**, 822 (2000)

Diffraction on current fluctuations in 2D gas

Coupling beam k_c along x on 2D plane, probe along z

Look at far-field diffraction pattern





$$I_{sc}(\vec{q}) \propto C_{xx}(\vec{q}) = \langle J_x^\dagger(\vec{q})\, J_x(\vec{q}) \rangle$$

$$C_{xx}(q_x \to 0,\, q_y = 0) = m\, n\, k_B T$$

$$C_{xx}(q_x = 0,\, q_y \to 0) = m\, n\, k_B T\, f_n$$

$$\left| \frac{\Delta I_{sc}}{I_{inc}} \right|_{q \to 0} \approx c^{te}\; T\; T_{deg}\, \frac{\gamma_e^2}{\Omega_C^4}\, [f_n]\, \Delta\theta_x\, \Delta\theta_y$$

Alternative (perhaps easier) experiment: stimulated Bragg scattering on current fluctuations
Method not limited to bosons: vortices and superfluidity in Fermi clouds ??

In conclusion...

Semiclassical-MC study of weakly interacting 2D Bose gas:

- B-E condensation due to finite-size effect, supefluid jump smeared out by finite size
- density of thermal vortices: activation law, correlations of vortex locations
- but density fluctuations significant: smooth crossover in T

  → Seems hard to extract more information on KT from vortices (at least continuous space without lattice)
- current-current correlations: clear info on f_n

  → Hopefully experimentally accessible by slow-light imaging

L. Giorgetti, IC and Y. Castin, *A semi-classical field method for the equilibrium Bose gas and application to thermal vortices in two dimensions,* PRA in the press (2007)

M. Artoni and IC, *In situ velocity imaging of ultracold atoms using slow light*, PRA **67**, 11602R (2003)

and thanks to ...

Luca Giorgetti

Support from: