Non-equilibrium Bose-Einstein condensation phenomena in microcavity polariton systems

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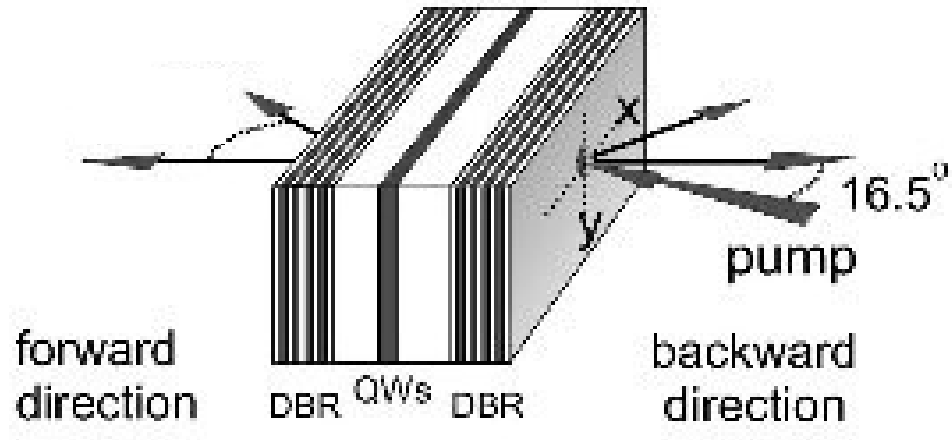
Introduction: What is a polariton?



Cavity photon

- DBR: stack λ/4 layers (e.g. GaAs/AlAs)
- Cavity layer → confined photonic mode
- Cavity mode delocalized along 2D plane

$$\omega_C(\boldsymbol{k}) = \omega_C^0 \sqrt{1 + \boldsymbol{k}^2 / k_z^{\,2}}$$

Quantum well exciton

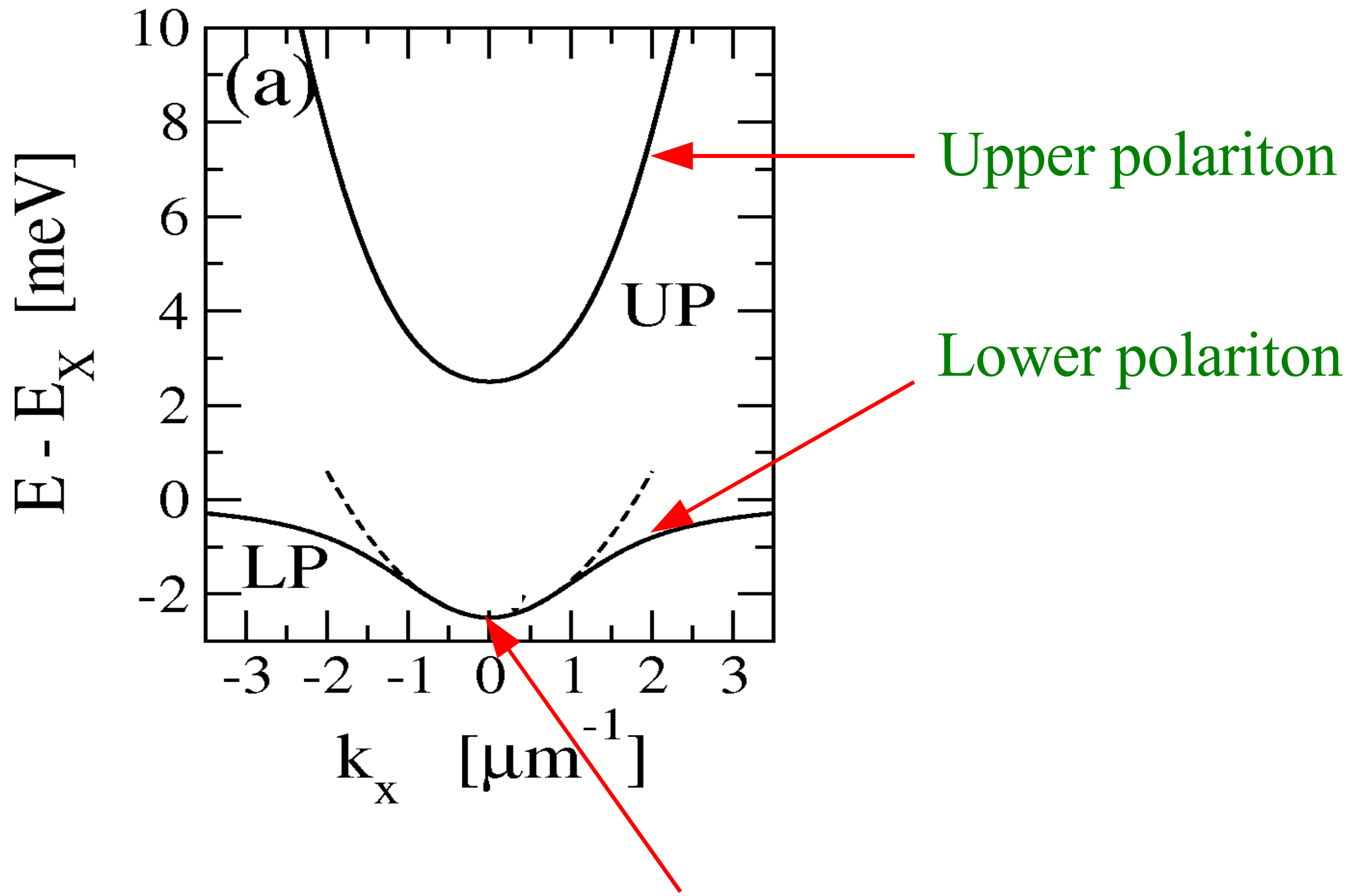
- e and h confined in QW layer (e.g. InGaAs)
- e-h pair: sort of H atom → **exciton**
- Excitons bosons if $n_{exc}\, a^2_{Bohr} \ll 1$
- Flat exciton dispersion $\omega_x(\mathbf{k}) \approx \omega_x$

Radiative coupling between excitonic transition and cavity photon at same in-plane k

Eigenmodes: bosonic superpositions of exciton and photon, called **polaritons**

Why polariton BEC ?

polariton dispersion in *k*-space



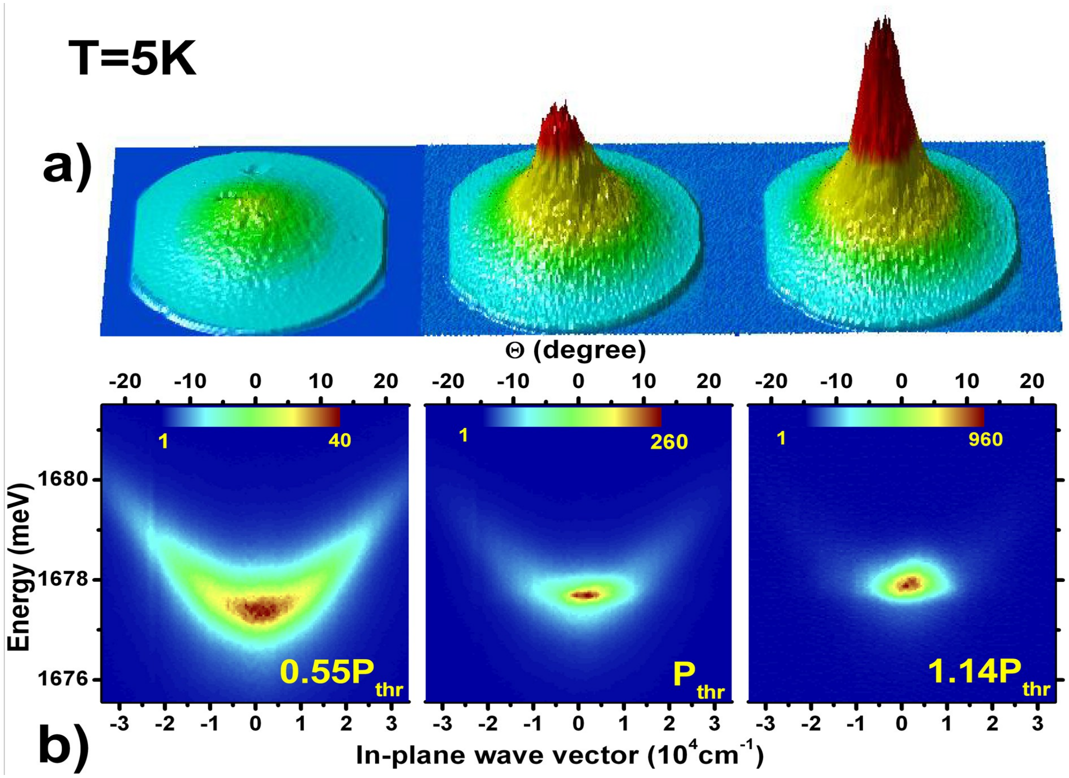
Small polariton mass $m_{pol} \approx 10^{-4}\ m_e$:

→ high $T_{BEC} \approx 30$ K for typical densities

→ for comparison, $m_{Rb} = 1.7 \cdot 10^5\ m_e$, $T_{BEC} \approx 10$ nK

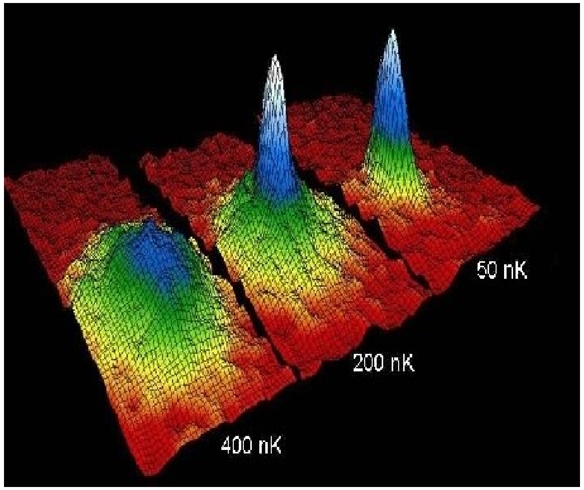
Many experimental signatures of polariton BEC...

1 – Narrowing of the momentum distribution



Bose-Einstein condensate of exciton polaritons

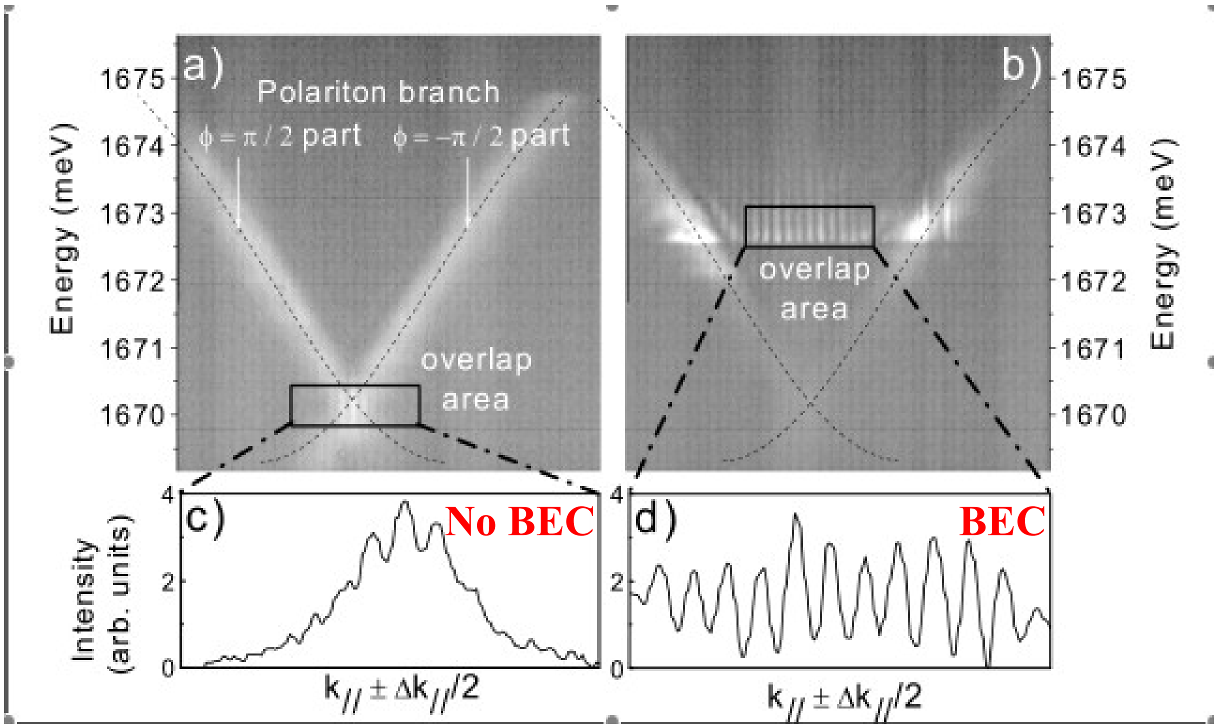
Kasprzak et al., Nature **443**, 409 (2006)



The first atomic BEC

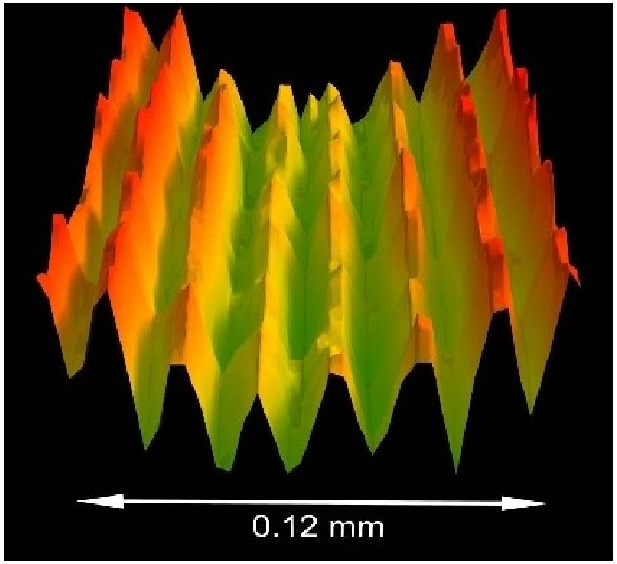
M. H. Anderson et al. Science **269**, 198 (1995)

2 – First order coherence: Young two-slit experiment



Interference pattern of emitted light from a polariton BEC

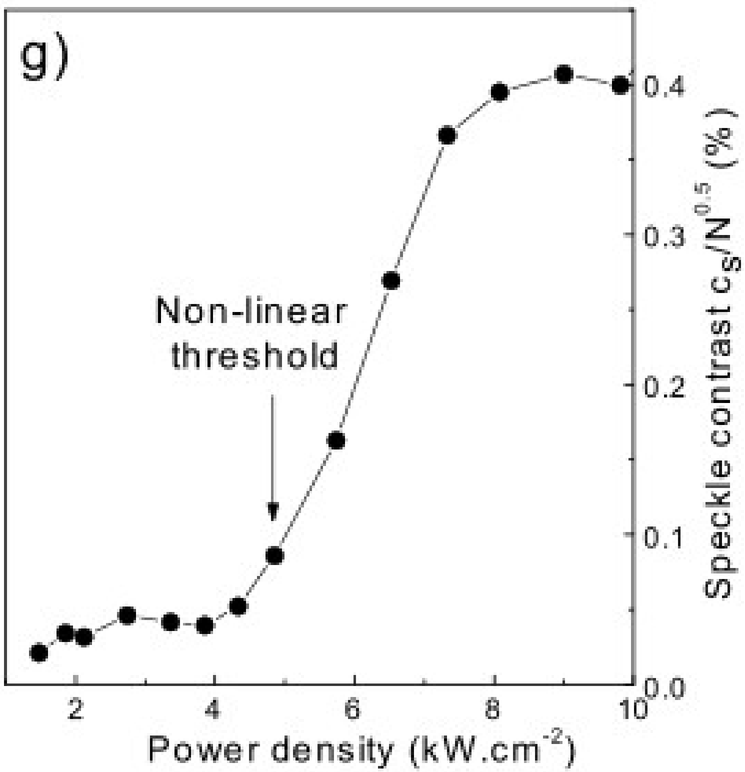
M. Richard et al., PRL **94**, 187401 (2005)



Interference pattern of two expanding atomic BECs

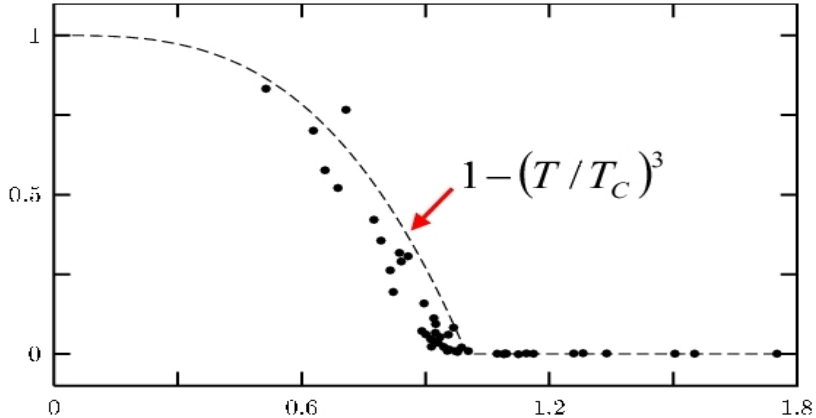
M. R. Andrews, Science **275**, 637 (1995)

3 – Threshold behaviour



Coherence suddenly appears
at threshold

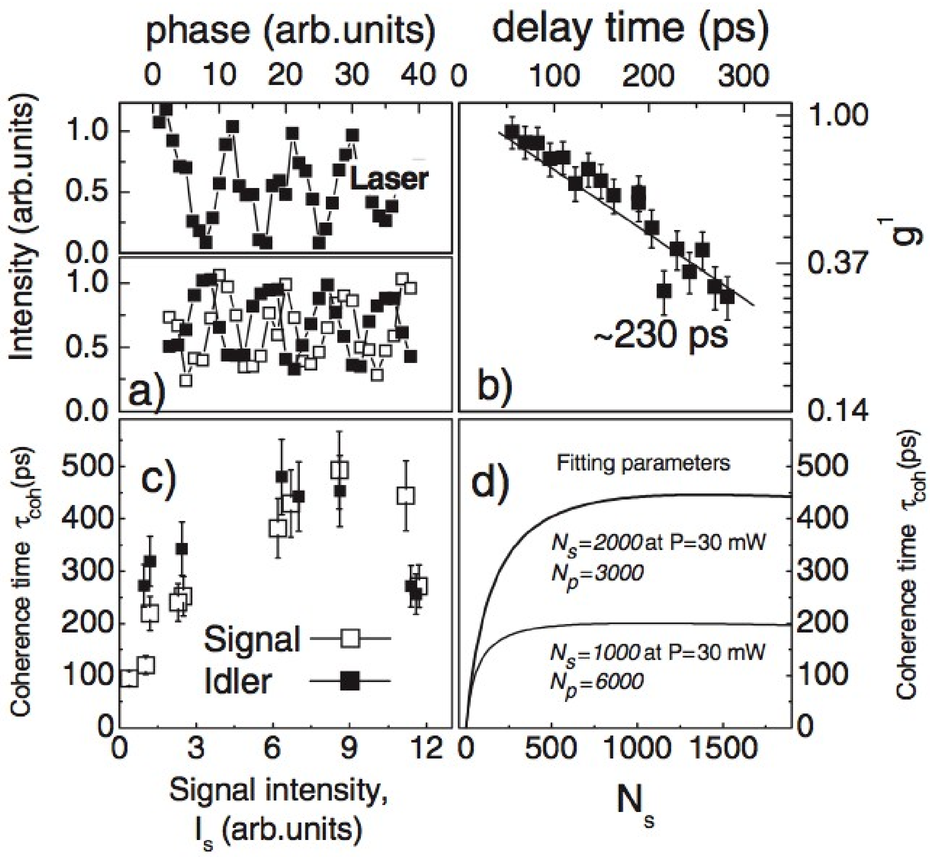
M. Richard et al., PRL **94**, 187401 (2005)



Bose condensed fraction

J. R. Ensher et al. PRL 77, 4984 (1996)

4 – Long coherence time

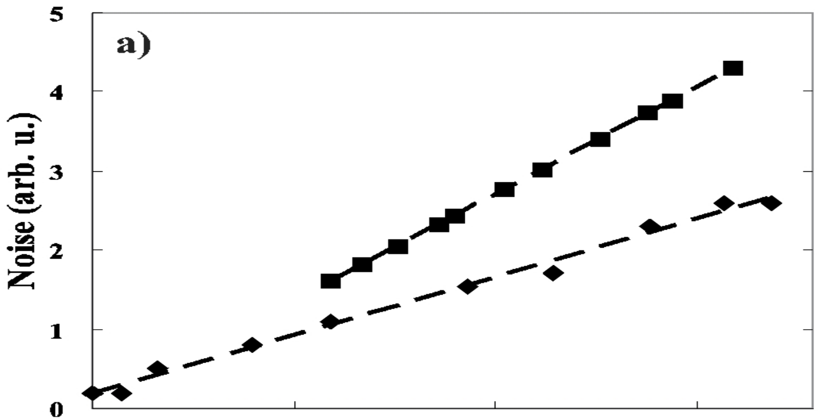


D. N. Krizhanovskii, D. Sanvitto, A. P. Love, M. S. Skolnick, D. M. Whittaker, and J. S. Roberts, PRL **97**, 097402 (2006)

Condensate emission:

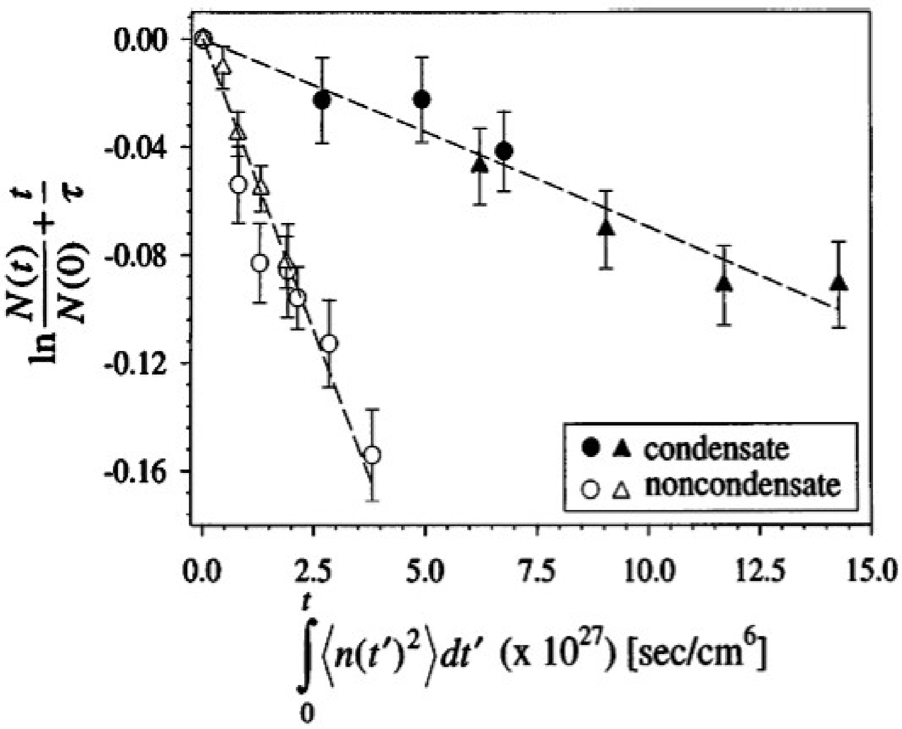
- phase coherent for long times
- what determines decoherence?
- role of interactions?

5 – Noise reduction in the condensed phase



Razor-blade experiment to mesure $g^{(2)}$

A. Baas et al., PRL **96**, 176401 (2006)

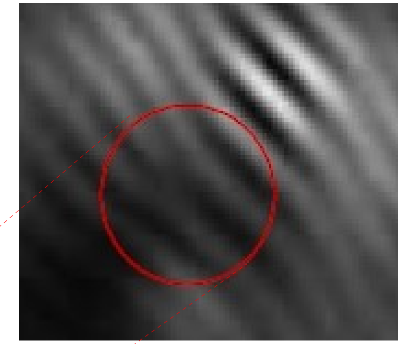


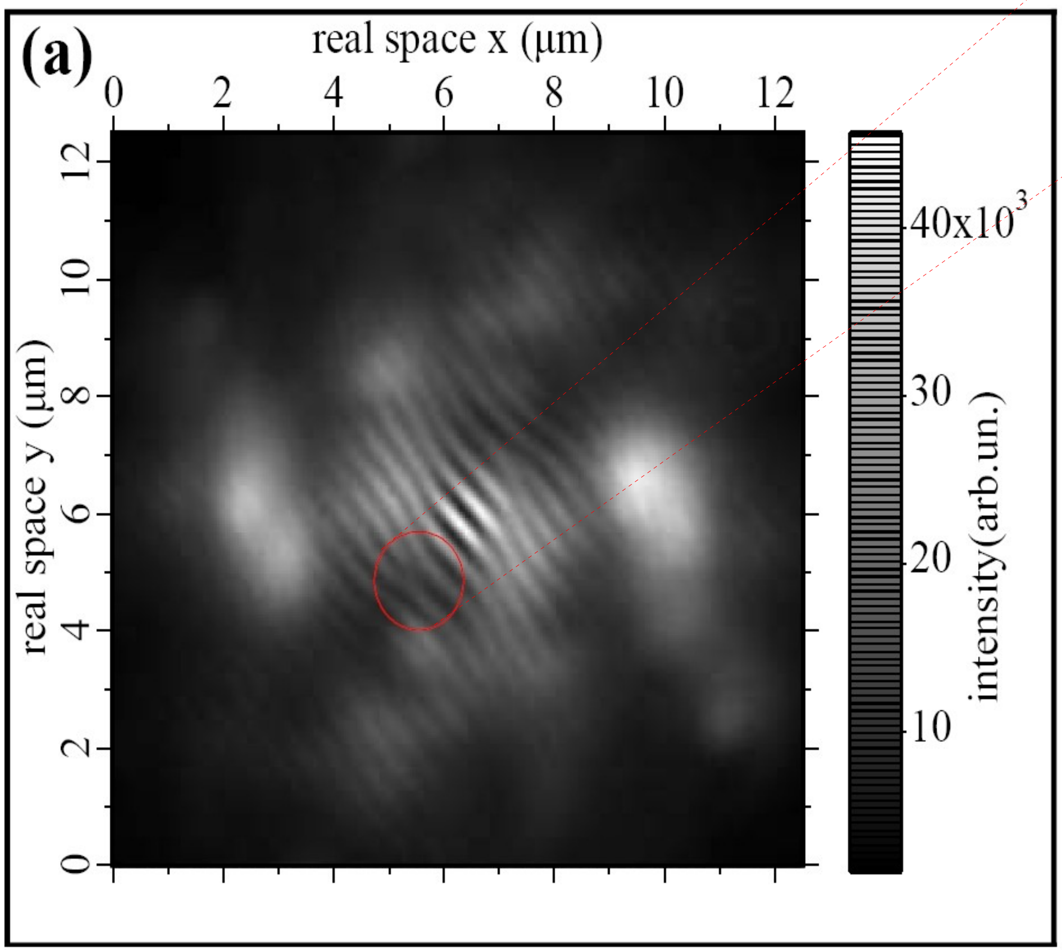
Suppressed density fluctuations by 3!

→ reduced 3-body recombination rate

E. A. Burt et al. PRL **79**. 337 (1997)

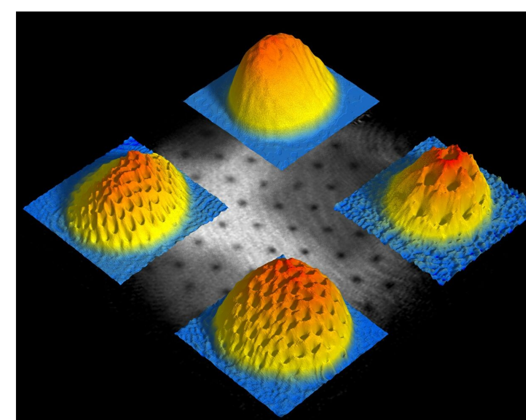
6 – Quantized vortices





K. G. Lagoudakis, M. Wouters, M. Richard, A. Baas, IC, R. André, Le Si Dang, B. Deveaud-Pledran, *Quantised Vortices in an Exciton Polariton Fluid,* preprint arXiv/0801.1916.

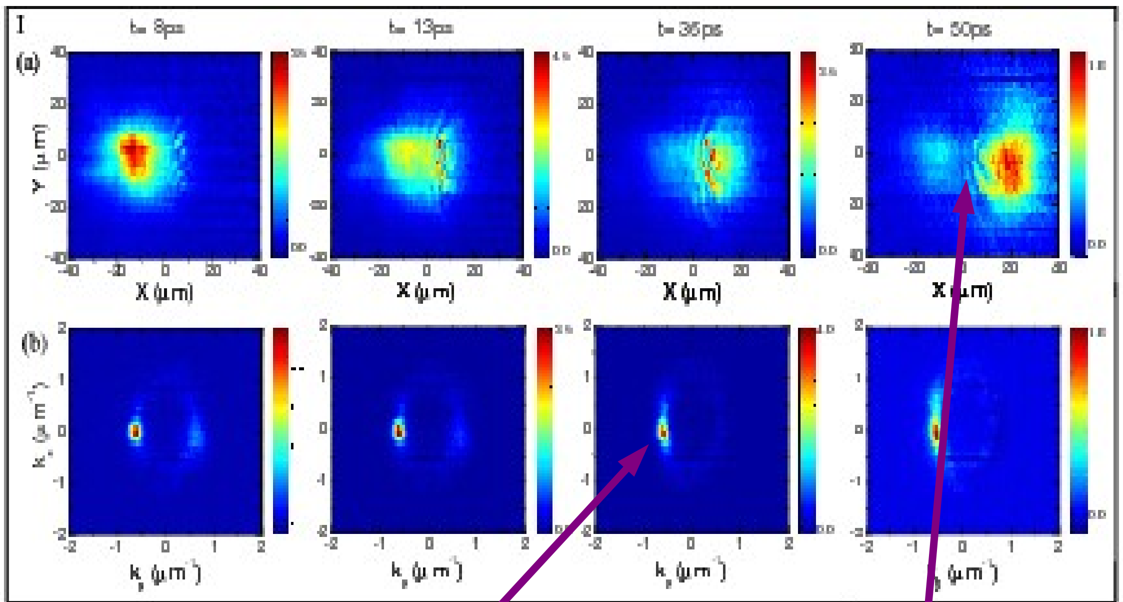
- dislocation in interference pattern
- appear spontaneously without need for stirring
- pinned to defects
- T=0 effect



Vortex lattice density profile in atomic BEC

Abo-Shaeer *et al*. Science **292**, 476 (2001)

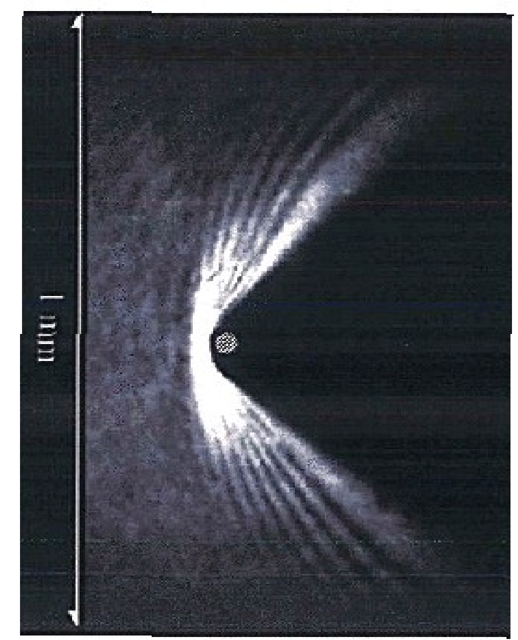
7 – Challenging superfluidity effects



A. Amo *et al.*, arxiv/0711.1539

Superfluid (?) probe

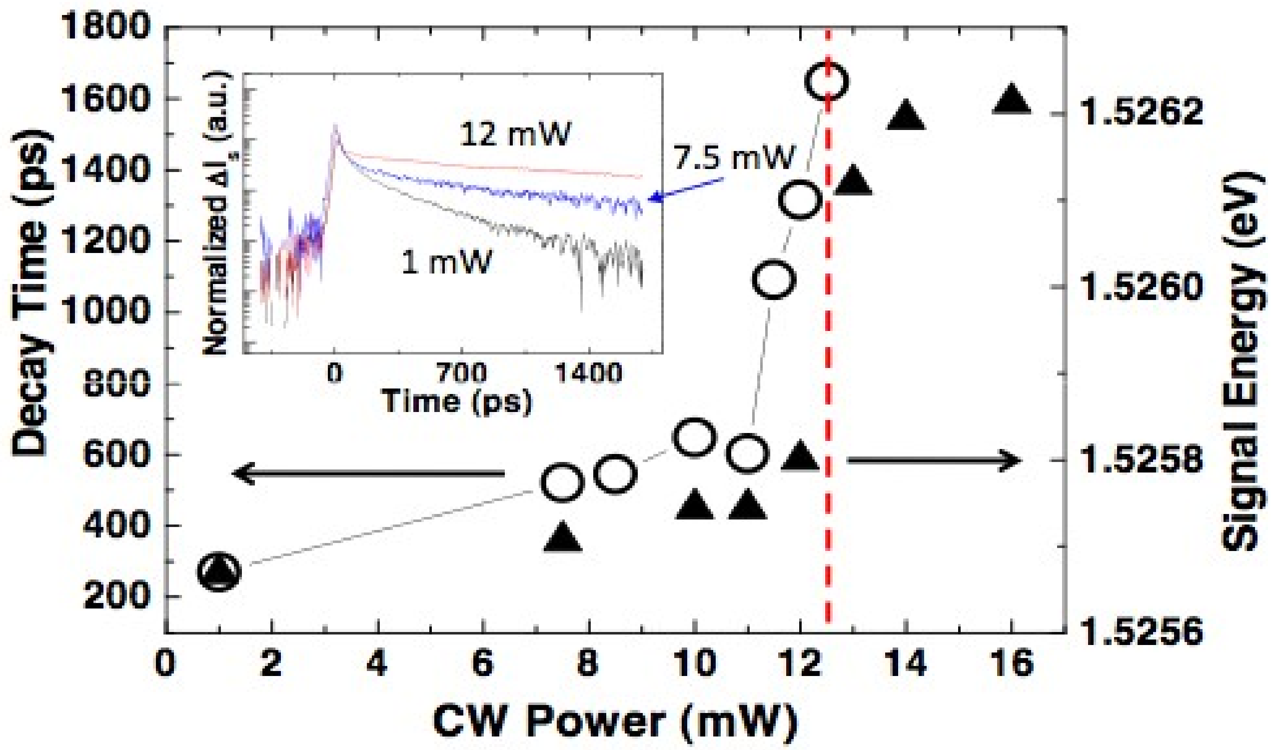
Non-superfluid pump



Atomic BEC against defect

IC, S. X. Hu, L. A. Collins, A. Smerzi, PRL **97**, 260403 (2006)
(expt. data from JILA)

8 – Critical slowing down



Decay time of response to probe pulse:

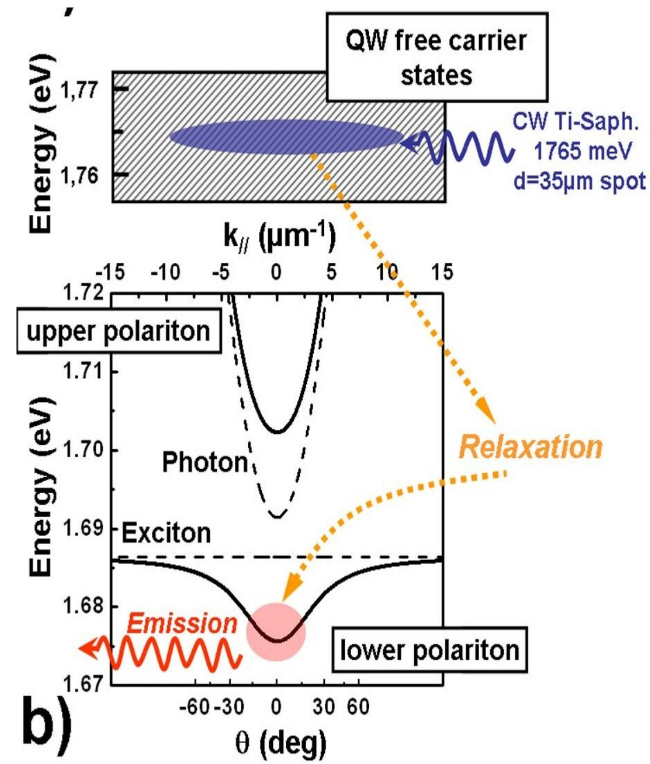
- diverges at BEC threshold
- remains high above threshold, possible signature of BEC Goldstone mode

D. Ballarini, D. Sanvitto, A. Amo, L. Viña, M. Wouters, IC, A. Lemaitre, J. Bloch,
Observation of long-lived polariton states in semiconductor microcavities across the parametric threshold
preprint arXiv:0807.3224

...and also a new feature: non-equilibrium BEC

- Optical injection
- Relaxation: polariton-polariton and polariton-phonon scattering
- Stimulation of scattering to lowest states

- Losses: particle number NOT conserved
- NO thermodynamical equilibrium
- Steady-state determined by dynamical balance of driving and dissipation



(Figure from Kasprzak et al., Nature 2006)

- Standard concepts of equilibrium statistical mechanics are not applicable
- Physics may be strongly different from usual equilibrium BEC

Some questions that naturally arise...

- What new physics can be learnt from polaritons that was not possible with “classical” systems such as liquid Helium and ultracold atoms ?

- Easier diagnostics via coherence properties of emitted light

- What more in polariton BEC than standard lasing operation ?

  strong polariton interactions, significant quantum fluctuations...

- Can it lead to completely new states of matter ?

  e.g., non-equilibrium strongly correlated gases...

- What are the consequences on applications to optoelectronic devices?

Today: three selected problems:

1 - the condensate shape

2 - elementary excitations and superfluidity properties

3 – effect of fluctuations in reduced dimensionality

1 – the condensate shape

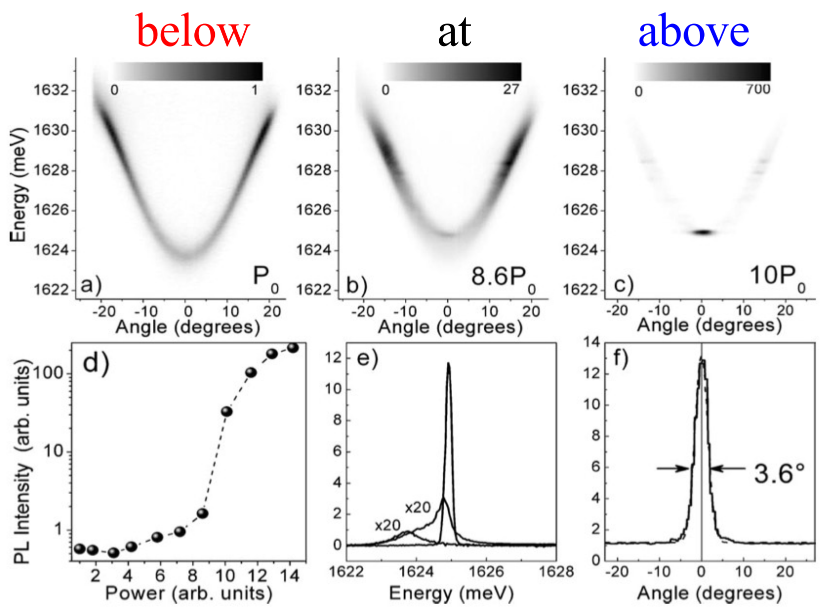
Some intriguing experimental data

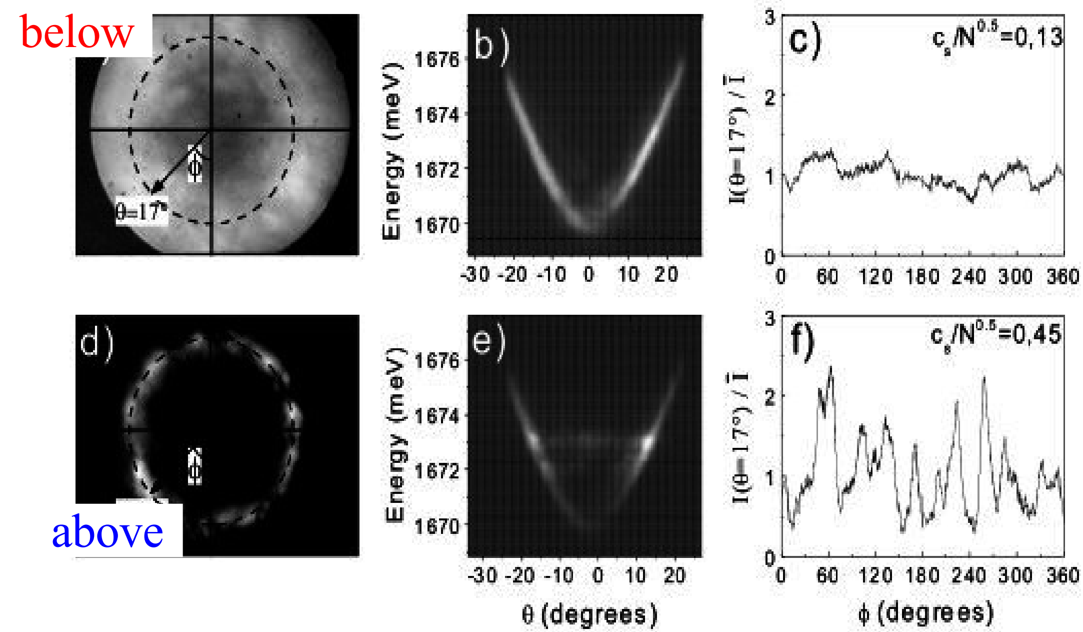
wide pump spot: 20 μm

M. Richard et al., PRB **72**, 201301 (2005)

small pump spot: 3 μm

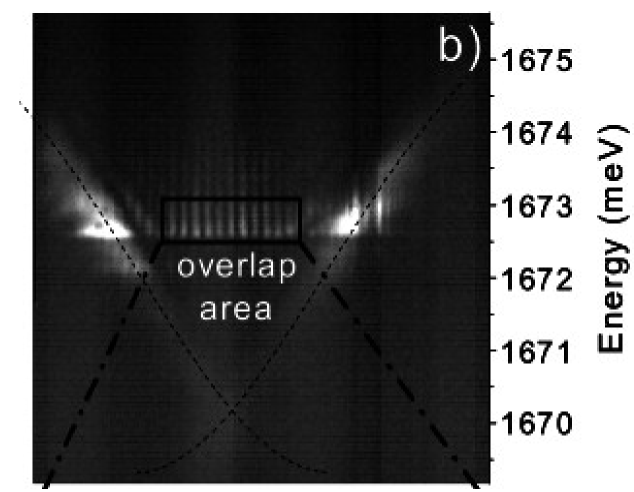
M. Richard et al., PRL **94**, 187401 (2005)



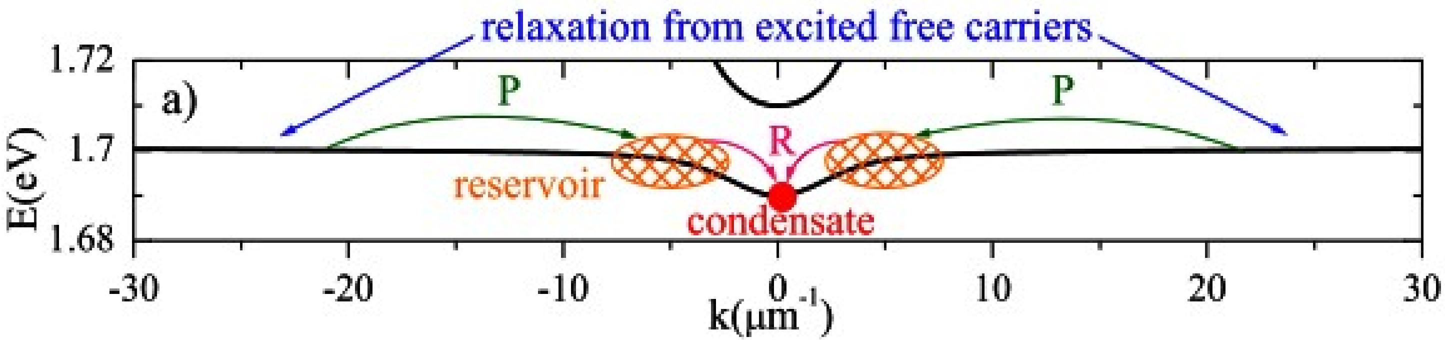


Experimental observations:

- condensate formation under non-resonant pump
- shape dramatically depends on pump spot size
- condensate fully coherent and not fragmented



A generalized GPE for non-resonantly pumped BECs



- **Condensate** : mean-field approx., GPE with losses / amplification

$$i\frac{\partial}{\partial t}\psi=\left[-\frac{\hbar^2\nabla^2}{2m_{LP}}-i\gamma/2+\frac{i}{2}R\left(n_B\right)+g\left|\psi\right|^2+2\tilde{g}n_B\right]\psi$$

macroscopic wavefunction $\psi(x)$, loss rate γ, amplification $R(n_B)$

- **Incoherent reservoir** : rate equation for density $n_B(x)$

$$\frac{\partial}{\partial t}n_B=P-\gamma_B\bar{n}_B-R\left(n_B\right)\left|\psi\left(x\right)\right|^2+\frac{D}{2}\nabla^2n_B$$

pumping rate P, spatial diffusion D, thermalization rate γ_B

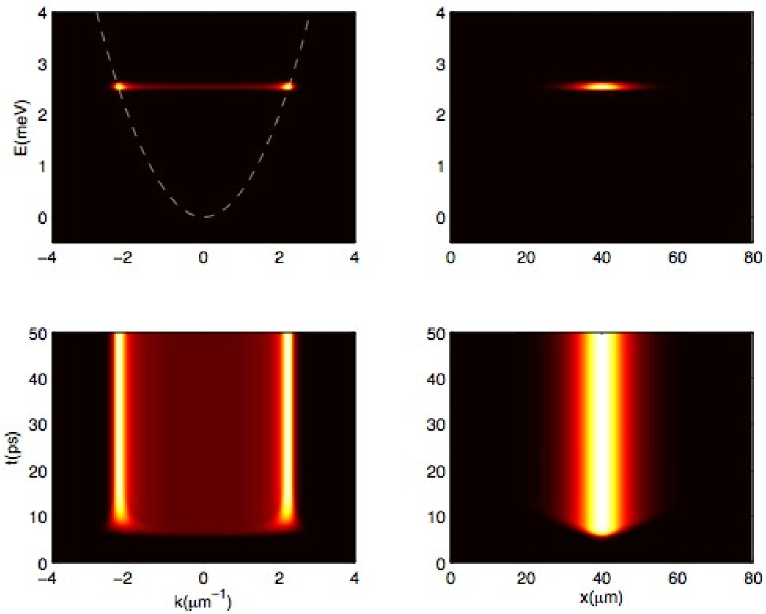
Fast reservoir limit: reduces to a **Complex-Ginzburg Landau Equation**

analogous to semiclassical equations of laser

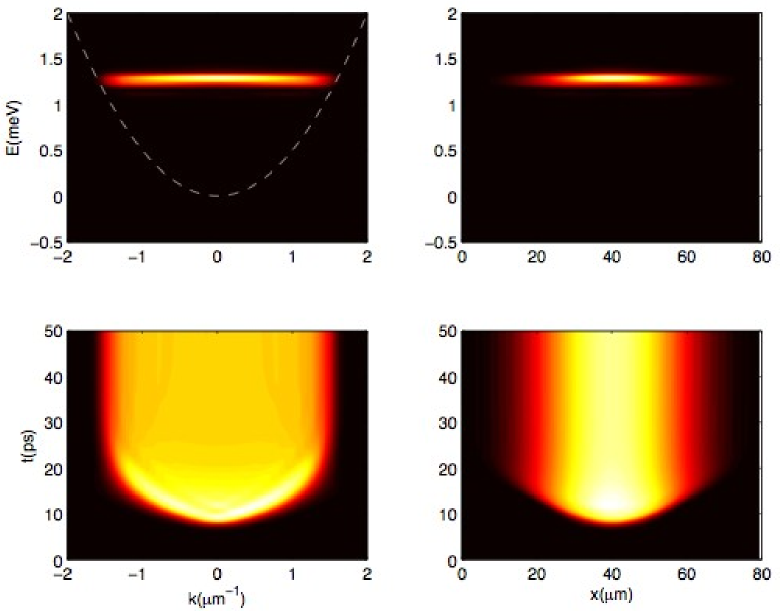
Numerical integration of non-equilibrium GPE

- Equilibrium, harmonic trap: Thomas-Fermi parabolic profile
- Non-equilibrium: dynamics affects shape. Stationary flow possible

related work on flow patterns in J. Keeling and N. G. Berloff, PRL **100**, 250401 (2008)



Narrow pump spot: σ = 5μm
Emission on ring at finite k



Wide pump spot: σ = 20 μm
Broad emission centered at k=0

Good agreement with experiments !!

M. Wouters, IC, and C. Ciuti, *Spatial and spectral shape of inhomogeneous non-equilibrium exciton-polariton condensates*, PRB **77**, 115340 (2008)

Physical interpretation of condensation at k≠0

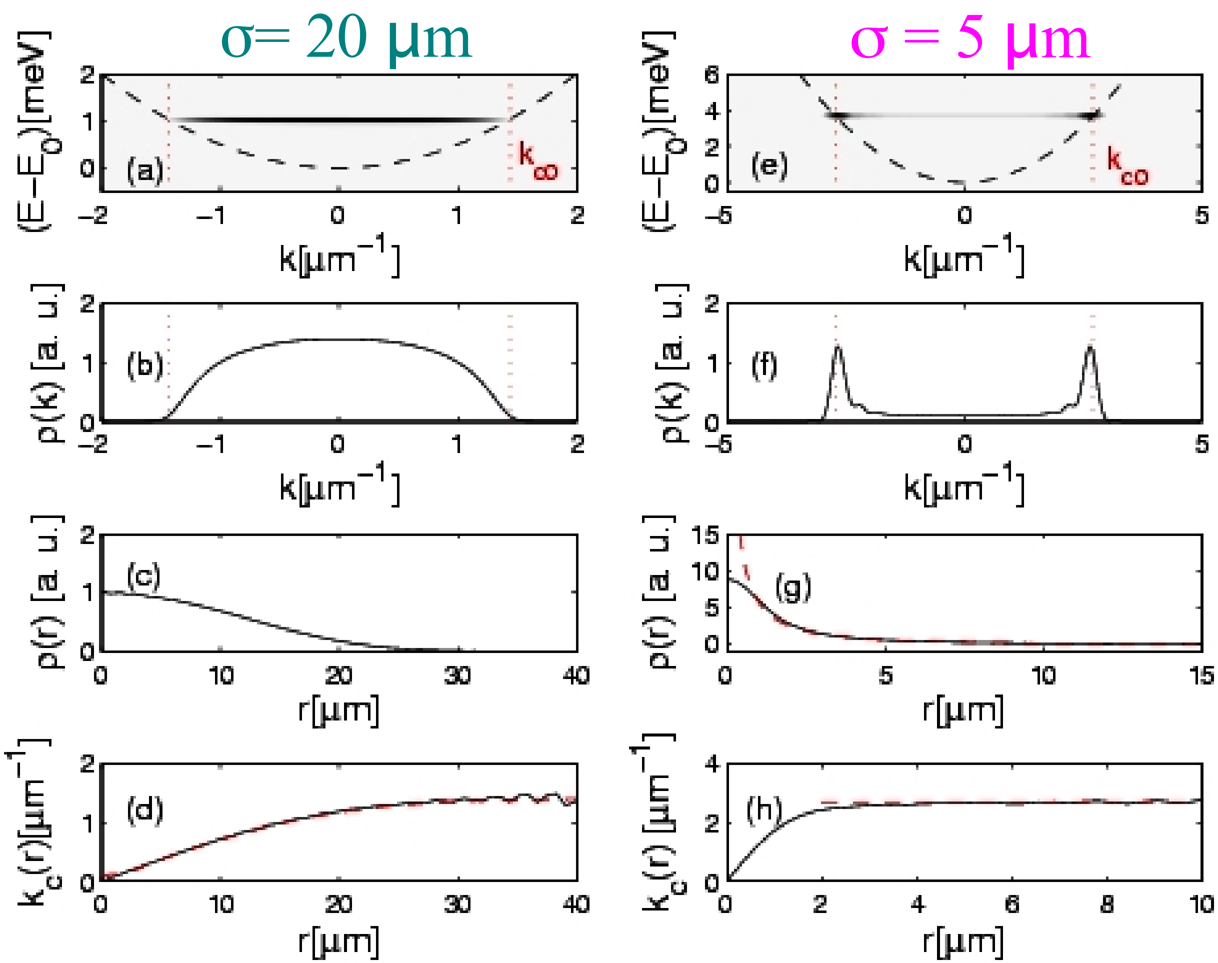
Repulsive interactions

- outward radial acceleration
- energy conservation

$$E = k^2/2m + U_{int}(r)$$

→ radially increasing local flow velocity

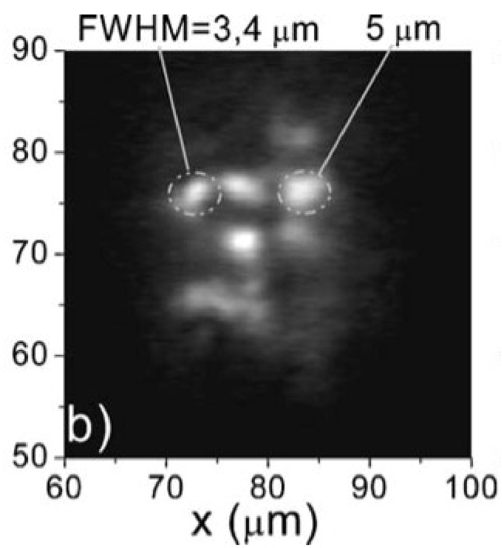
→ coherent ballistic flow



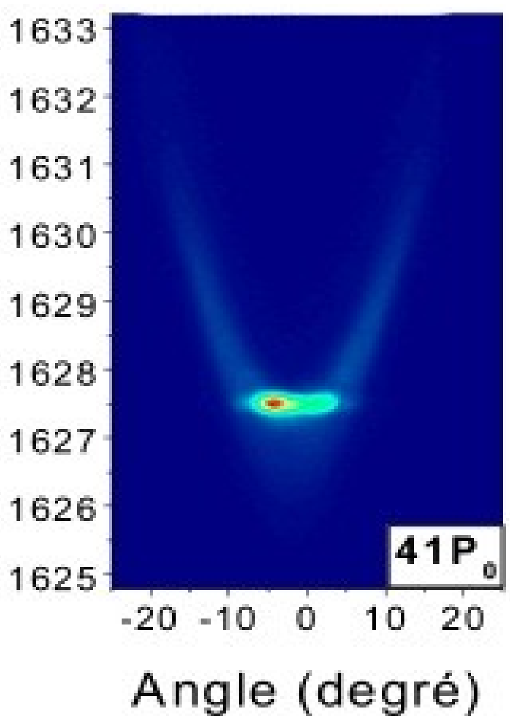
Narrow spot:

- ballistic free flight outside pump spot $U_{int}(r)=0$
- emission mostly on free particle dispersion

A closer look at experimental data: disorder effects



Experiment with large pump spot



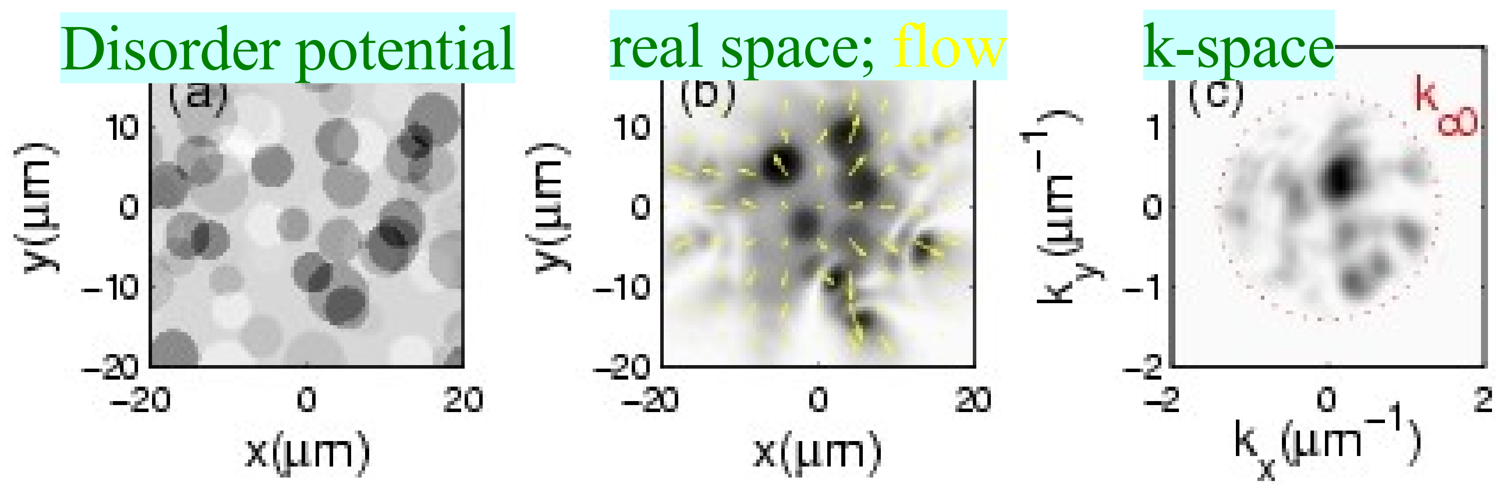
Steady-state of non-equilibrium GPE

- random disorder potential in GPE
- k-space: speckle modulation over broad profile

  peak at k≠0: signature of non-equilibrium

  speckles indicate long-range coherence
- real space: outward flow, modulation roughly

  follows disorder potential



M. Wouters, IC and C. Ciuti, PRB **77**, 115340 (2008)

2 – elementary excitations and superfluidity properties

Bogoliubov modes under non-resonant pumping

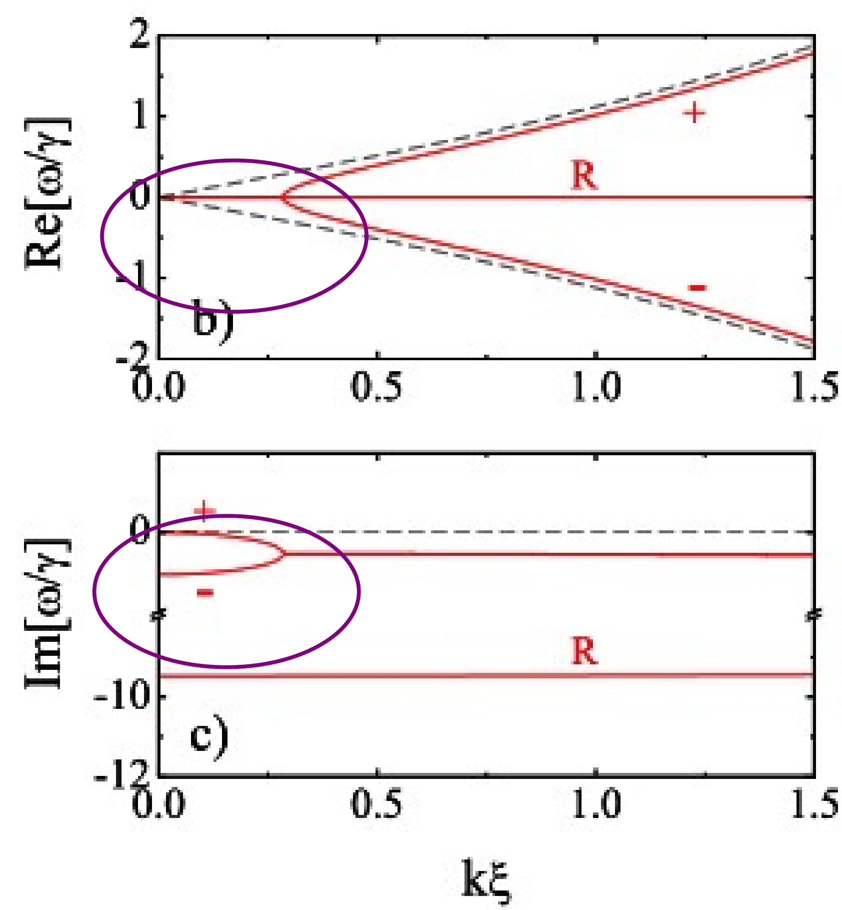
Linearize GPE around steady state

- Reservoir R mode at $-i\gamma_R$
- Condensate modes ± at:

$$\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{[\omega_{Bog}(k)]^2 - \frac{\Gamma^2}{4}}$$

with:

$$\omega_{Bog}(k) = \sqrt{\frac{\hbar k^2}{2m_{LP}}\left(\frac{\hbar k^2}{2m_{LP}} + 2\,\mu\right)}$$

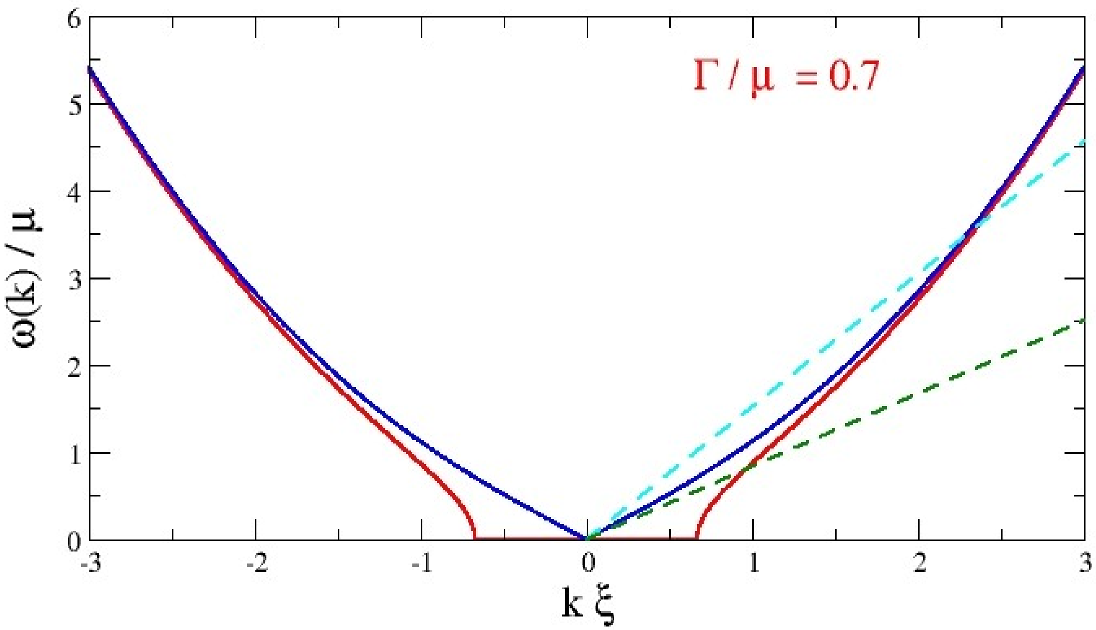


Goldstone mode related to U(1) spontaneous symmetry breaking is diffusive

no sound branch, density and phase decoupled

M. H. Szymanska, J. Keeling, P. B. Littlewood, *Nonequilibrium Quantum Condensation in an Incoherently Pumped Dissipative System*, PRL 96, 230602 (2006)
M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, Phys. Rev. Lett. **99**, 140402 (2007)
M. Wouters, IC, *The Goldstone mode of planar optical parametric oscillators*, Phys. Rev. A 76, 043807 (2007)

Consequences on superfluidity of polariton BECs



Naïf Landau argument:

- Landau critical velocity $v_L = \min_k [\omega(k) / k] = 0$ for non-equilibrium BEC
- Any moving defect expected to emit phonons

But nature is always richer than expected...

Low v :

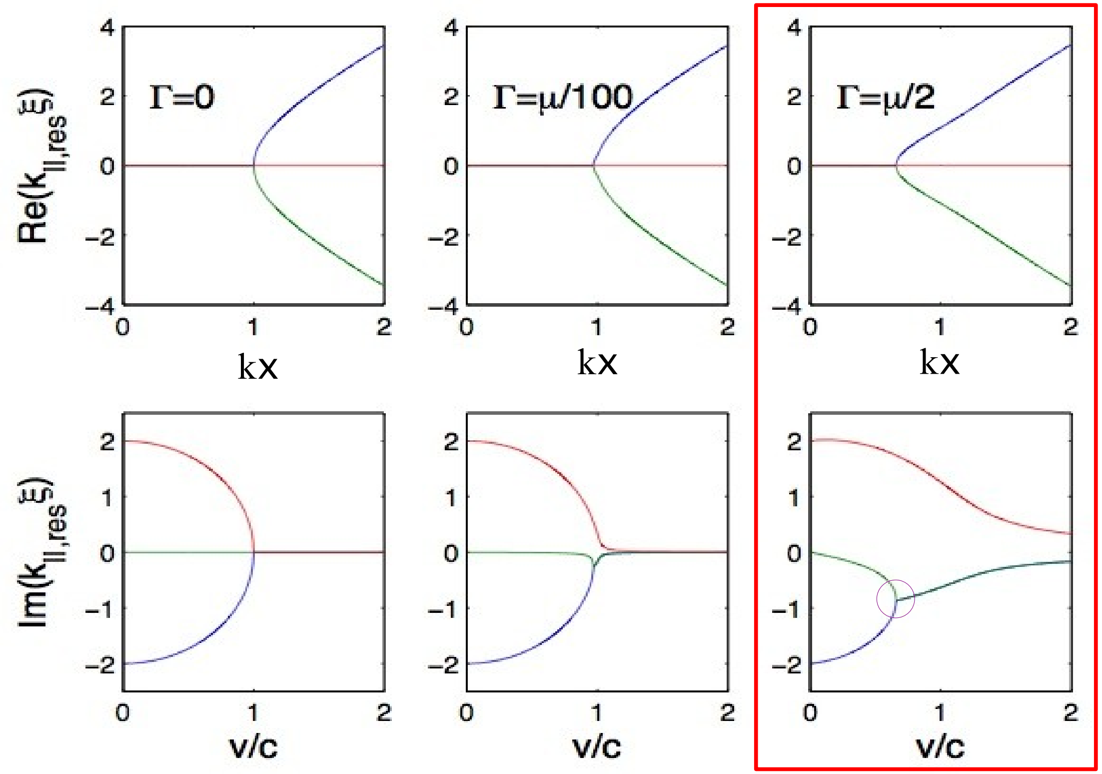
- emitted $k_\parallel$ purely imaginary
- no real propagating phonons
- localized perturbation around defect

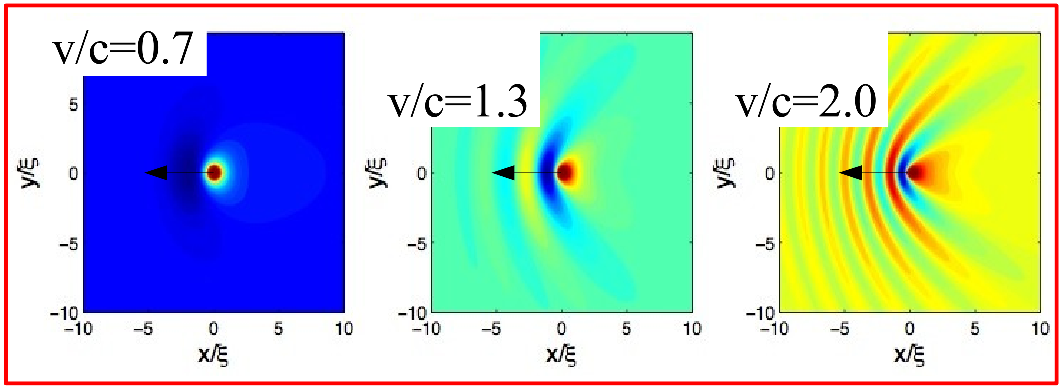
Critical velocity $v_c < c$:

- corresponds to bifurcation point
- decreases with Γ / μ

High v:

- emitted propagating phonons:
  - → Cerenkov cone
  - → parabolic precursors
- spatial damping of Cerenkov cone





M. Wouters and IC, *Excitations and superfluidity in non-equilibrium Bose-Einstein condensates of exciton-polaritons*, Superlattices and microstructures **43**, 524 (2008).

3 – effect of fluctuations in reduced dimensionality

Reduced dimensionality in BEC: equilibrium case

- 3D: BEC transition at finite T_c
- 2D: K-T transition at finite T_{KT} related to vortex pair unbinding :
  - algebraic decay of coherence for $T<T_{KT}$
  - exponential decay of coherence for $T>T_{KT}$
- 1D: exponential decay of coherence for $T>0$
- Hohenberg-Mermin-Wagner theorem sets $d_c=2$ for U(1) SSB

  in the thermodynamical limit

Note: Finite-size effects: BEC possible in all dimensionality.

T_c depends on size L: as L^{-1} in 1D, logarithmically in 2D below T_{KT}

What happens far from equilibrium ?

BEC from parametric oscillation in 1D photonic wire

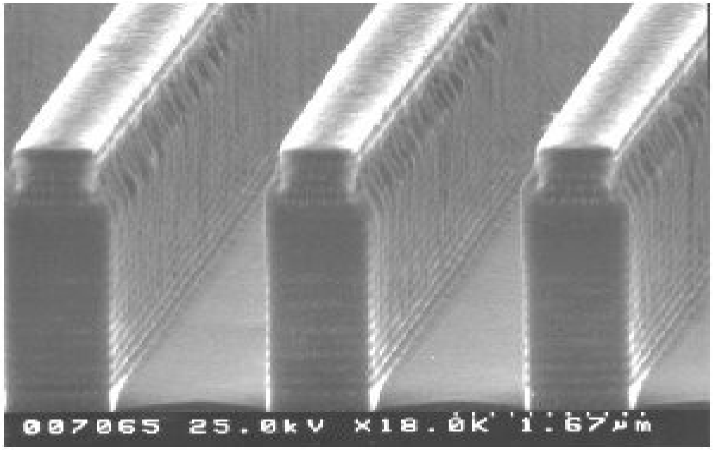
Wigner-QMC results

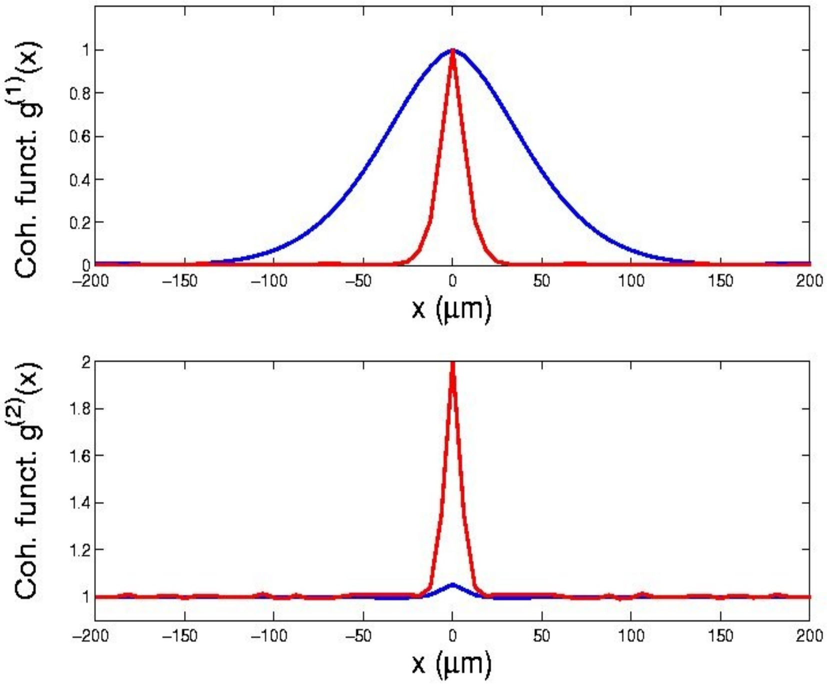
Below threshold:

- incoherent luminescence
- short range coherence

Above threshold:

- intensity fluctuations suppressed
- coherence length much longer
  but always finite





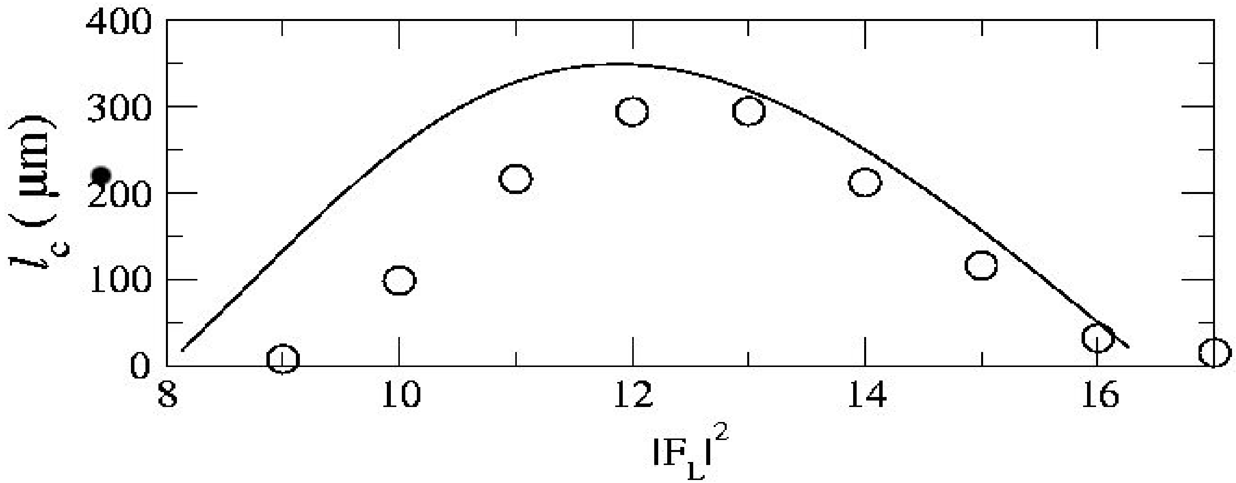
IC and C. Ciuti, *Spontaneous microcavity-polariton coherence across the parametric threshold: Quantum Monte Carlo studies*, PRB 72, 125335 (2005)

Analytical results

Integration of Wigner-Bogoliubov stochastic equations around pure BEC

Exponential decay of coherence (as at equilibrium).

- ➔ significant polariton-polariton interactions enhance fluctuations
- ➔ damping plays role of temperature
- ➔ coherence length l_c experimentally accessible, important for applications!



M. Wouters and IC, *Absence of Long-Range Coherence in the Parametric Emission from Photonic Wires*, PRB **74**, 245316 (2006)

In conclusion...

Bose-Einstein condensates offer many exciting surprises

Unique non-equilibrium phase transition: interactions vs. driving/dissipation

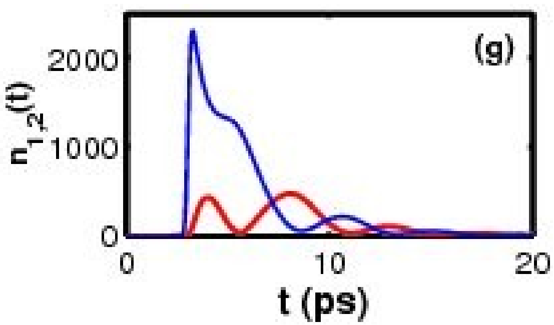
Peculiar condensate shapes in real and momentum space

Fluctuations can be very important, especially in low D

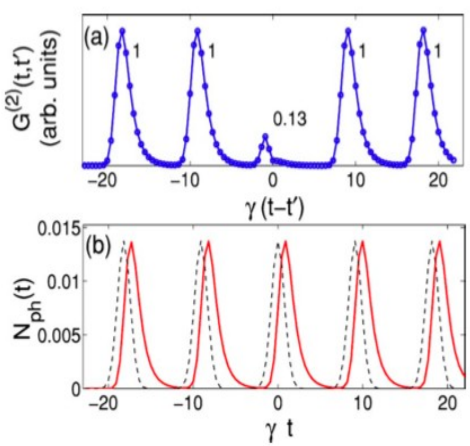
Diffusive Goldstone mode

Superfluidity effects beyond the Landau criterion

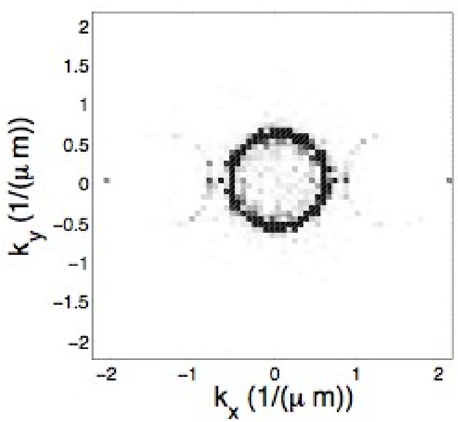
... and much more ...



Josephson oscillations and self-trapping



Single-photon emitter by polariton blockade



Quantum correlated states in BEC collision

Hic sunt leones...

- Short term (on the way: stay tuned!)

  meaning of polariton superfluidity and links with non-equilibrium BEC

  Wigner-QMC study of condensation under incoherent pump

  non-equilibrium strongly correlated phases in array of coupled polariton dots

- Medium term (hard but feasible)

  quantum hydrodynamics, e.g. dumb holes, Hawking emission

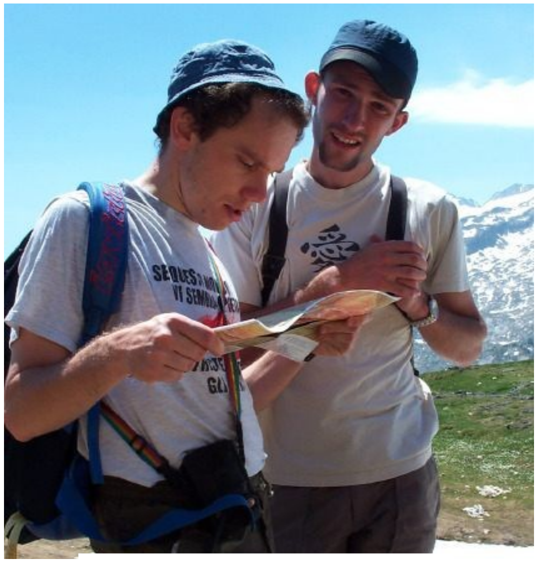
  Polariton Feshbach resonances on bi-exciton states (see Michiel's recent work)

- Long term (science-fiction)

  non-equilibrium statistical mechanics of 2D (K-T ?) transition

  non-equilibrium BEC phase transition with disorder, link with random lasing

Thanks to my brave coworkers!!



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Cristiano Ciuti

and the young crew

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(EPFL Lausanne)

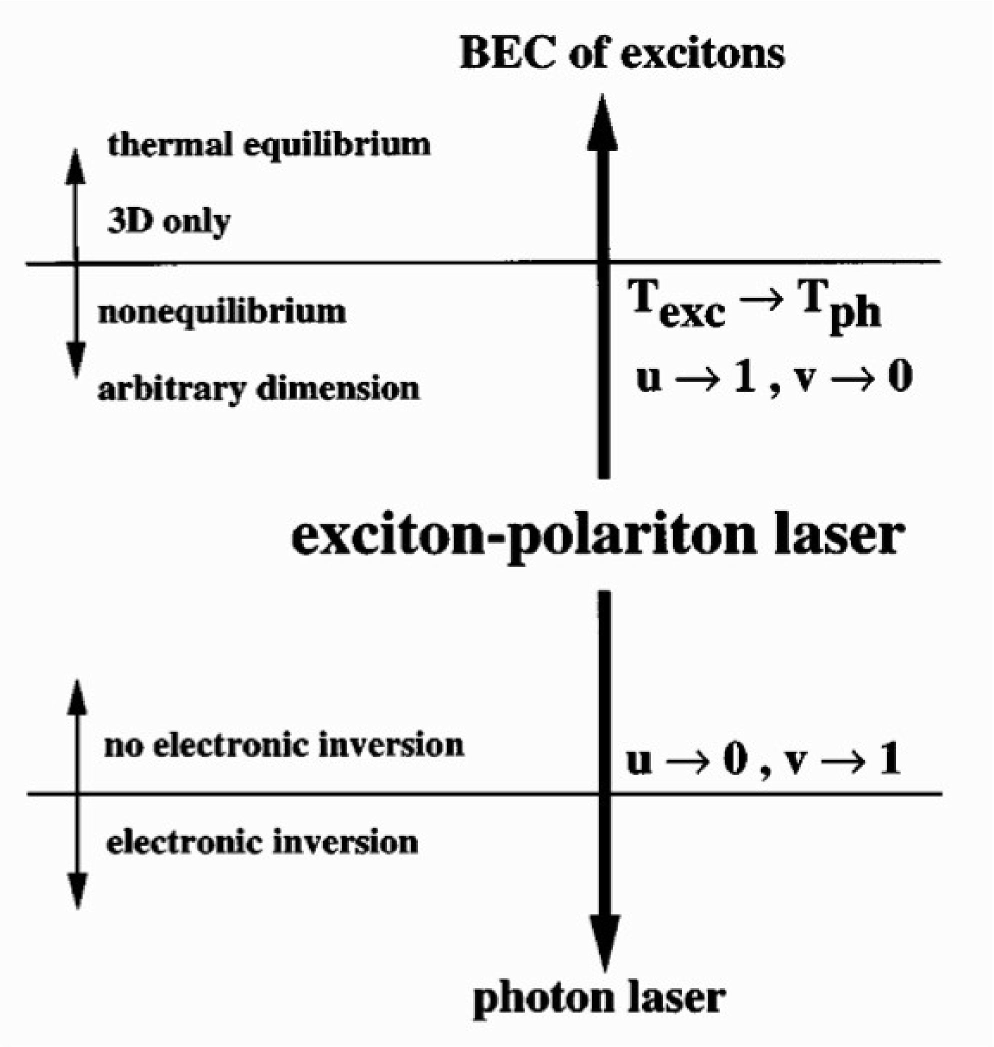
Maxime Richard
Le Si Dang
(Grenoble)

Zeno Gaburro
Lorenzo Pavesi
(Univ. Trento)

Much more than standard BEC...

Non-equilibrium exciton-polariton BEC

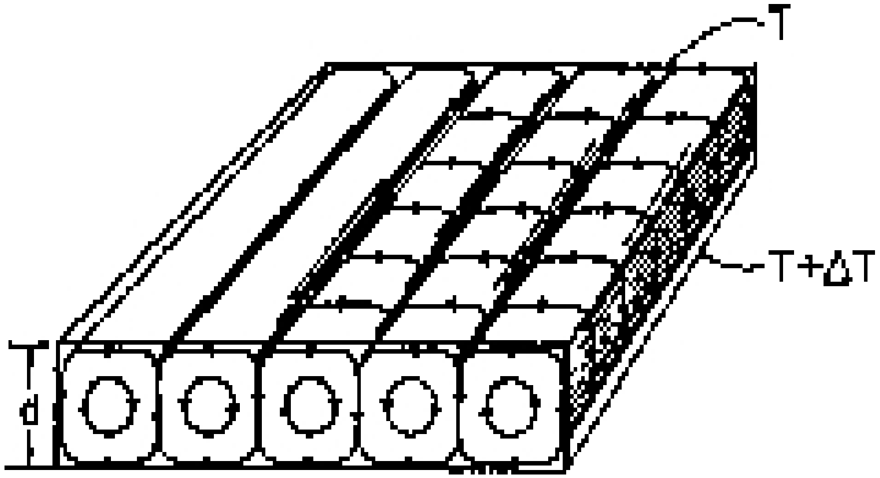
- Non-equilibrium system
  (not a standard BEC)
- Collisional interactions
  (not even a standard laser)
- Coherence functions accessible
  from emitted light
- Differently from standard laser:
  strong interactions and
  significant quantum fluctuations



From: A. Imamoglu et al. PRA 53, 4250 (1996)

Phase transitions in non-equilibrium systems as well !

Bénard cells in heat convection:



- dynamical equilibrium between driving (DT) and dissipation (viscosity)
- for $\Delta T > \Delta T_c$ translationally invariant state is dynamically unstable
- spontaneous breaking of translational symmetry: periodic pattern of convection rolls

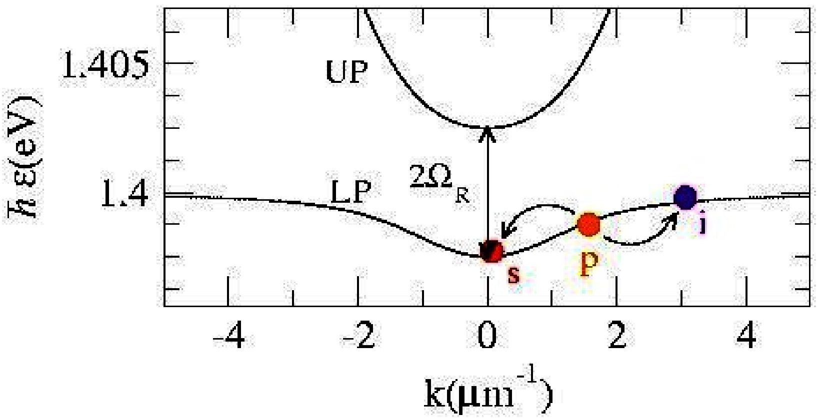
Other examples:

- Belousov-Zhabotinsky chemical reaction
- Coat patterns of mammalians
- Driven lattice gas

Ways to generate macroscopic coherence

Direct injection by resonant pump laser

- coherence not spontaneous, imprinted by pump
- close relation with nonlinear optics, still interesting superfluidity properties



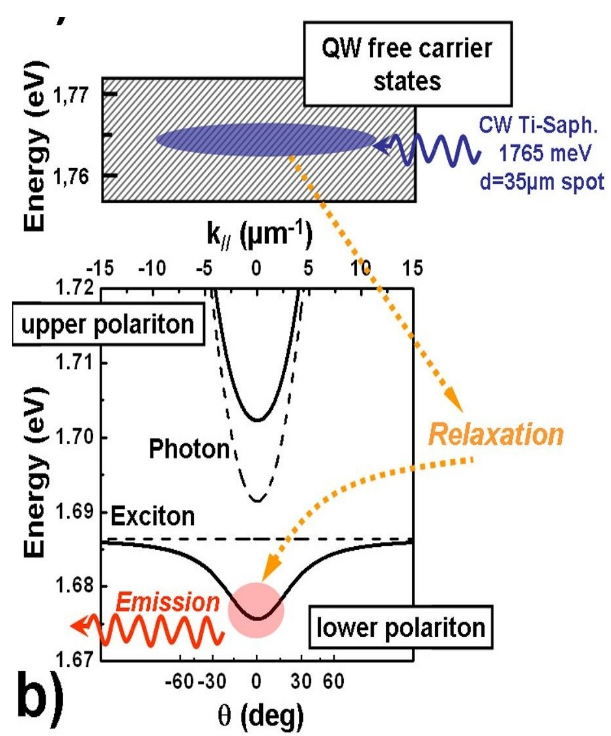
OPO process

- stimulated scattering into signal/idler modes
- spontaneous coherence, not locked to pump
- same spontaneous symmetry breaking

ab initio theoretical description by stochastic GPE

Non-resonant pumping

- thermalisation due via polariton-polariton collisions, quasi-equilibrium condition
- coherence spontaneously created via BEC effect
- hard to theoretically model *ab initio*



(Fig. from Kasprzak et al., Nature 2006)

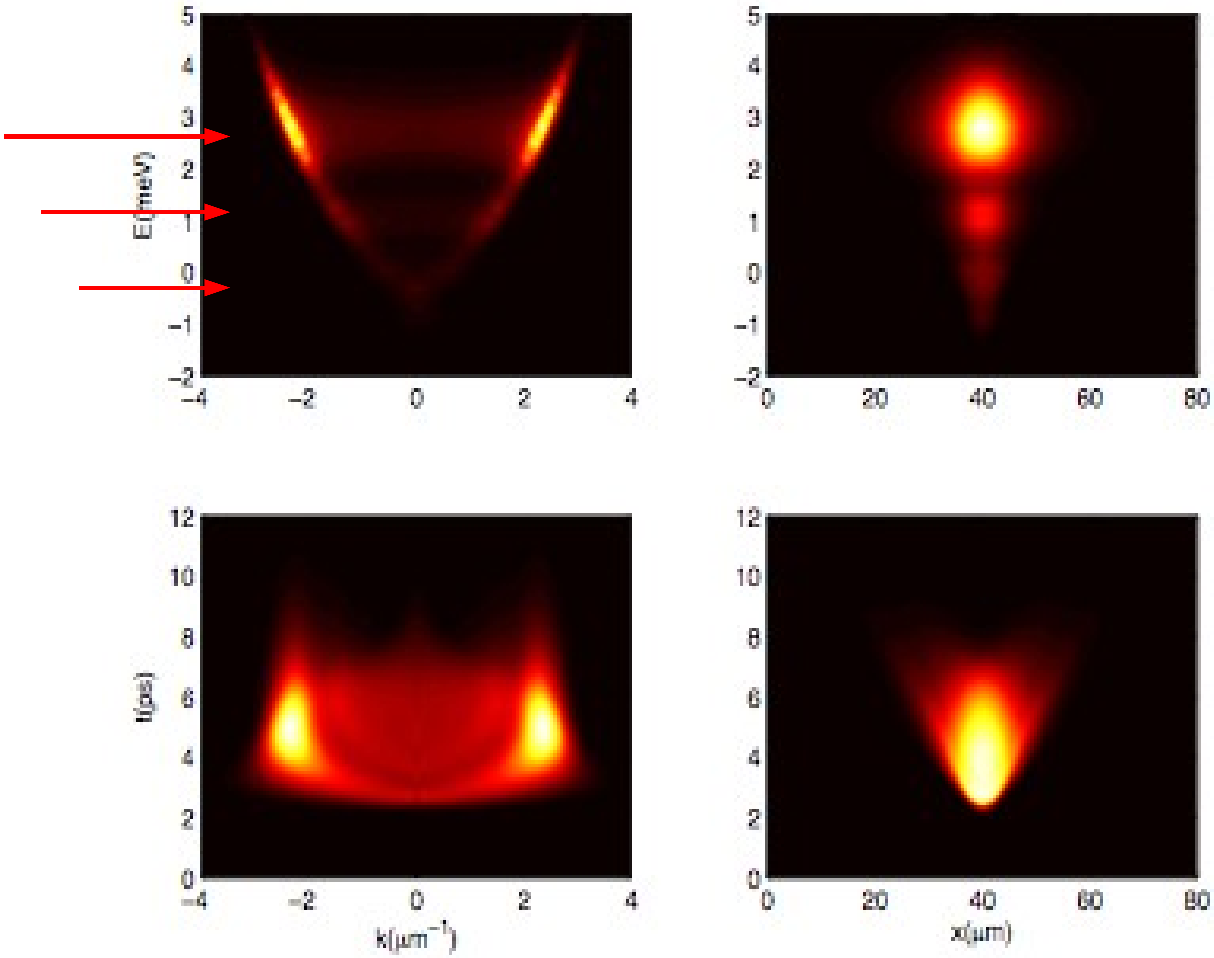
Simulations for pulsed excitation

- Non-trivial time evolution:

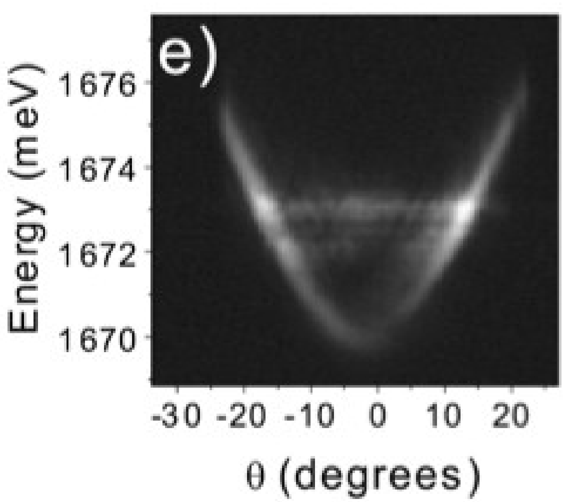
  first k=0, then expands

- Emission concentrated

  at several E's



Also in expt's !!!



M. Wouters, IC and C. Ciuti,, preprint arXiv:0707.1446 (2007)

Wigner-QMC

Generalizes truncated-Wigner method for BECs (Lobo, Sinatra, Castin)

Time evolution: stochastic Gross-Pitaevskii equation

$$i\,d\begin{pmatrix}\psi_X(\mathbf{x},t)\\ \psi_C(\mathbf{x},t)\end{pmatrix} = \left[\mathbf{h}^0 + \begin{pmatrix} V_X(\mathbf{x}) + g(|\psi_X(\mathbf{x},t)|^2 - 1/dV) - i\gamma_X & 0 \\ 0 & V_C(\mathbf{x}) - i\gamma_C\end{pmatrix}\right]\begin{pmatrix}\psi_X(\mathbf{x},t)\\ \psi_C(\mathbf{x},t)\end{pmatrix} dt + $$
$$+ \begin{pmatrix} 0 \\ k\,\mathcal{E}_p(\mathbf{x},t)\end{pmatrix} dt + \frac{1}{\sqrt{4\Delta V}}\begin{pmatrix}\sqrt{\gamma_X}\,dW_X(\mathbf{x},t)\\ \sqrt{\gamma_C}\,dW_C(\mathbf{x},t)\end{pmatrix}$$

Single particle Hamiltonian
$$\mathbf{h}^0 = \begin{pmatrix}\omega_X(-i\nabla) & \Omega_R \\ \Omega_R & \omega_C(-i\nabla)\end{pmatrix}$$

Losses $\gamma_{X,C}$. Fluctuation-dissipation: white noise

$$\overline{dW_i(\mathbf{x},t)\,dW_j(\mathbf{x}',t)} = 0$$
$$\overline{dW_i(\mathbf{x},t)\,dW_j^*(\mathbf{x}',t)} = 2\,dt\,\delta_{\mathbf{x},\mathbf{x}'}\,\delta_{ij}$$

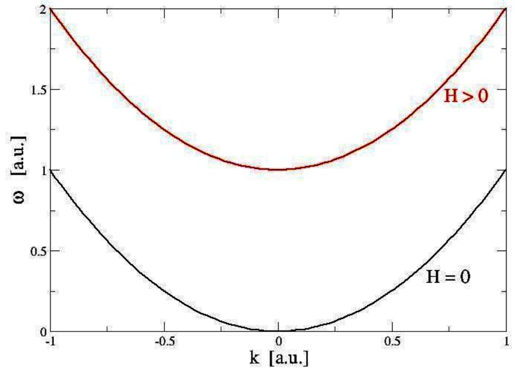
Observables: MC averages over noise
$$\langle\psi_i^*(\mathbf{x})\,\psi_i(\mathbf{x})\rangle_W = \frac{1}{2}\left[\langle\hat\Psi_i^\dagger(\mathbf{x})\,\hat\Psi_i(\mathbf{x})\rangle + \langle\hat\Psi_i(\mathbf{x})\,\hat\Psi_i^\dagger(\mathbf{x})\rangle\right]$$

- → not linearized theory, full account of large fluctuations around critical point
- → any geometry can be simulated, full time-dynamics

→ Accurate *ab initio* description of OPO transition

IC and C. Ciuti, PRL, **93** 166401 (2004); PSSb **242**, 2224 (2005); PRB 72, 125335 (2005)

Pinning the signal/idler phase



Goldstone mode in ferromagnets

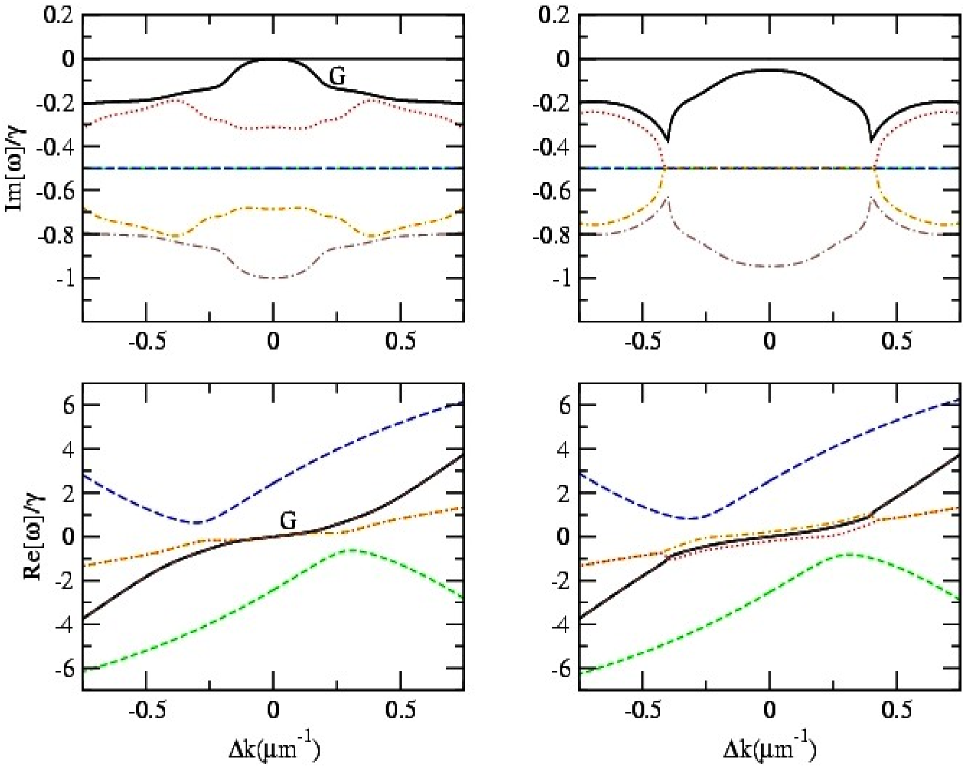
- magnons: wavy oscillations in spin orientation
- spin orientation can be pinned by external B
- gap in Goldstone spectrum opens

Goldstone mode of OPOs:

- slow rotation of signal/idler phases

Seed laser driving signal:

- stimulates signal emission, phase pinned
- phase symmetry explicitely broken
- gap opens in imaginary part of $\omega_G(k)$

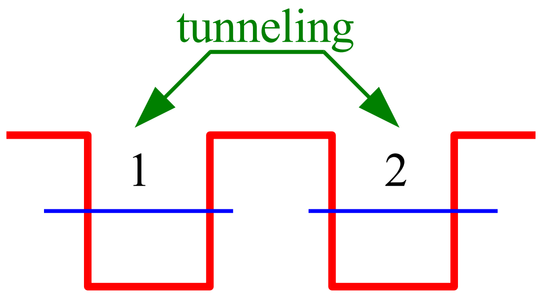


M. Wouters, IC, *The Goldstone mode of planar optical parametric oscillators*, Phys. Rev. A 76, 043807 (2007)

Two-well geometry: Josephson effect

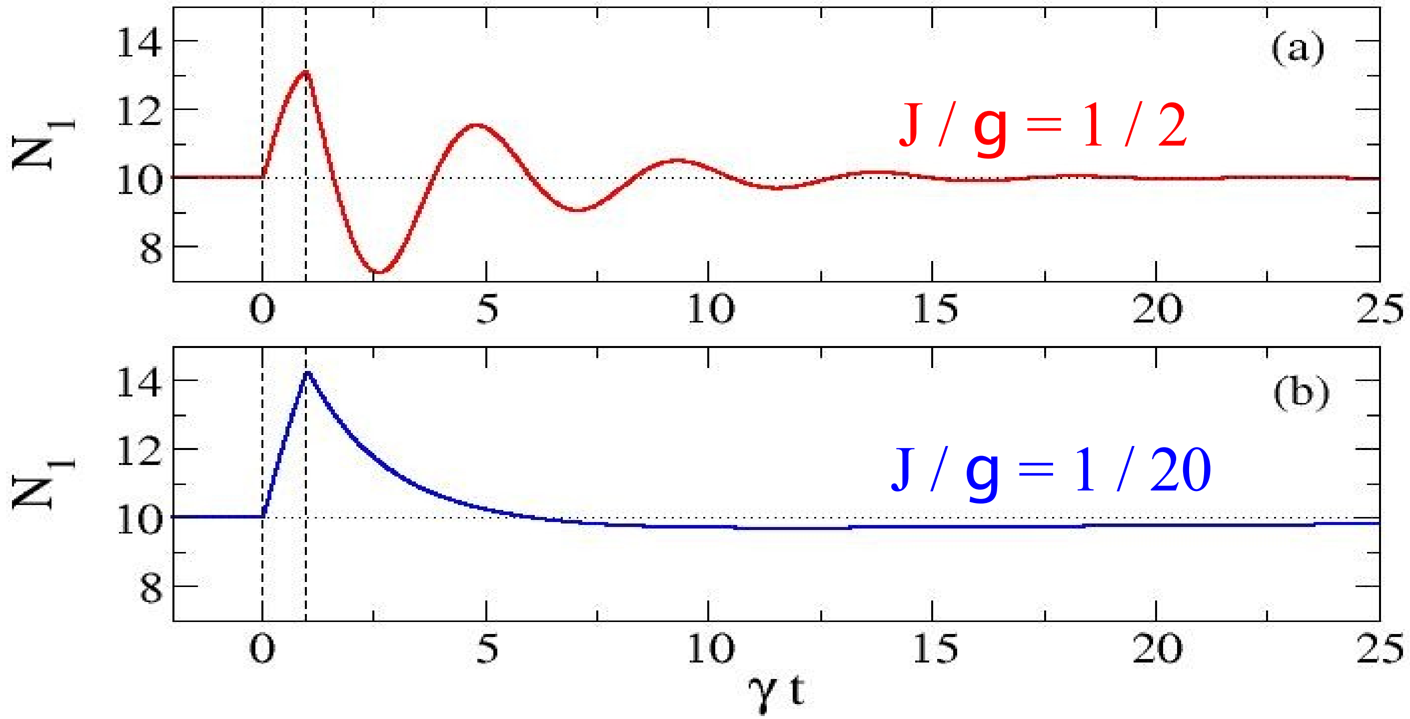
ψ_i → amplitude in i-th well; population $N_i = |\psi_i|^2$

n_i → reservoir density behind i-th well



$$i\frac{d\psi_j}{dt} = -J\,\psi_{3-j} + U\,|\psi_j|^2\,\psi_j + \frac{i}{2}\left[R(n_j) - \gamma\right]\psi_j$$
$$\frac{dn_j}{dt} = P_j - \gamma_R\, n_j - R(n_j)|\psi_j|^2.$$

Exp. with polariton traps:
El Daif *et al.*, APL '06
Baas, Richard *et al.*, '07 (ICSCE-3)

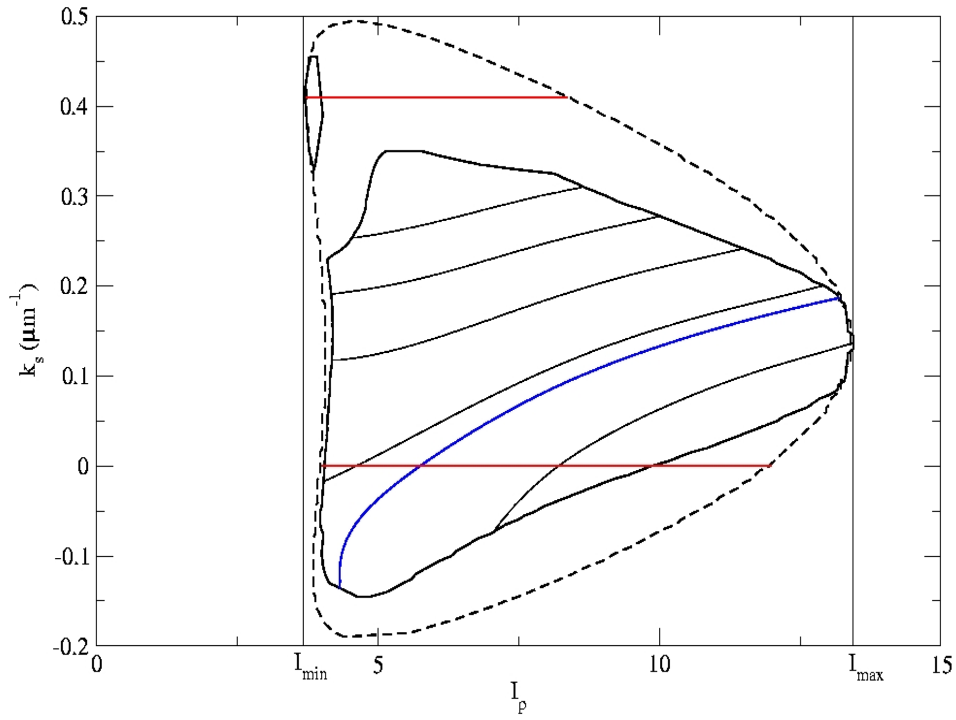


Josephson oscillations

overdamped Josephson oscillations

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, cond-mat/0702413, to appear on PRL

Finite spot effects



Equilibrium:

BEC in lowest energy state

Non-equilibrium:

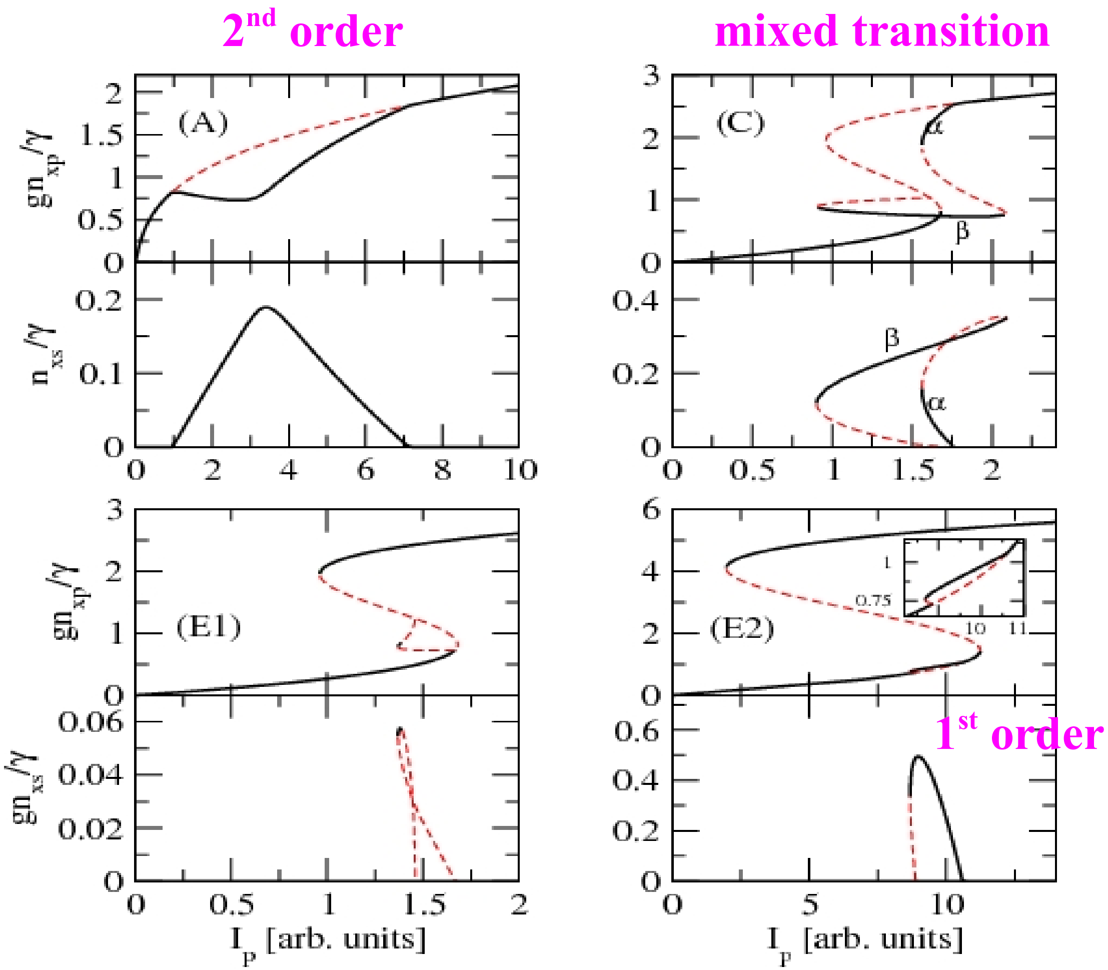
- no free-energy available
- k_s dynamically selected
- methods of pattern formation in nonlinear dynamical systems

- Finite excitation spot: absolute vs. convective instability
- Single w_s, inhomogeneous broadening of k_s due to spatially varying pump intensity profile: change in k_s

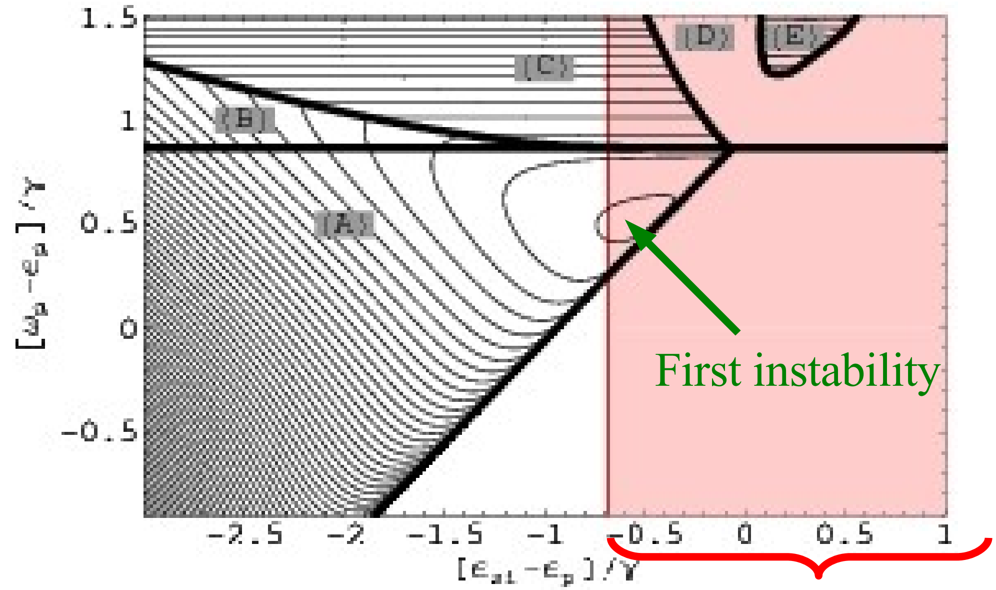
Richer physics than simple Thomas-Fermi profile of equilibrium BECs!!

M. Wouters and IC, *Pattern formation effects in parametric oscillation in semiconductor microcavities*, in preparation

Not only second-order phase transition...



Contours: value of threshold intensity



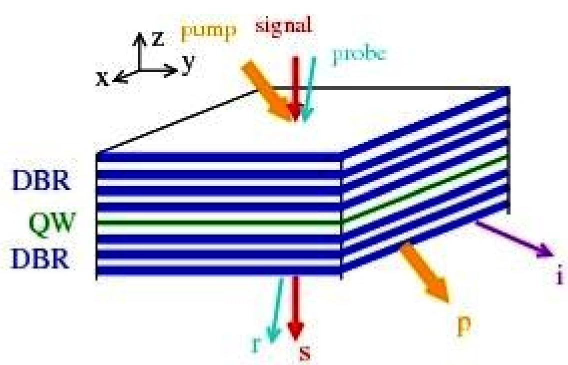
Available region depends on pump angle

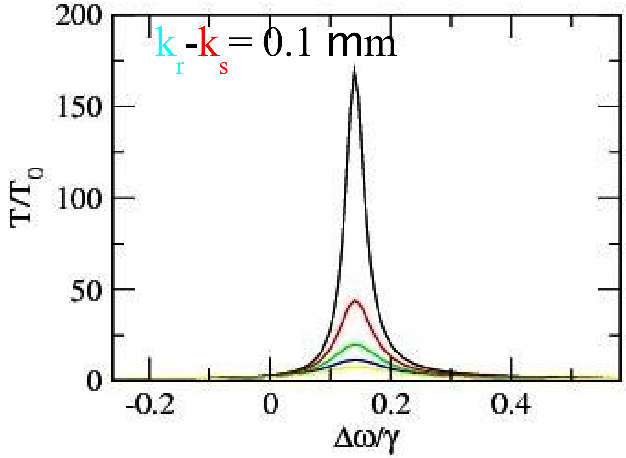
“Magic angle” condition slightly modified !!

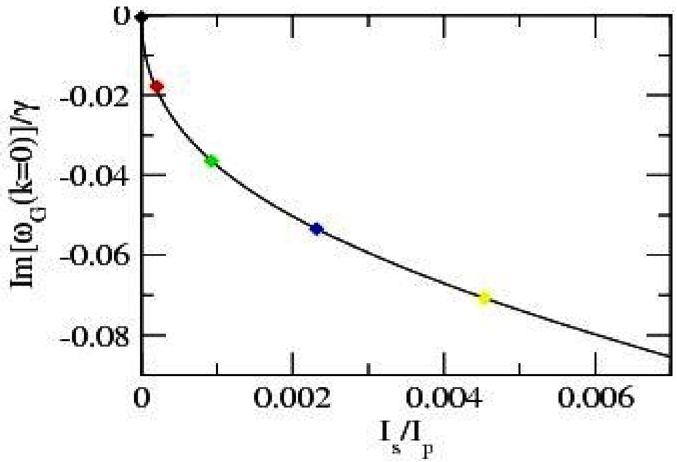
M. Wouters and IC, *The parametric oscillation threshold of semiconductor microcavities in the strong coupling regime*, PRB **75**, 075332 (2007)

Observing the Goldstone mode

- Goldstone mode: peak in probe transmission at angle close to signal
- amplified transmission w/r to unloaded cavity resonant transmission
- when phase pinned by signal laser: peak broadened and suppressed







Hard to do with atoms because of atom number conservation

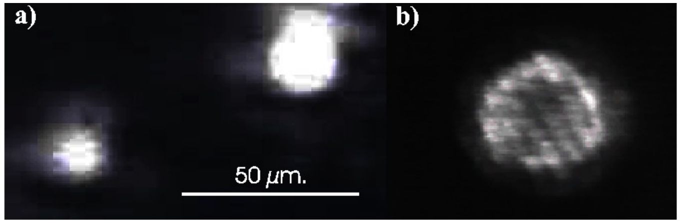
M. Wouters, IC, *The Goldstone mode of planar optical parametric oscillators*, cond-mat/0606755, to appear on PRA

Experimental observations

Correlation functions of emission reproduce those of cavity polaritons

$g^{(1)}(x) \rightarrow$ Young-like experiment

- light from two paths interferes
- above threshold: fringes observed



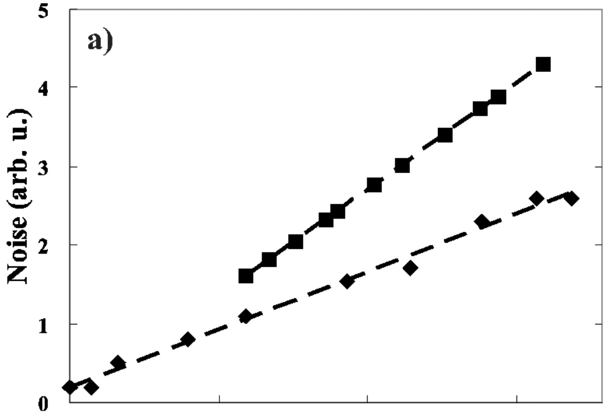
$g^{(2)}(x) \rightarrow$ noise-correlation experiment

- output beam cut by razor blade
- above threshold: linear dependence

  means single spatial mode

- slope means excess noise over standard

  quantum limit



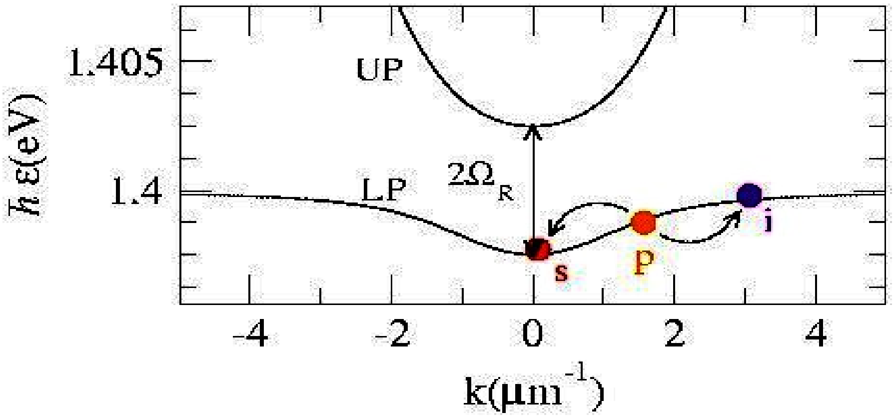
(figs. from Baas et al., PRL 2006)

Good agreement with theory !!

Let's go beyond mean field...

Coherent pump around "magic angle":

- resonant parametric scattering
  into signal/idler modes
- strong pumping: scattering is stimulated



Above threshold for **parametric oscillation**:

- spontaneous appearance of coherence, phase not locked to pump
- same spontaneous U(1) symmetry breaking as in BEC

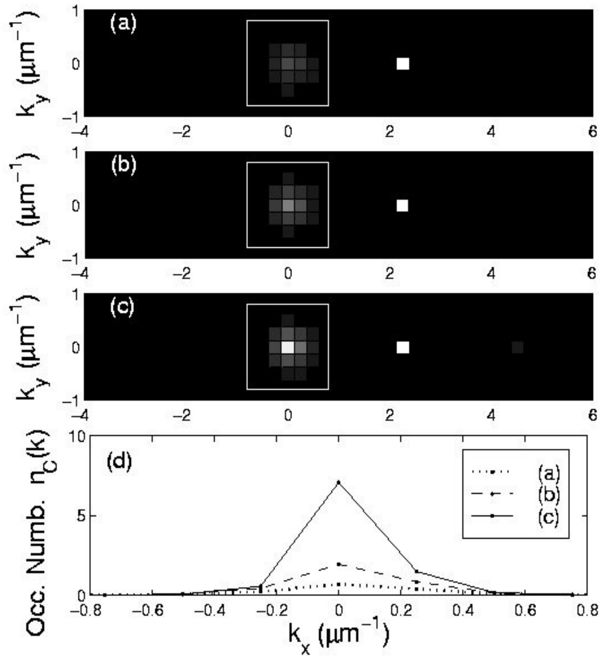
Ab initio theoretical description available by stochastic GPE - Wigner QMC

- → any geometry can be simulated, full time-dynamics
- → not linearized theory, full account of fluctuations even around critical point

IC and C. Ciuti, *Spontaneous microcavity-polariton coherence across the parametric threshold: Quantum Monte Carlo studies*, PRB 72, 125335 (2005)

The parametric oscillation threshold

Pump beam close to magic angle for OPO process

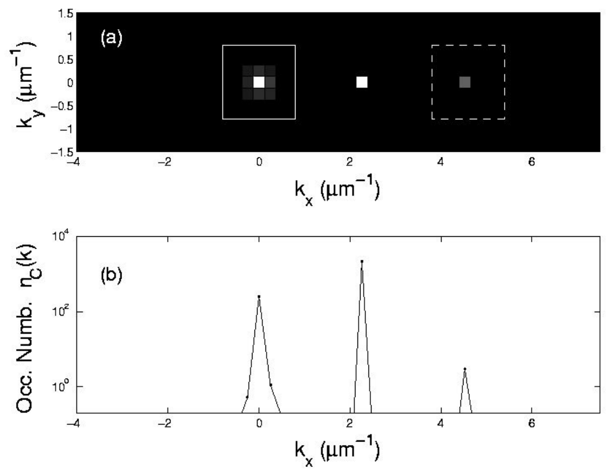


Below threshold:

- coherent emission from pump mode
- quantum fluctuations: many-mode incoherent luminescence
- strongest for signal and idler around phase-matching

Approaching threshold:

- signal/idler intensity increases, linewidth narrows



Above threshold:

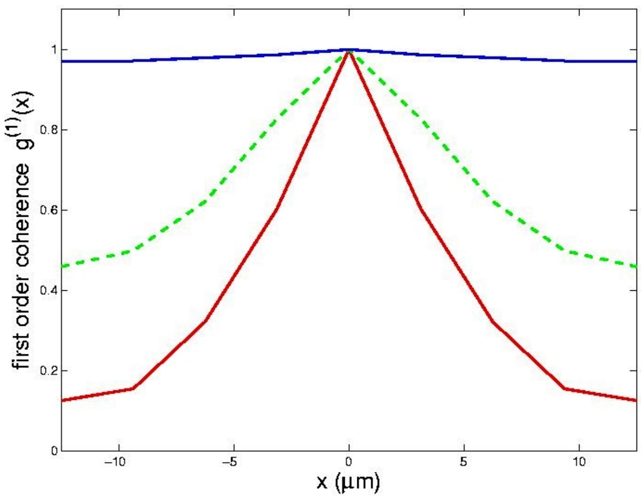
- single signal/idler pair selected
- emission becomes macroscopic
- signal/idler phases still random, only their sum fixed

IC and C. Ciuti, *Spontaneous microcavity-polariton coherence across the parametric threshold: Quantum Monte Carlo studies*, PRB 72, 125335 (2005)

Signal/idler coherence across threshold

First-order coherence $g^{(1)}(x)$

- approaching threshold from below : l_c diverges
- above threshold: long-range coherence
- BEC according to Penrose-Onsager criterion
- coherence NOT inherited from pump

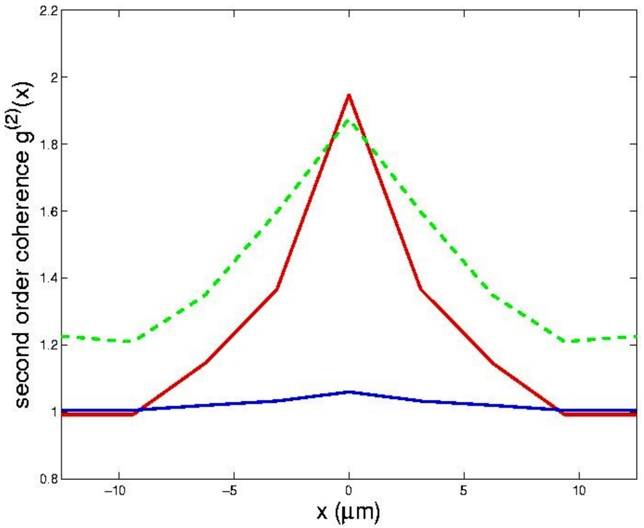


Second-order coherence $g^{(2)}(x)$

- below threshold: HB-T bunching

$$g^{(2)}(0)=2, \quad g^{(2)}(\text{large } x)=1$$

- above threshold: suppression of fluctuations: $g^{(2)}(x)=1$



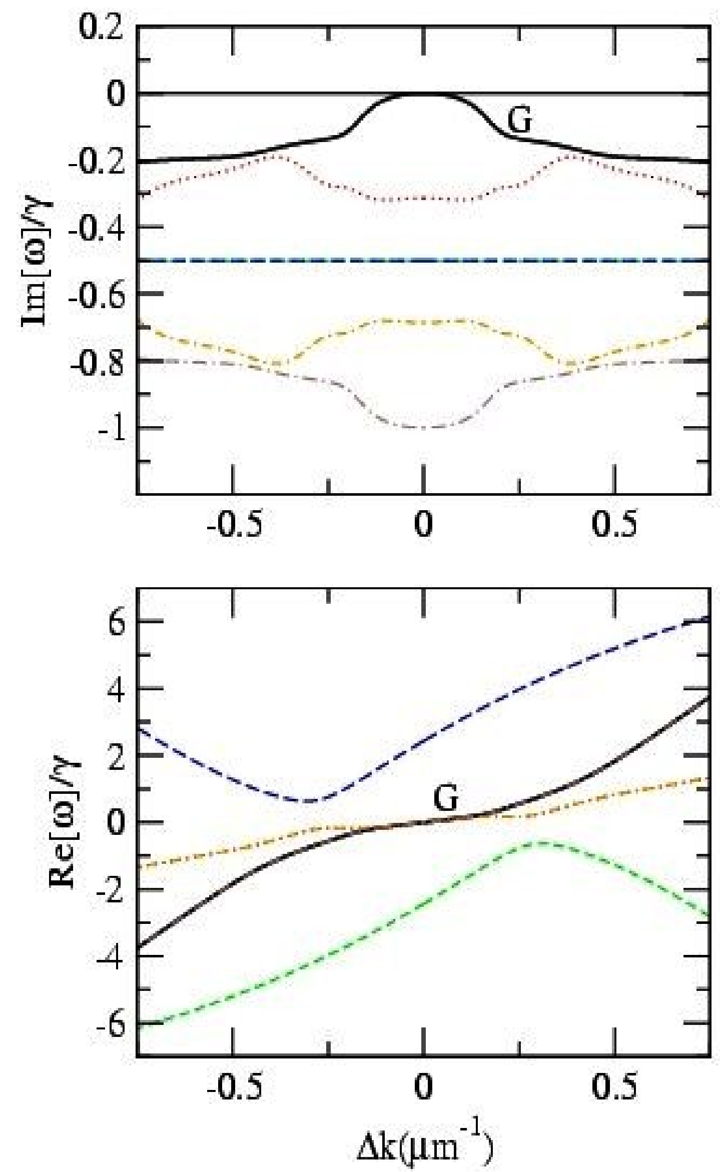
As in atomic gases at thermal equilibrium !

A closer look at the Bogoliubov modes

Steady-state above threshold:

- coherent signal/idler beams
- U(1) symmetry spontaneously broken
- soft Goldstone mode $\omega_G(k) \to 0$ for $k \to 0$
- corresponds to slow signal-idler phase rotation

  → as Bogoliubov phonon at equilibrium !!!



Fundamental physical difference:

→ Goldstone mode diffusive,

not propagating like sound

M. Wouters, IC, *The Goldstone mode of planar optical parametric oscillators*, Phys. Rev. A 76, 043807 (2007)

Simultaneously to our work:

M. H. Szymanska, J. Keeling, P. B. Littlewood, *Nonequilibrium Quantum Condensation in an Incoherently Pumped Dissipative System*, PRL 96, 230602 (2006)

Calculate Goldstone mode dispersion under non-resonant pumping

also in this case: **diffusive Goldstone mode** !!

- Is this a general result of non-equilibrium systems ?
- Simple physical interpretation ?