Quantum correlations of density fluctuations : from the radius of stars to the analog Hawking radiation

1) **Question** : How to measure the angular radius of Syrius ?

$R = 1{,}68\ R_\odot$ (solar radius $R_\odot = 6{,}95 \cdot 10^8\ m$)

$d = 5{,}24\ ly$ (1 light year $ly = 9{,}46 \cdot 10^{15}\ m$)

$$\Theta = \frac{2R}{d} = 4{,}7 \cdot 10^{-8}\ \text{rad} = 2{,}7 \cdot 10^{-6}\ \text{degrees}$$

= angular size of a person (2m) at distance of $4 \cdot 10^7\ m = 40000$ km ($\approx$ earth circumference)

Naive idea of measuring angular radius on telescope suffers of diffraction broadening.

collection device of radius R_D

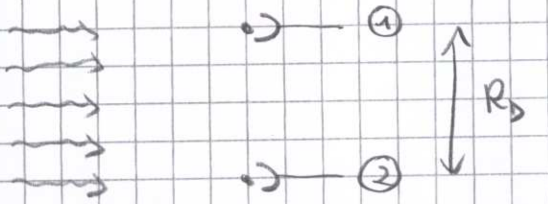
plane wave diffracted into cone of angular aperture $\theta_D \approx \lambda / R_D$

to have $\theta_D \leq \Theta$, one requires $R_D \gtrsim \lambda/\Theta$

for visible light $\lambda = 600$ nm, $R_D \gtrsim 13$ m to resolve angular radius of Syrius.

$\rightarrow$ Requires position stability of λ at distances R_D $\longrightarrow$ HARD!

Intensity correlations



Two photodetectors at distance R_D

Average photodetector signal:

$$I_{1,2} \propto \langle E^{+}(r_{1,2})\, E(r_{1,2}) \rangle .$$

What is noise on $I_{1,2}$?

Are noises at $r_{1,2}$ correlated?

$$g^{(2)}(r_1, r_2) = \frac{1}{I_1 \cdot I_2} \cdot \langle E^{+}(r_1)\, E^{+}(r_2)\, E(r_2)\, E(r_1) \rangle$$

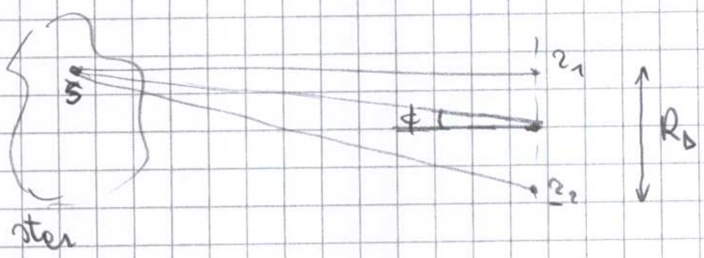
correlation signal: probability of simultaneously detecting photons at r_1 and r_2
"Glauber coherence function"

By Wick Theorem (assuming star = incoherent light source)

$$C = \frac{1}{I_1 \cdot I_2} \cdot \Big[\langle E^{+}(r_1)\, E(r_1) \rangle \langle E^{+}(r_2)\, E(r_2) \rangle + \cancelto{0}{\langle E^{+}(r_1)\, E^{+}(r_2) \rangle} \cancelto{0}{\langle E(r_2)\, E(r_1) \rangle} + \langle E^{+}(r_1)\, E(r_2) \rangle \langle E^{+}(r_2)\, E(r_1) \rangle \Big] =$$

$$= 1 + \frac{1}{I_1 \cdot I_2} \left| \langle E^{+}(r_1)\, E(r_2) \rangle \right|^2$$

Electric field at r results from all points on surface of star:



$$E(r) = \int d^2s \; A(s) \; e^{i\frac{\omega}{c}|r-s|}$$

As $|r_1 - r_2| \ll R$, $\quad \frac{\omega}{c}|r_2 - s| - \frac{\omega}{c}|r_1 - s| \simeq R_D \cdot \phi(s) \frac{\omega}{c}$

$$\langle E^+(r_1) E(r_2)\rangle = \int d^2s \int d^2s' \; A^+(s) A(s') \cdot e^{i\frac{\omega}{c}(|r_1-s| - |r_2-s'|)}$$

$$\simeq \int d^2s \; J(s) \; e^{i\frac{\omega}{c}\left[|r_1-s| - |r_2-s|\right]} \simeq$$

$\hookrightarrow$ star emission is spatially incoherent

$$\simeq \int d^2s \; J(s) \cdot \exp\left[i \frac{\omega}{c} R_D \phi(s)\right].$$

* if $\frac{\omega}{c} R_D \phi(s) \ll 1$ for $\forall s \longrightarrow \langle E^+(r_1) E(r_2)\rangle \simeq I_{1,2}$

* if $\frac{\omega}{c} R_D \phi(s) \gtrsim 2\pi \longrightarrow \langle E^+(r_1) E(r_2)\rangle$ drops to 0.

Crossover at $R_D \simeq \frac{\lambda}{2\pi \phi(s)} \longrightarrow$ similar to diffraction angle

<u>But</u>: no need for precise stabilisation of detectors!

Experiment:

Measure $g^{(2)}(r_1, r_2)$ for different $|r_1 - r_2| = R_D$.

From characteristic R_D at which $g^{(2)} \to 0$, one can extract the value of ϕ, angular radius of the star

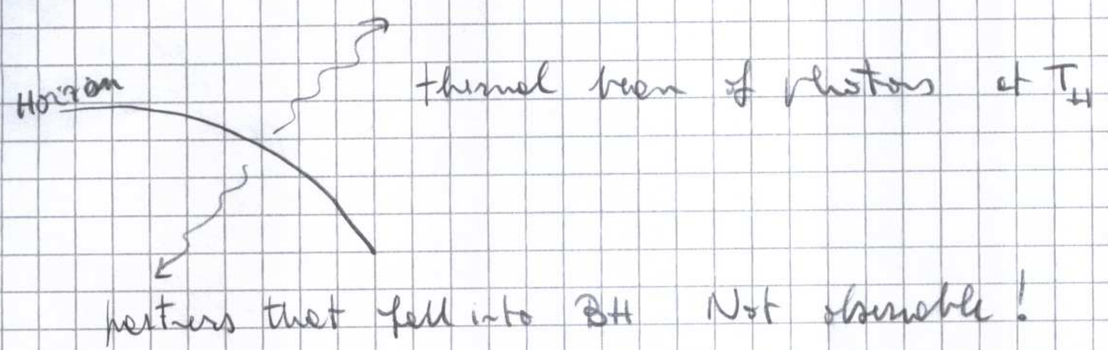
exercise $\longrightarrow$ how to extract the R ?

Ref: Hanbury-Brown and Twiss, Nature 178, 1046 (1956).

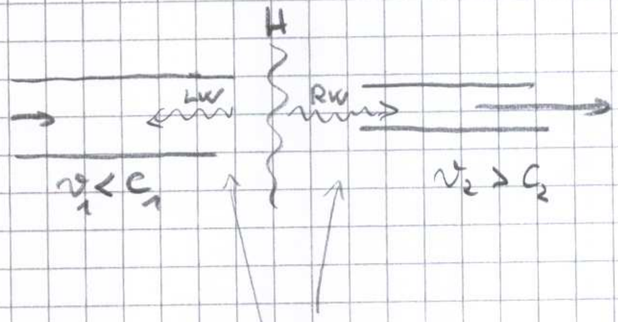
Other technique (normally used): surface brilliance $\approx$ black body, distance known from parallax, total observed luminosity $\longrightarrow$ radius.

2) Correlations in Hawking radiation

2-a) Astrophysical BH's



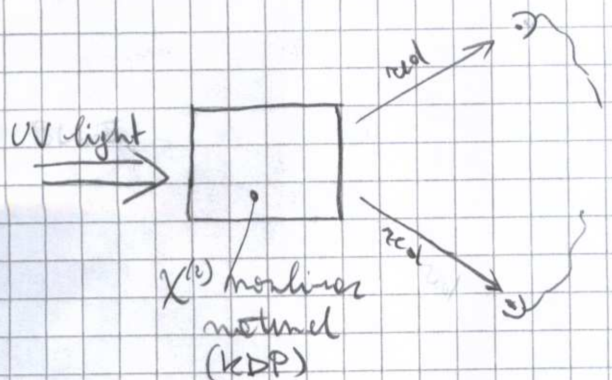
2-b) BEC-based analog models



pairs of phonons emitted from horizon

* when only look at LW phonon $\longrightarrow T_H$
* (quantum) correlations between emission times of RW/LW phonons

Same phenomenon as in parametric down-conversion:



Arrival times strongly correlated [a]

Detection of photons on arm 1 project state onto single photon state on 2 [b]

Ref: Friberg, Hong, Mandel PRL 54, 2011 (85) (a)

Rarity, Tapster, Jakeman, Opt. Comm 62, 201 (87) (b)

3) How to detect phonons in a BEC ?

Bogoliubov theory : BEC + weak fluctuations

$$\hat{\psi}(r) = \phi_0 + \int \frac{d^3k}{(2\pi)^3} \left[U_k e^{ikr} \hat{a}_k + V_k e^{-ikr} \hat{a}_k^\dagger \right]$$

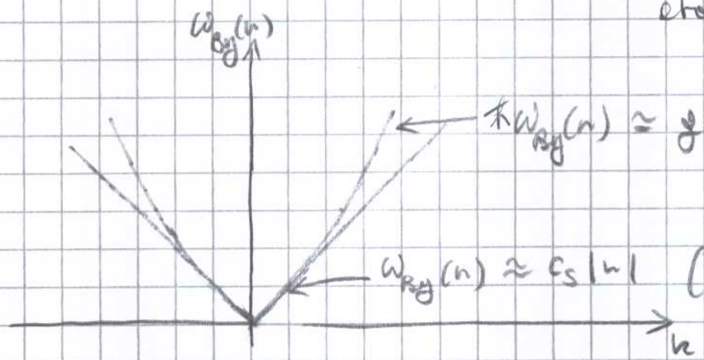
BEC of density $|\phi_0|^2$

$\hat{a}_k, \hat{a}_k^\dagger$ family of bosonic field operators describing elementary excitations on top of ground state

$$\hat{H} = E_0 + \int \frac{d^3k}{(2\pi)^3} \hbar\omega_{Bog}(k) \; \hat{a}_k^\dagger \hat{a}_k$$

in BEC at rest $\hbar\omega_{Bog}(k) = \sqrt{\frac{\hbar^2k^2}{2m}\left(\frac{\hbar^2k^2}{2m} + 2g|\phi_0|^2\right)}$

atom-atom interactions.



$\hbar\omega_{Bog}(k) \simeq g|\phi_0|^2 + \frac{\hbar^2k^2}{2m}$ (single particle)

$\omega_{Bog}(k) \simeq c_S |k|$ (phonons).

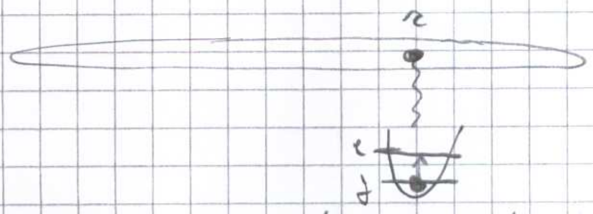
$$\hat{n}(r) = n_0 + \sqrt{n_0} \int \frac{d^3k}{(2\pi)^3} (U_k + V_k) \left[e^{ikr} \hat{a}_k + e^{-ikr} \hat{a}_k^\dagger \right]$$

where $U_k \pm V_k = \left(\frac{\frac{\hbar^2k^2}{2m}}{\omega_{Bog}(k)} \right)^{\pm 1/2}$ "Bogoliubov coefficients"

Note similarity with $E_\perp(r)$ field in QED :
(within Coulomb gauge)

$$E_\perp(r) = \int \frac{d^3k}{(2\pi)^3} \sum_\varepsilon i\sqrt{2\pi\hbar c k}\left[\hat{\varepsilon}\, e^{ikr}\,\hat{a}_\varepsilon(k) - \hat{\varepsilon}^*\, e^{-ikr}\,\hat{a}^\dagger_\varepsilon(k)\right]$$

- Phono-detector :



Atomic quantum dot coupled to density fluctuations
at position r

AQD "clicks" $g \to e$ while absorbing a
phonon of energy $\hbar\omega_{eg} = E_e - E_g$
from fluid.

Ref. Fedichev, Fischer PRL 91, 240407 (2003)

→ As standard photo-detector, it is sensitive to
positive frequency part only of density fluctuations

$$\hat{\delta n}(r) = \hat{\delta n}^{(+)}(r) + \hat{\delta n}^{(-)}(r)$$

involves $\hat{a}(k)$ involves $\hat{a}^\dagger(k)$

- Image of density (e.g. absorption imaging or single-atom detection, ...)

  → directly measures $n(r)$ including both $n^\pm(r)$

  → instantaneous measurement, not frequency selective

  → includes zero-point density fluctuations of BEC

Ref: Fölling, Gerbier, Widera, Mandel, Gericke, Bloch, Nature 434, 481 (2005) → fig. 2

↳ very noisy image of $n(r)$, apparently structureless

($\langle n(r) \rangle$ could be extracted by repeating expt. many times)

↳ interesting information in $\langle n(r)\, n(r') \rangle$

Noise is the interesting quantity!

Our idea → extract info on $\langle a_{Rw}\, a_{Lw} \rangle$ from

$\langle \delta n(r)\, \delta n(r') \rangle$ with r, r' on opposite sides of horizon

$$\langle \delta n(r)\, \delta n(r') \rangle = \# \langle (a_{Rw} + a^\dagger_{Rw})(a_{Lw} + a^\dagger_{Lw}) \rangle =$$

$$= \ldots + \langle a_{Rw}\, a_{Lw} \rangle \text{ as required}$$

4) gravitational - style calculation of $\langle \delta n(x)\, \delta n(x') \rangle$

Balbinot, Fabbri, Fagnocchi, Recati, Carusotto PRA 78, 021603 ('08)

Assumptions:

* dilute BEC ($n a^3 \ll 1$, or $n \xi^d \gg 1$),

  weak density fluctuations

* only low-k phononic modes matter,

  e.g. $\omega = c_s |k|$ in co-moving frame

* hard to deal with BH formation

  Boundary conditions to be imposed by hand

  (different kinds of vacua)

* 1+1 dimensional calculation for BH geometry

$\Longrightarrow$ explicit form for $g^{(2)}(x,x')$ with x, x' far from horizon on opposite sides.

$$g^{(2)}(x,x') = 1 - \frac{k^2 \xi_u \xi_d}{16\pi\, c_u c_d} \, \frac{1}{\sqrt{n\xi_u}\,\sqrt{n\xi_d}} \, \frac{c_u c_d}{(c_u - v)(v - c_d)} \cdot \frac{1}{\cosh^2\left[\frac{k}{2}\left(\frac{x}{c_u - v} + \frac{x'}{v - c_d}\right)\right]}$$

with $k = \frac{1}{2v}\frac{d}{dx}(c^2 - v^2)\Big|_H$ = surface gravity

Features:

* peaked on $\frac{|x|}{c_u - v} = \frac{|x'|}{v - c_d}$

with $x \to$ upstream and $x' \to$ downstream

$c_u - v > 0, \quad v - c_d > 0$ (BH conditions)

↳ phonons emitted simultaneously → correl. in arrival times

* signal strongest if $\kappa = \frac{1}{2v} \frac{d}{dx}(c^2 - v^2)\Big|_H \sim \frac{c}{\xi}$

$\Longrightarrow g^{(2)}$ peak $\approx \frac{1}{n\xi} \cdot \#$ (universal form)

with $\# \approx 10^{-2}$ depends on details (speeds, etc.)

* width of tongue $\approx \frac{c_u - v}{\kappa}, \quad \frac{v - c_d}{\kappa}$

inversely proportional to surface gravity

5) Numerical experiments

Carusotto, Fagnocchi, Recati, Balbinot, Fabbri, NJP 10, 103001 (2008)

5.a) Why a full ab initio approach necessary?

* trans-Planckian problem.
  - HR emerges from modes that on horizon are infinitely blue-shifted, i.e. have very large k's.
  - in this regime, HD approx that underlies gravitational-style calculations is no longer valid

* formation in time of horizon
  - boundary conditions automatically imposed during time-evolution
    (no doubt about Boulware / Hartle-Hawking / ...)

* further check of stability of BH.

5-b) <u>Truncated Wigner approach</u> [Sinatra, Lobo, Castin, J Phys B 35, 3599 ('02)]

quantum field $\hat{\psi}(r) \longrightarrow$ replaced by stochastic classical field $\psi(r)$

works for dilute gas, better in low-D where UV problems are less severe

<u>$t=t_0$</u> : sample <u>initial state</u> with a set of random fields $\{\psi_i\}$ with distribution $P_0[\psi]$ chosen to reproduce initial state at temperature T

<u>Bogoliubov approx</u> for homogeneous gas (at $T \ll T_{deg}$) :

$$\psi_0(x) = e^{ik_0x}\left[\sqrt{n_0} + \sum_{k\neq 0} \alpha_k U_k e^{ikx} + \alpha_k^* V_k e^{-ikx}\right]$$

with α_k random gaussian variables such that :

$$\langle \alpha_k \rangle = 0 \quad , \quad \langle \alpha_k^2 \rangle = 0$$

$$\langle |\alpha_k|^2 \rangle = \frac{1}{2\tanh\left(\frac{\hbar\omega_{Bog}(k)}{2k_BT}\right)} =$$

$$\simeq \begin{cases} 1/2 & \text{for } \hbar\omega_{Bog}(k) \gg k_BT \text{ (zero point fluctuations)} \\ \dfrac{k_BT}{\hbar\omega_{Bog}(k)} & \text{for } \hbar\omega_{Bog}(k) \ll k_BT \text{ (thermal fluctuations)} \end{cases}$$

Evolve $\psi(x,t)$ starting from $\psi(x, t=t_0) = \psi_0(x)$

according to eq. formally identical to GPE:

$$i\hbar\,\partial_t \psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x,t)\,\psi + g(x,t)\,|\psi|^2\psi$$

At each time t, stochastic averages of ψ's give

expectation value of symmetrically-ordered observables, e.g.

$$\langle \psi^*(r)\,\psi(r')\rangle_{stoch} = \frac{1}{2}\left[\langle \psi^+(r)\,\psi(r')\rangle_q + \langle \psi(r')\,\psi^+(r)\rangle_q\right] =$$

$$= \langle \psi^+(r)\,\psi(r')\rangle_q + \frac{1}{2}\,\delta(r-r')$$

and so on.

$$\langle n(r)\,n(r')\rangle = \langle\, |\psi(r)|^2\, |\psi(r')|^2 \,\rangle_{stoch} + \text{corrections}.$$

Physical idea: stochastic noise "mimics" quantum noise

Among the family of quasi-classical distribution functions, Wigner has the good level of noise to best reproduce quantum physics.

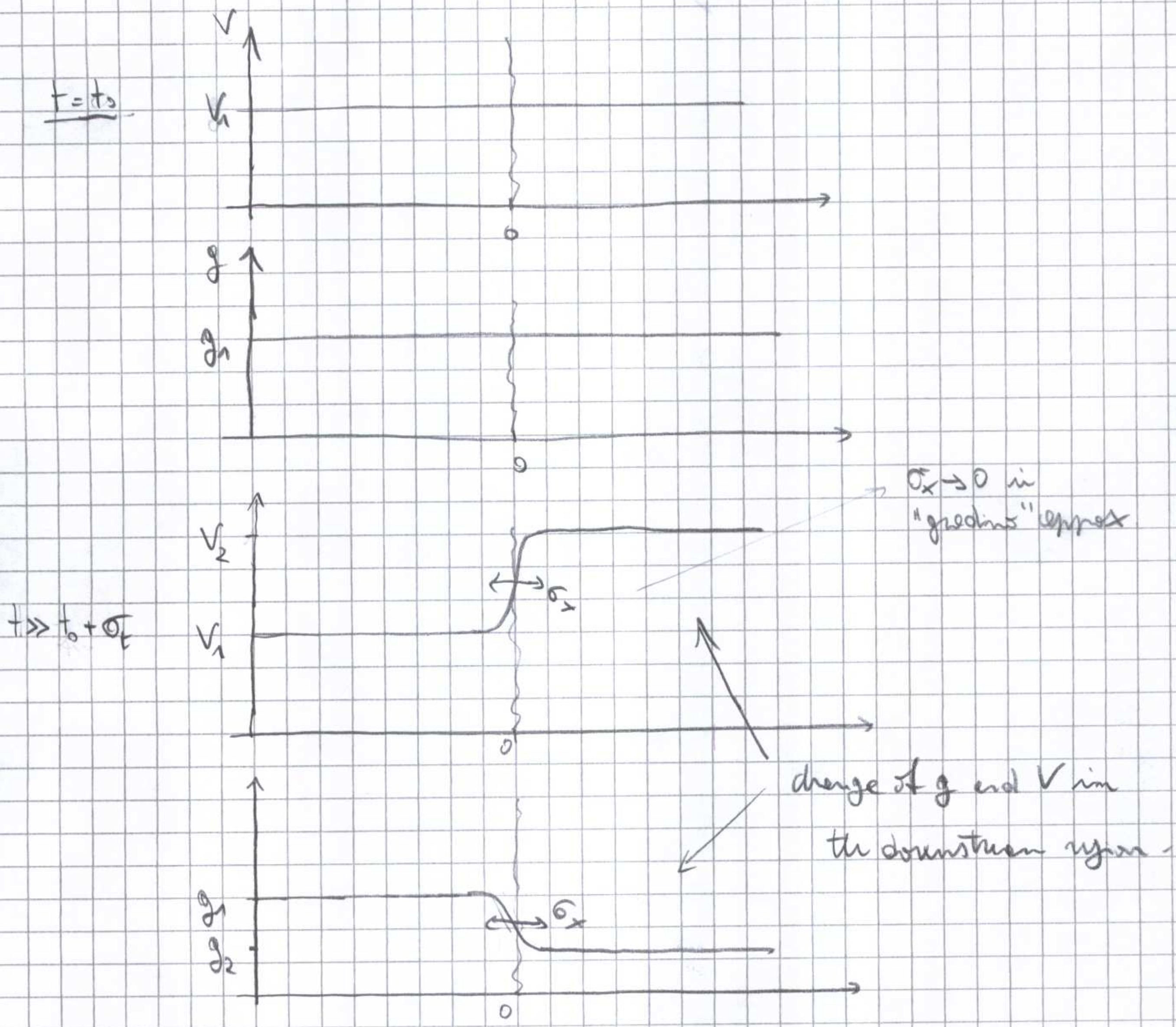
Exact, but computationally heavier schemes in

Carusotto, Castin, Dalibard PRA 63, 23606 (2001)

5-c) What configuration to consider?

- initial state : homogeneous gas at low $T \ll T_{deg}$, moving at v
  $\longrightarrow$ Bogoliubov sampling

- BH formed by ramping $V(x,t)$, $g(x,t)$



keeping $V(x,t) + n_0\, g(x,t) = \text{constant} \ \forall x, \forall t$

$c(x,t) = \sqrt{\dfrac{n_0\, g(x,t)}{m}}$ such that $c_2 < v < c_1$

Choice of keeping $V + n_0 g = c^{te}$ allows for trivial solution of GPE at all times:

$$\psi(x,t) = \sqrt{n_0}\; e^{i k_0 x}\; e^{-i\omega_0 t}$$

↳ Eliminates non-trivial BEC dynamics (soliton shedding, Cerenkov, ...)

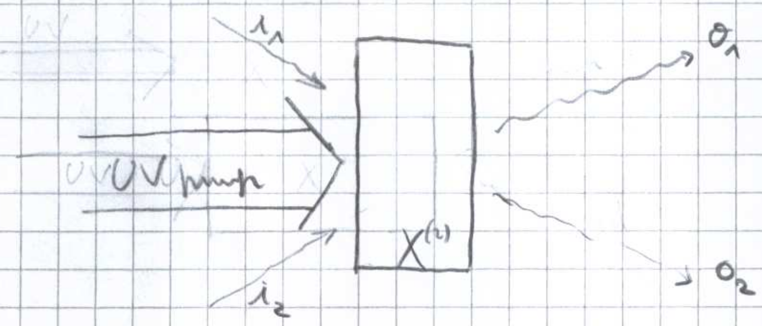
↳ What is left comes from fluctuations only

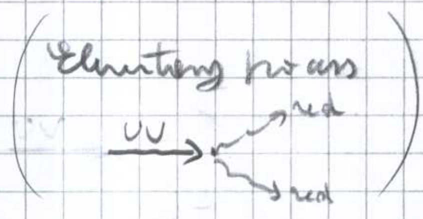
↳ Other configurations possible, but numerically much more demanding

5-d) <u>numerical data</u>

5-e) <u>Explanation of additional features</u>

<u>Parametric down-conversion model:</u>





$\chi^{(2)}$ that converts incident quantum fluctuations in $i_{1,2}$ into real radiation (photons) in $o_{1,2}$

$$\begin{pmatrix} \hat{a}_{o_1} \\ \hat{a}^{\dagger}_{o_2} \end{pmatrix} = \begin{pmatrix} \cosh\beta & \sinh\beta \\ \sinh\beta & \cosh\beta \end{pmatrix} \begin{pmatrix} \hat{a}_{i_1} \\ \hat{a}^{\dagger}_{i_2} \end{pmatrix}$$ input-output formalism of quantum optics

β depends on UV pump amplitude.

$$\langle a^\dagger_{o_1} a_{o_1} \rangle = \langle (\cosh\beta\, a^\dagger_{i_1} + \sinh\beta\, a_{i_2}) \cdot$$

$$\cdot (\cosh\beta\, a_{i_1} + \sinh\beta\, a^\dagger_{i_2}) \rangle =$$

for incoming vacuum $\langle a^\dagger_{i_1} a_{i_1} \rangle = \langle a^\dagger_{i_1} a^\dagger_{i_2} \rangle = 0$

$$= \sinh^2\beta \, \langle a_{i_2} a^\dagger_{i_2} \rangle = \sinh^2\beta > 0 .$$

$\longrightarrow$ parametric luminescence.

$$\langle a_{o_1} a_{o_2} \rangle = \langle (\cosh\beta\, a_{i_1} + \sinh\beta\, a^\dagger_{i_2}) (\cosh\beta\, a_{i_2} + \sinh\beta\, a^\dagger_{i_1}) \rangle$$

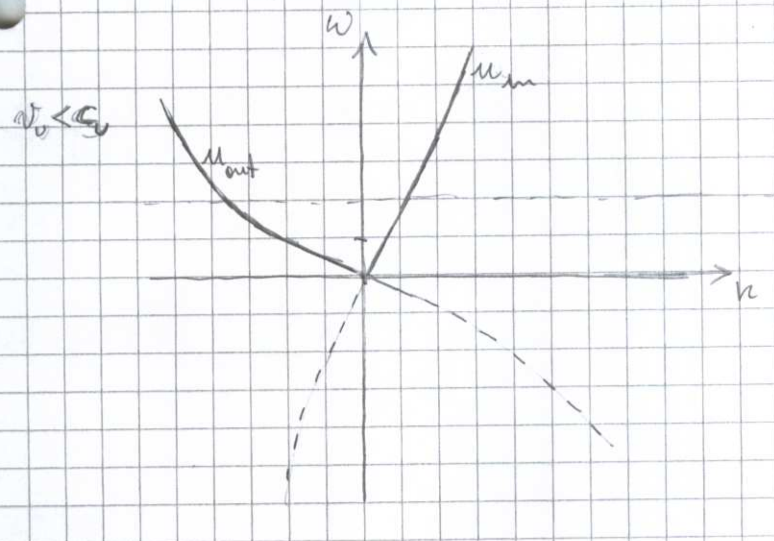
$$= \cosh\beta \sinh\beta$$

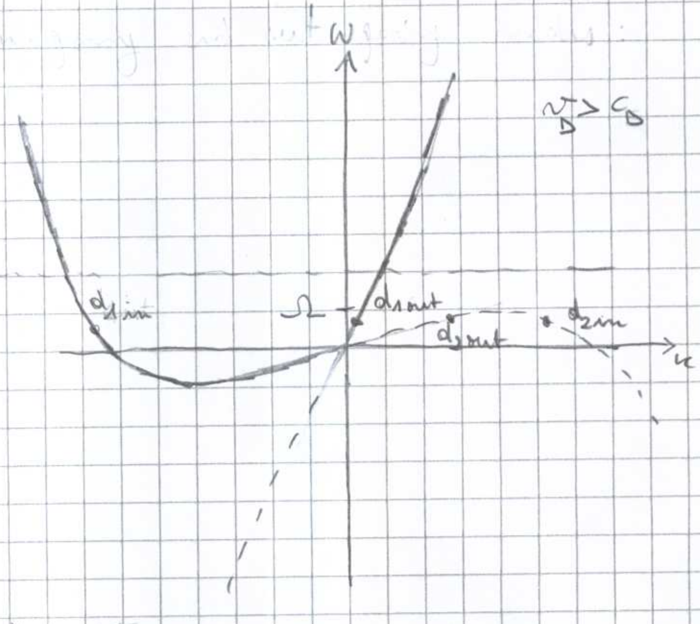
$\longrightarrow$ non-trivial correlations in the output from different ports.

More complete model including time/frequency shows that correlations are limited to photons <u>emitted</u> simultaneously:

$$\langle E_{o_1}(x_1, t_1)\, E_{o_2}(x_2, t_2) \rangle = f\left[(x_1 - ct_1) - (x_2 - ct_2) \right]$$

BH configuration :





→ system invariant under time-translations → ω conserved

→ input-output connects operators evolving at a given ω

→ { dashed line is dispersion of $a^\dagger$ operators,
whole line is dispersion of a operators.

→ in- or out-going character of modes is determined by the group velocity of mode, $v_g = \frac{d\omega}{dk}$.

Ref. Recati, Pavloff, Carusotto PRA 80, 043603 ('09)

two cases {
(i) $0 < \omega < \Omega$: $(u_{in}, d_{1in}, d_{2in}) \longrightarrow (u_{out}, d_{1out}, d_{2out})$
(ii) $\Omega < \omega$: $(u_{in}, d_{1in}) \longrightarrow (u_{out}, d_{1out})$
$\omega < 0$ gives hermitian conjugate eqs.

(ii)

$$\begin{pmatrix} \hat{a}_{u\text{-}out}(\omega) \\ \hat{a}_{d1\,out}(\omega) \\ \hat{a}^{+}_{d2\,out}(\omega) \end{pmatrix} = S(\omega) \begin{pmatrix} \hat{a}_{u\text{-}in}(\omega) \\ \hat{a}_{d1\,in}(\omega) \\ \hat{a}^{+}_{d2\,in}(\omega) \end{pmatrix}$$

$\hat{a}_{u\text{-}in}(\omega)$ etc. are short-hands for

$\hat{a}_{u\text{-}in}(k)$ with $\omega = \omega_{Bog}(k)$ for the u-in branch.

Introduction to input-output theory in:

Ciuti, Carusotto, PRA 74, 033811 ('06)

Walls, Milburn "Quantum Optics"

Gardiner, Zoller "Quantum Noise"

Hawking radiation:

$$\langle a^{+}_{u\text{-}out}(\omega)\, a_{u\text{-}out}(\omega) \rangle = \left| S_{u,d_2}(\omega) \right|^2 \cdot \left[1 + \bar{n}^{o}_{d_2}(\omega) \right]$$

thermal behaviour of HR $\Longrightarrow$ $\left| S_{u,d_2}(\omega) \right|^2 \sim 1/\omega$ at low ω.

analytically verified in the "gradient approx"

complete calculation by Parentani and coworkers.

Correlations:

$$\langle a_{u\text{-}out}\, a_{d2\text{-}out} \rangle = S_{ud2}\, S^{*}_{d2d2} \left[1 + \bar{n}^{o}_{d_2}(\omega) \right]$$

but also between other pairs of modes, namely (ud_1), $(d_1 d_2)$ ...

(ii) $\begin{pmatrix} a_{u\text{-}out} \\ a_{d1\text{-}out} \end{pmatrix} = S(\omega) \begin{pmatrix} a_{u\text{-}in} \\ a_{d1\text{-}in} \end{pmatrix}$

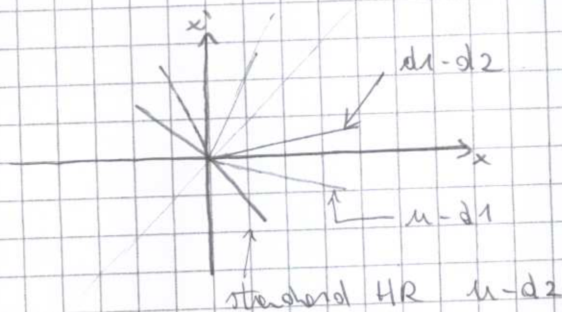
↳ no mixing of a, a^{+} operators.

↳ all normally-ordered observables have zero expectation value starting from vacuum

⟹ no HR for $\omega > \Omega$.

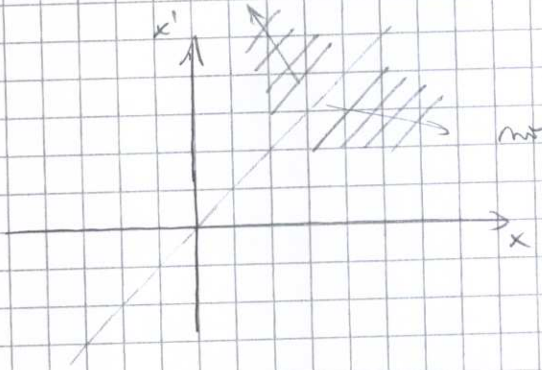
To interpret features in correlation diagram:

* emission occurs on different pairs of modes simultaneous
* they travel at group velocity.



* outgoing modes have small k
→ tongue-shaped features
* Stationary or steady-state

One more feature is transient after switch-on horizon:



generated at $t_0 = 0$.
move out-ward at $v \simeq 2c_d$
feature depends on $|x - x'|$.

Explanation: at $t=0$, $g = g_1 \to g_2$ in downstream region.

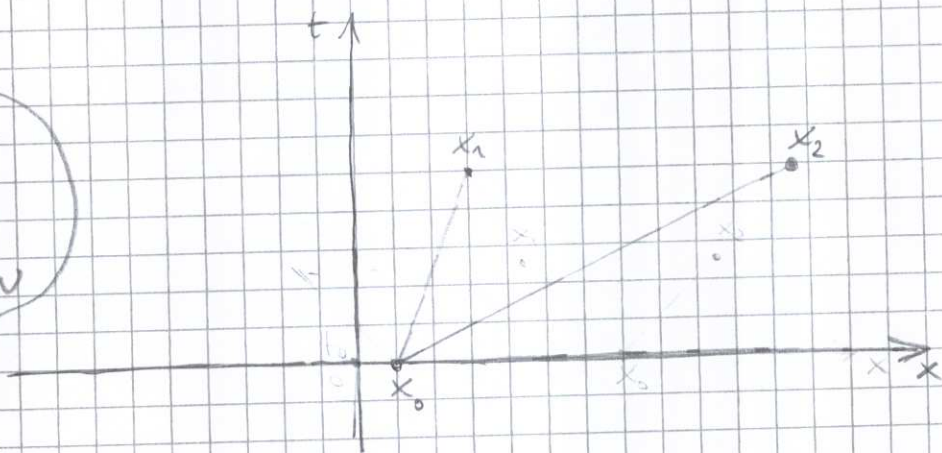
emission of pairs $\pm k$ of phonons at each spatial position

travel opposite direction at $v = v_0 \pm c_d$

DYNAMICAL CASIMIR EFFECT

or

COSMOLOGICAL PARTICLE PRODUCTION



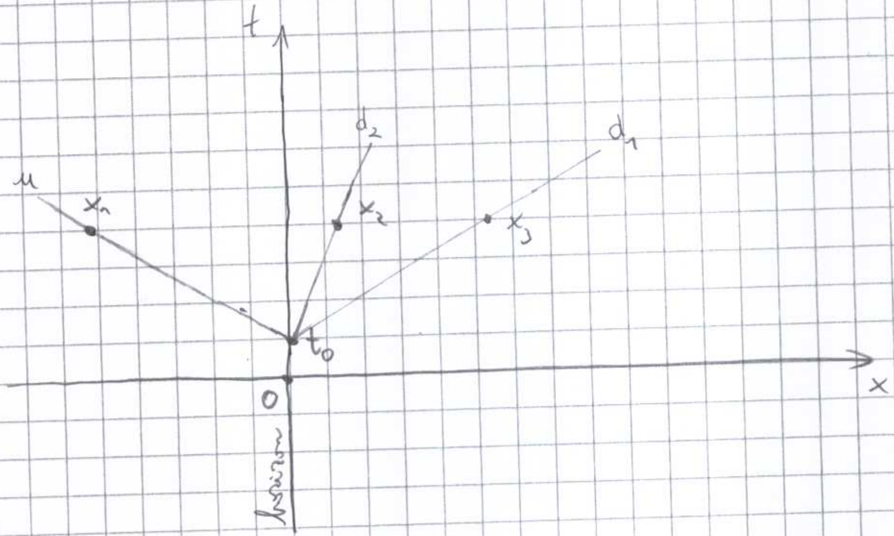
$\longrightarrow$ at each time t, density fluctuations are correlated if x_1, x_2 are reached by trajectories of correlated phonons emitted same location x_0 at switch time

Ref Carusotto, Balbinot, Fabbri, Recati, EPJD 56, 391 (10)

$$(x_1 - x_0) = (v - c_d)\,t \quad , \quad (x_2 - x_0) = (v + c_d)\,t$$

$$\Longrightarrow \quad (x_1 - x_2) = \left[(v - c_d) - (v + c_d)\right] t = 2c_d t .$$

Similar diagram for HR:



correlations if

$$x_1 = (v - c_u)(t - t_0)$$

$$x_2 = (v - c_d)(t - t_0)$$

$$x_3 = (v + c_d)(t - t_0)$$

$$\Longrightarrow \quad \frac{x_1}{x_2} = \frac{v - c_u}{v - c_d}. \qquad \text{etc.}$$

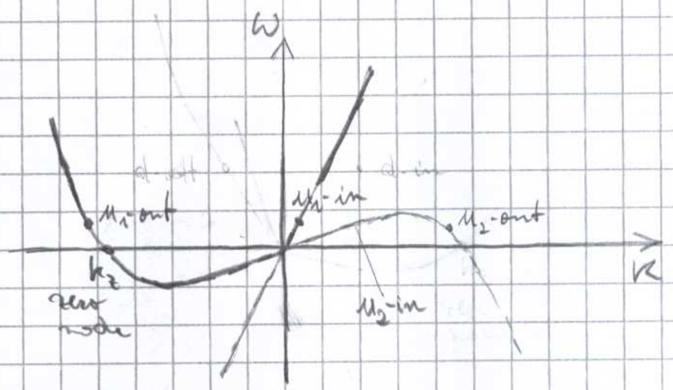
White holes

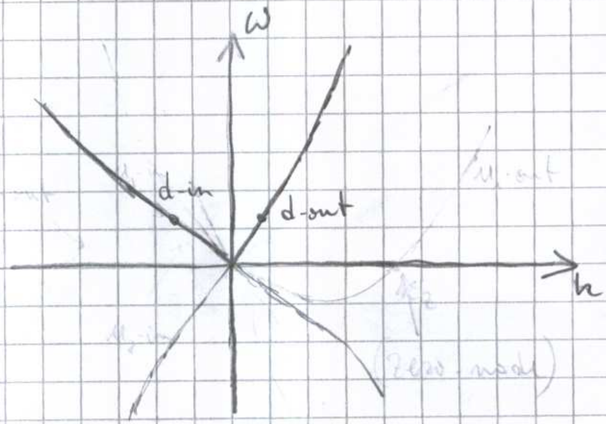
Similar configuration, reversed flow: Same condition $V_u + g_u \eta = V_d + g_d \eta$

$v > c_u$ } $v < c_u$

Super } sub

$\longrightarrow$ no phonon perturbation can penetrate Super-sonic region





As in BH: 3×3 input-output matrix (for $\omega < \Omega$).

$$\begin{pmatrix} Q_{d\text{-out}} \\ Q_{u1\text{-out}} \\ Q^{+}_{u2\text{-out}} \end{pmatrix} = S(\omega) \begin{pmatrix} Q_{d\text{-in}} \\ Q_{u1\text{-in}} \\ Q^{+}_{u2\text{-in}} \end{pmatrix}$$

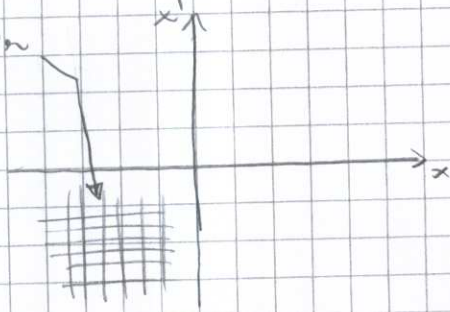
* exchanged roles of u-d regions and of in-out modes

$\Rightarrow S_{BH}(\omega) = \eta\, S^{T}_{WH}(\omega)\, \eta$

* naively one expects same physics but...

- correlation features at <u>high k</u> appear and have growing amplitude in time

$u_1 - u_2$ correlation
"checkerboard"



- d – $u_{1,2}$ correlations have <u>spatial modulation</u> and intuitive:
  ↳ $k_{u_{1,2}}(\omega \to 0) = k_z$.
  $v_{g,u_{1,2}}(\omega \to 0) = v_{g,z}$

- analytical calculation for $\langle a^{\dagger}_{d\text{-}out}(\omega)\, a_{d\text{-}out}(\omega) \rangle$ within the "gradino approx" gives a <u>constant (ω)</u> instead of $1/\omega$ thermal law of HR in BH's

- however $\langle a^{\dagger}_{u_{1,2}}\, a_{u_{1,2}} \rangle$ go as <u>$1/\omega$</u> as in Cono theory? / Weinfurtner? exp.

- crucial role of the $V_u + n g_u = V_d + n g_d$ condition in neutralizing Bogoliubov – Cerenkov emission of phonons in supersonic flow [Carusotto et al. PRL 97, 260403 ('07)]

  ↳ growth of checkerboard pattern is a remnant

  ↳ dynamical stability shown. Is this robust to more general flow patterns?

What is crucial to observe vacuum radiation effects ?
(i.e. conversion of zero-point fluctuations into observable radiation)

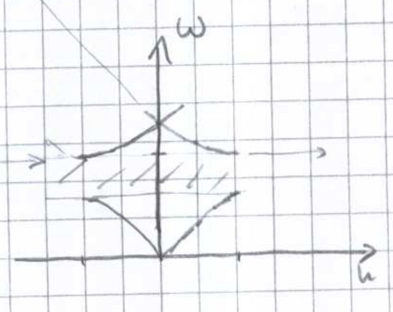
* mixing of a, a^+ modes by S connecting asymptotic flat regions

* existence of negative energy branch of Bogoliubov modes (at least 1 in- and 1-outgoing) in the asymptotic flat regions.

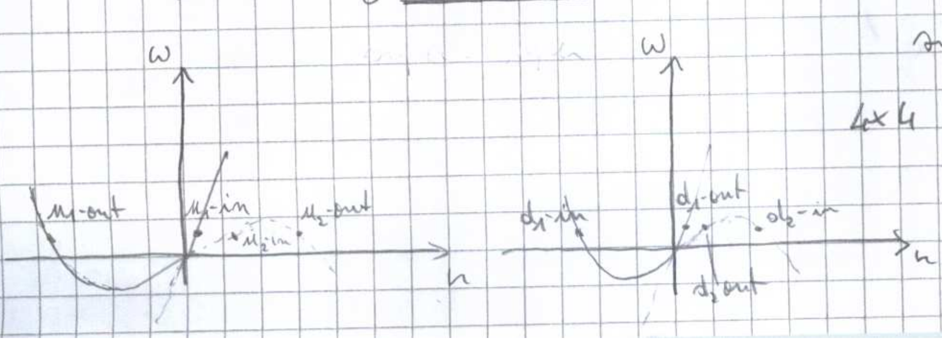
$\Longrightarrow$ no direct role of group velocity nor of group velocity horizon (may serve to reinforce matrix element)

example: spatially modulated DBR

$\to$ A-in mode totally reflected at group-velocity horizon where $\omega = \min_k \omega_c(k)$



$\Longrightarrow$ actually no role of horizon : vacuum radiation at super-super interface.
4x4 input-output matrix.
Ref: Finazzi-Parentani, 1102.1452



Addenda

Preliminary results for gradino approx (Mayoral) :

$$\begin{cases} S_{d_{1,2},\, d_{1,2}} \sim \sqrt{\omega} \\ S_{u_{1,2},\, d_{1,2}} \sim c^{te} \\ S_{d_{1,2},\, u_{1,2}} \sim c^{te} \\ S_{u_{1,2}\, u_{1,2}} \sim \sqrt{\omega} \end{cases}$$

Same side $\sim \sqrt{\omega}$ (reflection)

Opposite side $\sim c^{te}$ (transmission)

$\Longrightarrow$ Vacuum radiation exists <u>even in the absence of horizon</u>

but intensity of correlations ($\sim \omega$) much smaller.

<u>NOTE</u> : As comparison :

<u>BH</u> : $S_{i,\, d_{1,2}} \sim 1/\sqrt{\omega}$

$S_{i,\, u} \sim c^{te}$

out in

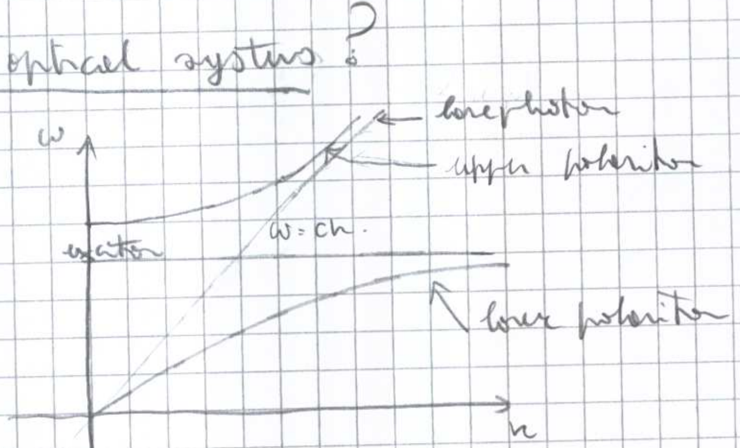
<u>WH</u> $S_{d_1,\, i} \sim c^{te}$

$S_{u_{1,2},\, i} \sim 1/\sqrt{\omega}$

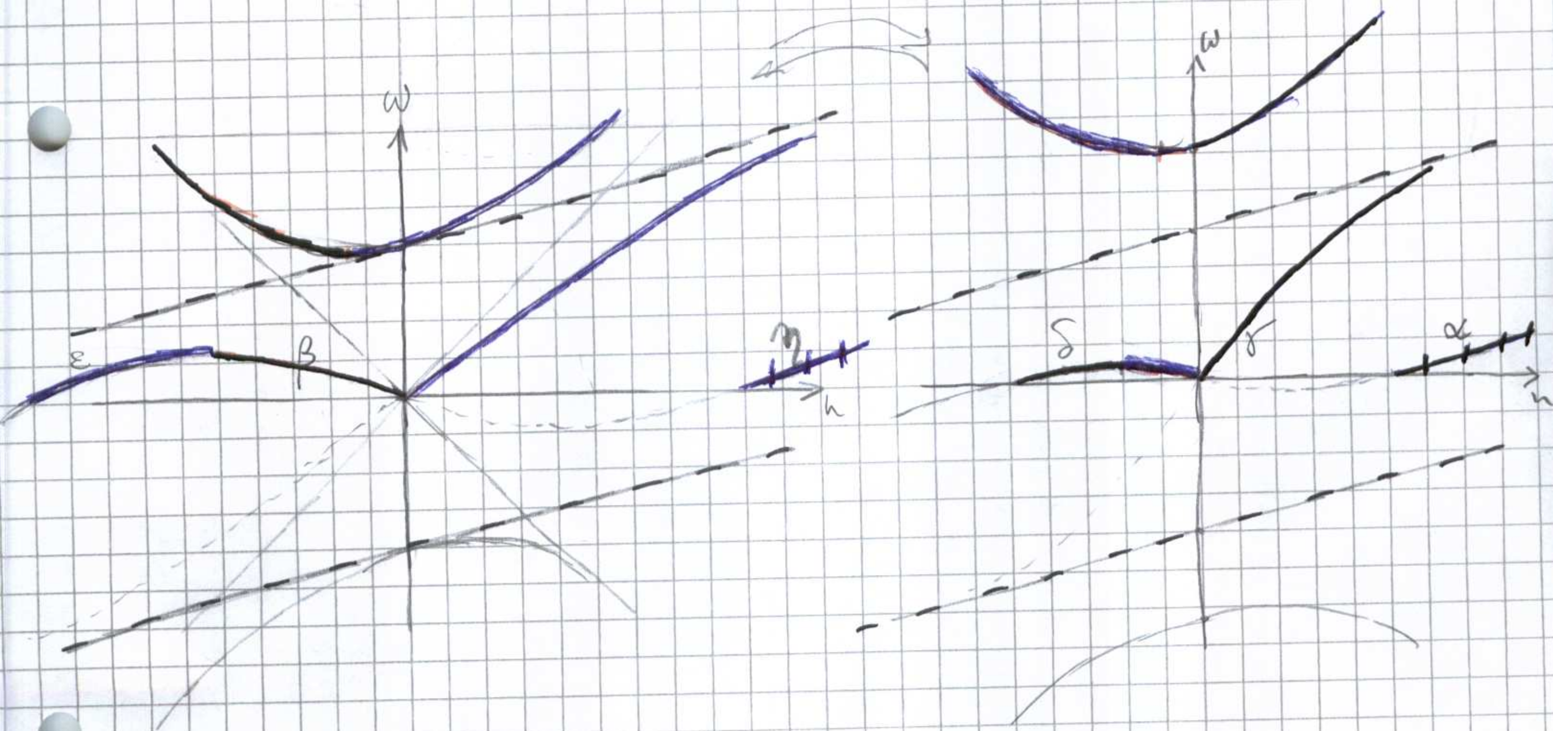
out in

What can be expected in optical systems?

generic optical medium:



(at rest)



In the pulse reference frame. Pulse moving leftwards.

* UP's don't play any role in vacuum radiation processes

* negative-norm (at) modes exist. On RH/LH are out/in-going

* vacuum radiation consists of α-β pairs (on different sides) or α-γ, α-δ on the same side.

Is any of this processes analog HR?

* negative-norm in-going mode η

* does not rely on horizon concept

* the α partner exists at finite k, as well as the δ one

* does not have any apparent wavelength stretching phenomenon.

* group horizon gives reflection of ε branch into β.

* note: physics changes when $v > \frac{c}{n(\omega=0)}$: a true horizon may appear!

Exercise: repeat this analysis for water tank exps.

Work in progress: using Hopfield Hamiltonian, calculate explicit form of $S(\omega)$ for the input-output theory of vacuum radiation from moving pulses.
[Cemmotto, et al. PRA 77, 063621 ('08)]