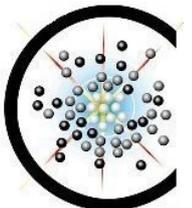


**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich



**BEC**   
BOSE EINSTEIN CONDENSATION

# Spectroscopy of a non-equilibrium Tonks-Girardeau gas of strongly interacting photons

Iacopo Carusotto

*BEC CNR-INFN and Università di Trento, Italy  
Institute of Quantum Electronics, ETH Zürich, Switzerland*

*in collaboration with*

Atac Imamoglu, Hakan Türeci  
*IQE-ETH*

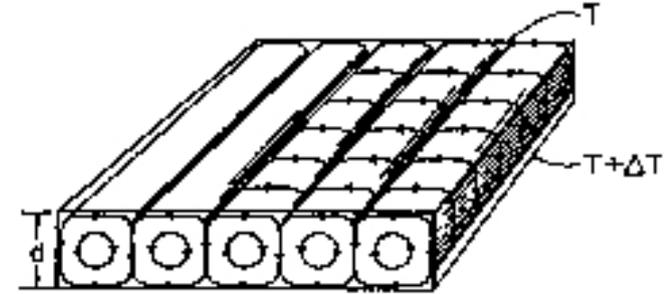
Dario Gerace  
*CNISM and Univ. di Pavia*

Simone De Liberato, Cristiano Ciuti  
*MPQ, Univ. Paris VII*

# The non-equilibrium world

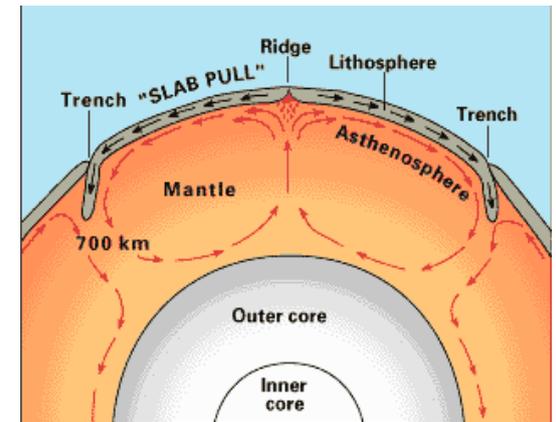
## Thermodynamical equilibrium:

- Boltzmann **probability distribution**  $\exp(-E/k_b T)$
- **fluctuation-dissipation** theorems of linear response theory
- general theorems on **phase transitions**, e.g. in low-D



## Non-equilibrium steady-states:

- **dynamical balance** of driving and dissipation
- no Boltzmann factor, **dynamics** plays major role



## Non-equilibrium phase transitions:

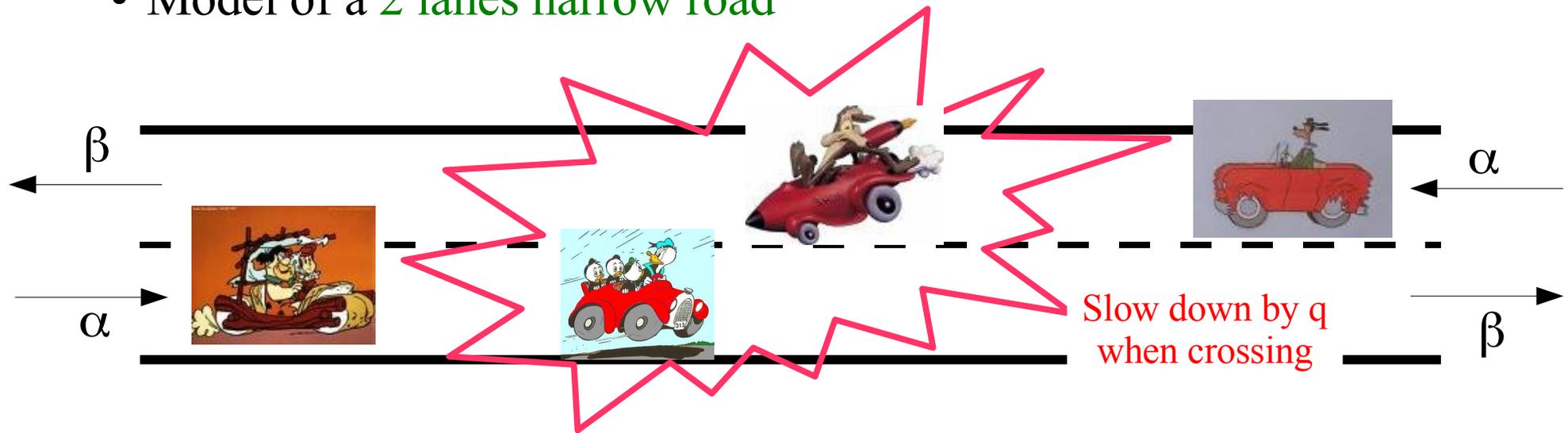
- **Bénard convection cells**: break translational symmetry
- **driven-diffusive lattice gases**: non-equilibrium Ising transition
- **lasing**: break U(1) symmetry, sort of non-equilibrium BEC
- interplay of **quantum physics** with **non-equilibrium** still to be explored

# Non-equilibrium phase transitions in 1D systems

General theorems forbid phase transitions in 1D equilibrium systems

Non-equilibrium world richer:

- Totally asymmetric, two component exclusion process (TASEP-2)
- Model of a 2 lanes narrow road



When traffic is heavy: road turns into effectively one-way

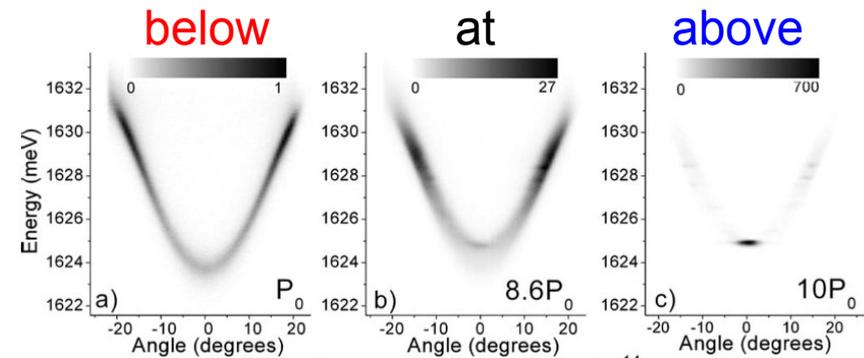
- Spontaneous left/right symmetry breaking
- Robust against fluctuations
- Reversal time grows exponentially with length of road

# Non-equilibrium many-body physics with light

## 1 - Shape of exciton-polariton Bose-Einstein condensates

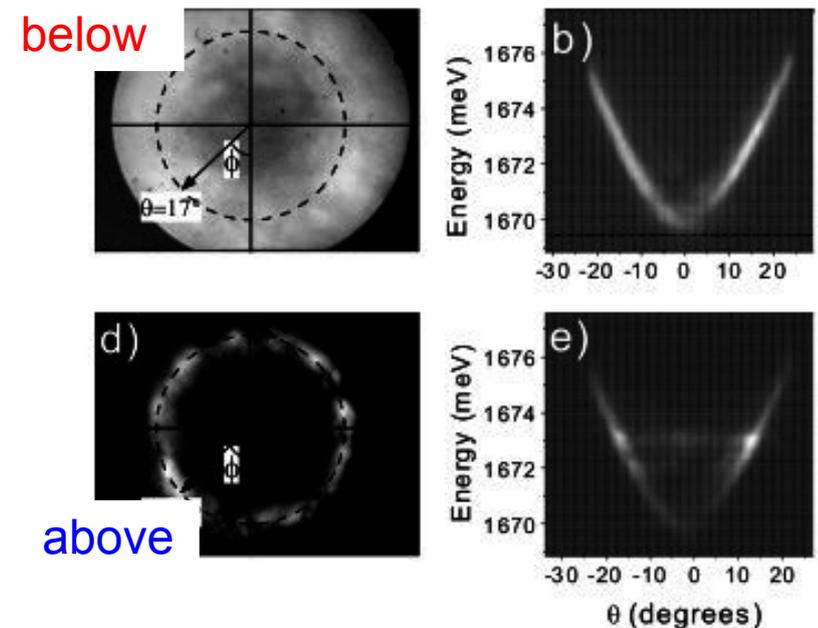
### Large pump spot:

- BEC into  $k=0$



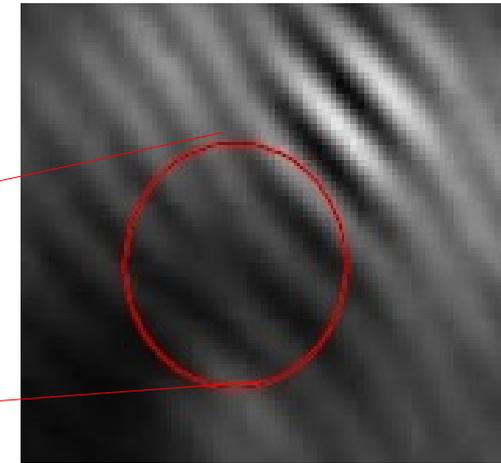
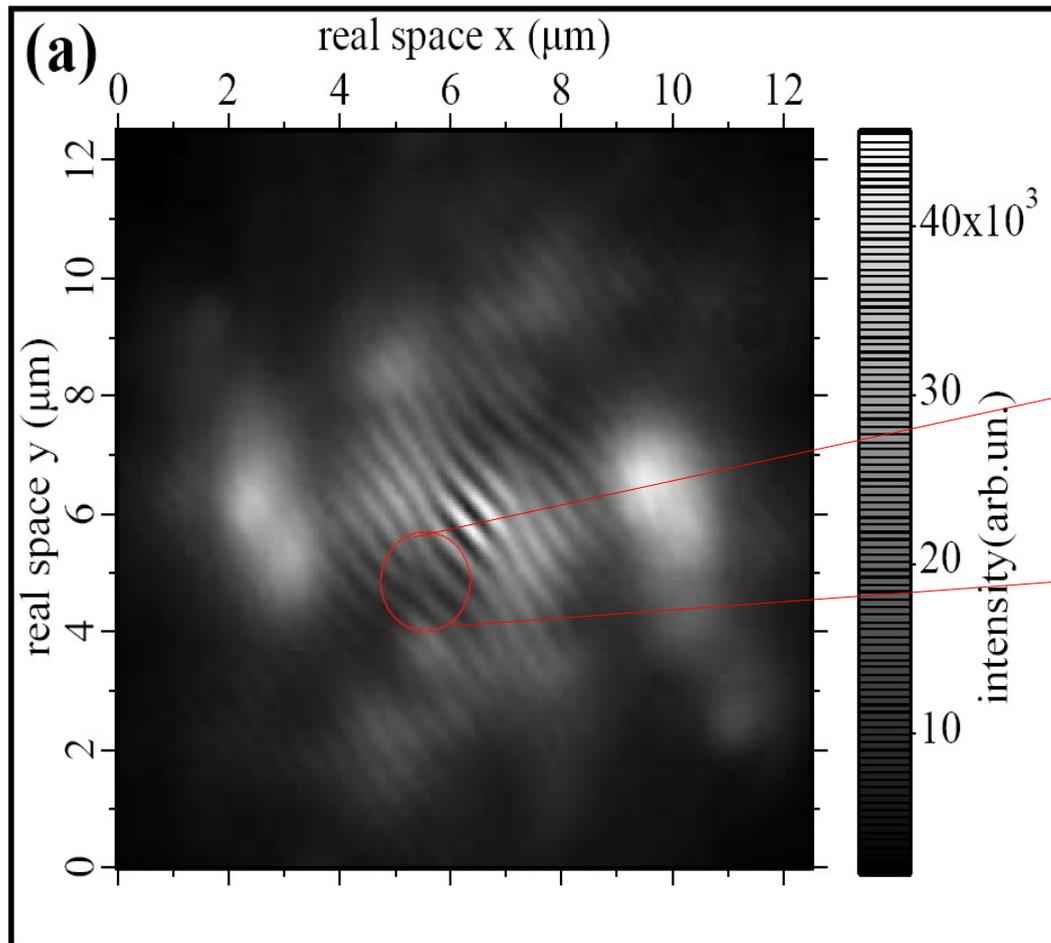
### Small pump spot:

- BEC on ring in  $k$ -space
- radial flow under effect of pump and interactions
- still fully coherent BEC
- close relation with lasing and random lasing



Figures from Grenoble group: M. Richard et al., PRB **72**, 201301 (2005); M. Richard et al., PRL **94**, 187401 (2005)  
Theory: M. Wouters, IC, and C. Ciuti, PRB **77**, 115340 (2008)

## 2 – Appearance of quantized vortices

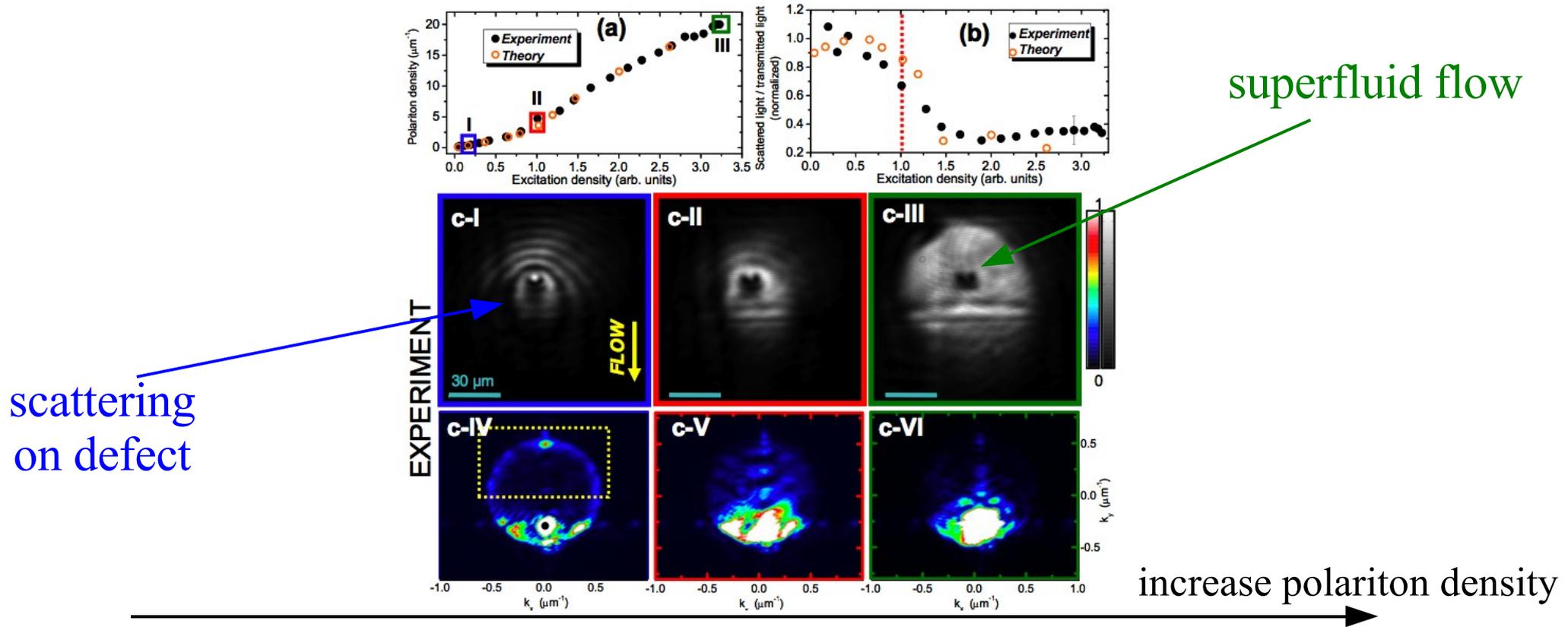


Fork dislocation in interference pattern with a plane wave

K. G. Lagoudakis, M. Wouters, M. Richard, A. Baas, IC, R. André, Le Si Dang, B. Deveaud-Pledran, *Quantised Vortices in an Exciton Polariton Fluid*, Nature Phys. **4**, 706 (2008).

- **phase singularity** in macroscopic wavefunction
- appears with **no need for stirring**
- interplay of **non-equilibrium flow**, **disorder** and **interactions**

### 3 – Polariton superfluidity according to Landau criterion



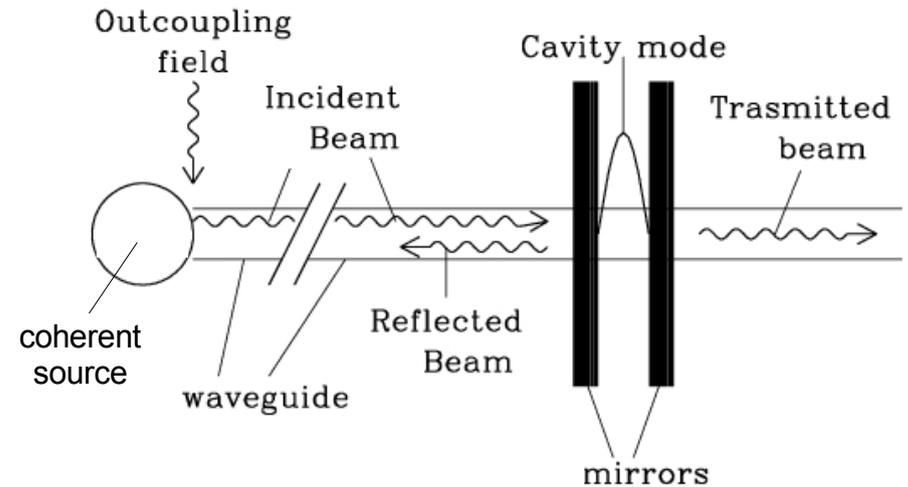
**Figure from LKB-P6 group:** J.Lefrère, A.Amo, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, submitted (2008)  
**Theory:** IC and C. Ciuti, PRL **93**, 166401 (2004).

.... but all these effects are at mean-field level !!

# Beyond mean-field effects

## Single-mode system:

- **coherent pump** provides driving
- **radiative losses** give dissipation
- **nonlinearity** gives interactions.



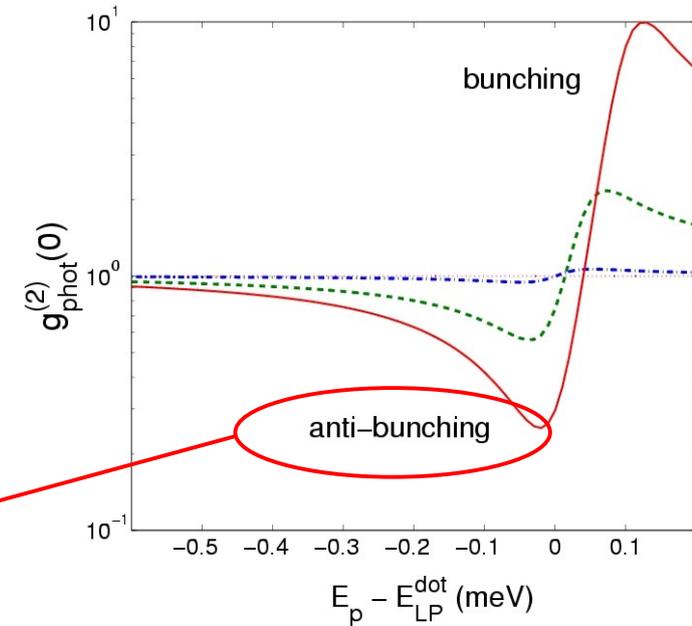
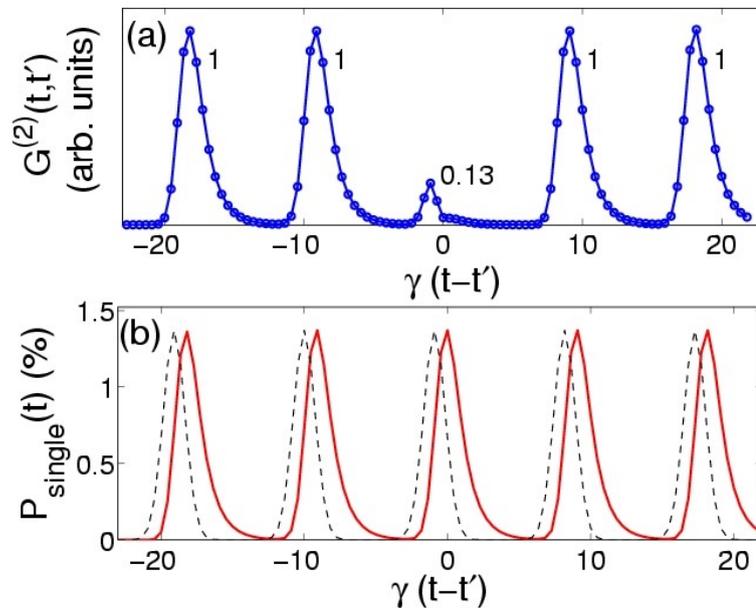
## Quantum features if $U \gg \gamma$

- **red-detuned pump**: negative feedback, **antibunching**  $g^{(2)}(\tau=0) < 1$
- **blue-detuned pump**: positive feedback, **bunching**  $g^{(2)}(\tau=0) > 1$

## Spatially extended, many-mode systems:

- complex **interplay** of **classical** and **quantum dynamics**
- **in-cavity quantum correlations** transfer into **statistics of emission**

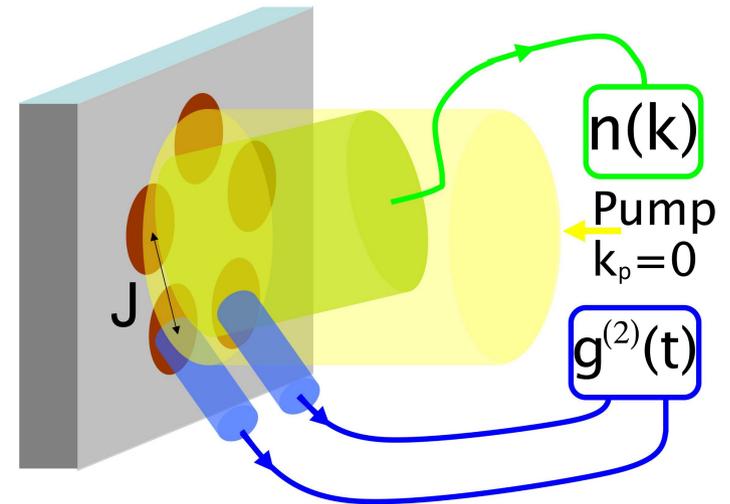
# Bunching vs. anti-bunching



A. Imamoglu, H. Schmidt, G. Woods, M. Deutsch, Phys. Rev. Lett. **79**, 1467 (1997)  
Figures here from: A. Verger, C. Ciuti, IC, Phys. Rev. B **73**, 193306 (2006).

# The many-cavity system: 1D array of coupled cavities

- periodic boundary conditions
- strong on-site repulsion  $U$
- Josephson tunnel coupling  $J$
- external driving  $F_p$ :
  - monochromatic at  $\omega_p$
  - plane wave shape  $k_p=0$



- weak losses  $\gamma \ll J, U$  but always crucial to determine steady-state
- light collected in reflection and/or transmission geometry
- far-field emission:  $k$ -selection from orbital angular momentum
- near-field emission: spatial resolution

# Impenetrable bosons ( $U=\infty$ ): Girardeau's Bose-Fermi mapping

Many-body wavefunction in terms of non-interacting Fermions

$$\psi(x_1, x_2, \dots) = \epsilon[\sigma(x_1, x_2, \dots)] \psi_F(x_1, x_2, \dots)$$

- ✓ density and density fluctuations preserved in Bose-Fermi mapping
- ✗ coherence function  $g^{(l)}(x, x')$  and momentum distribution not preserved

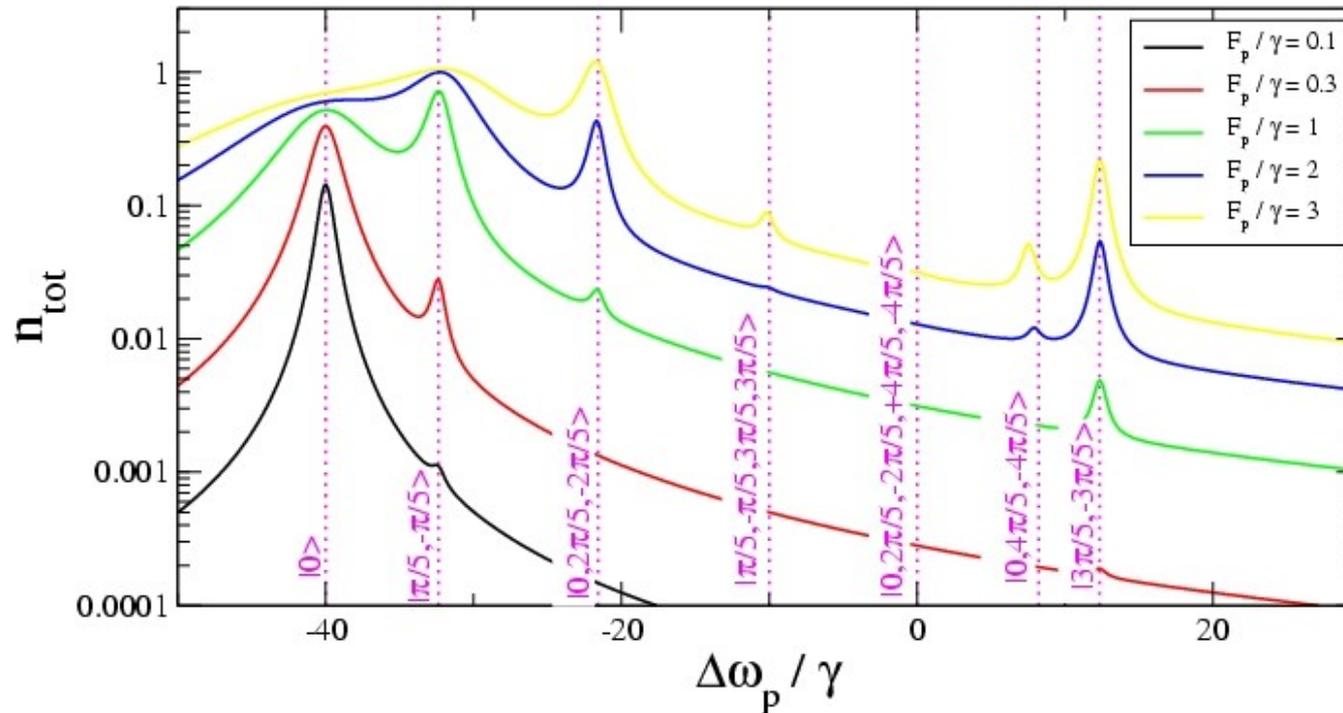
Many-body states labeled by momenta  $q$  of occupied Fermi orbitals  $|q_1, q_2, q_3, \dots\rangle$

Periodic boundary cond.: Fermi orbitals satisfy PBC/anti-PBC for odd/even  $N$

Energy (and momentum) preserved by mapping  $E = \sum_i \frac{\hbar^2 q_i^2}{2m}$

Example: two-particle states  $\psi(i_1, i_2) = \frac{1}{\sqrt{2L}} \sin\left(\frac{(2n+1)\pi}{L} |i_1 - i_2|\right)$

# Impenetrable bosons ( $U=\infty$ ): photoluminescence



Total luminescence as a function pump frequency for fixed pump intensity:

- each peak corresponds to a many-body state  $|q_1, q_2, q_3 \dots \rangle$
- $q_i$  quantized according to PBC/anti-PBC depending on  $N=\text{odd/even}$
- $N$ -particle state excited by  $N$  photon transition
- many-body energy  $E$  corresponds to peak at  $\Delta\omega_p = E / N$
- intensity of peak grows as  $(I_p)^N$

# Finite-U effects: analytics

Weakly-interacting limit  $U \ll J$  :

- mean-field approximation + perturbation theory valid
- lowest 2-body state: peak position  $\Delta\omega_p \approx -2J + U/M$   
almost factorizable wf

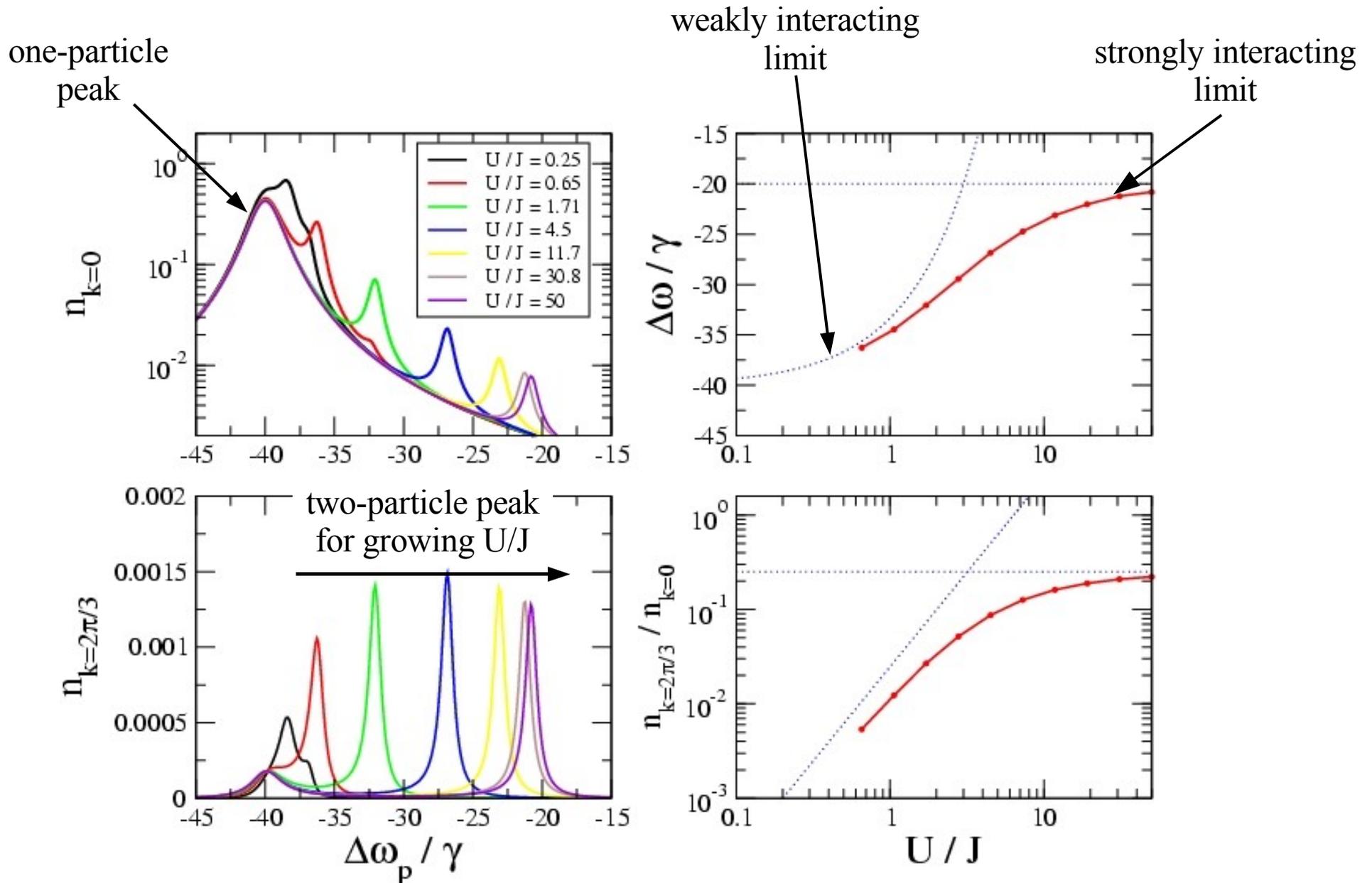
$$\psi(i_1, i_2) = \frac{1}{M} \left[ 1 - \frac{U}{9J} \cos\left(\frac{2\pi}{3}(i_1 - i_2)\right) \right]$$

Strongly-interacting limit  $U \gg J$  :

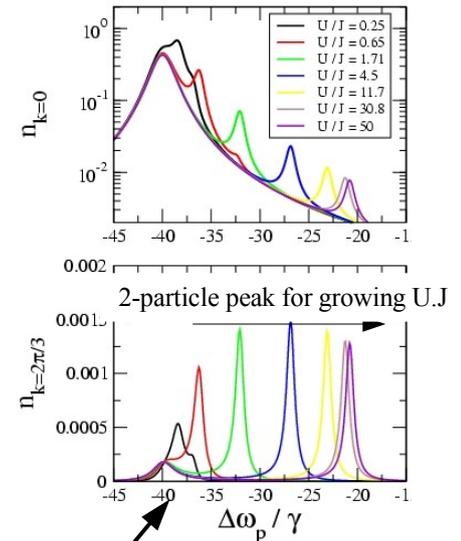
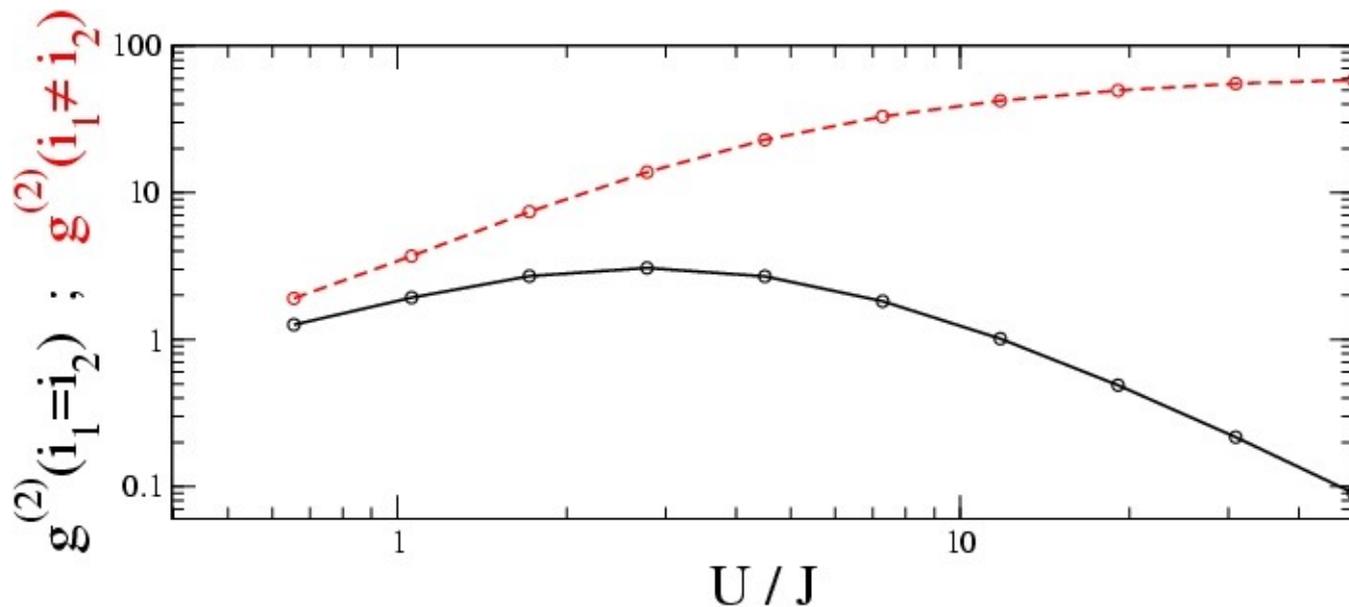
- strongly correlated Tonks-Girardeau gas
- lowest 2-body state: peak position  $\Delta\omega_p \approx -J$ .  
signature of anti-PBC of Fermi orbitals  
wf has many components

$$\psi(i_1, i_2) = \frac{1}{\sqrt{2}M} \sin\left(\frac{(2n+1)\pi}{L} |i_1 - i_2|\right)$$

# Finite-U effects: k-resolved photoluminescence

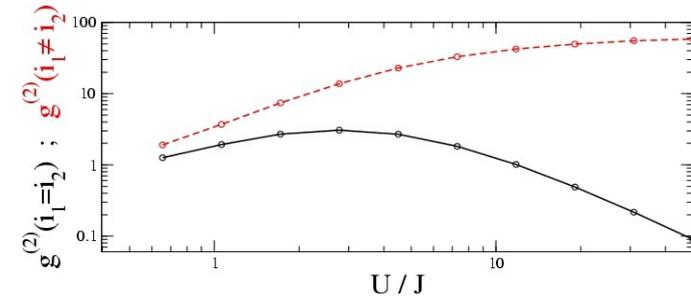
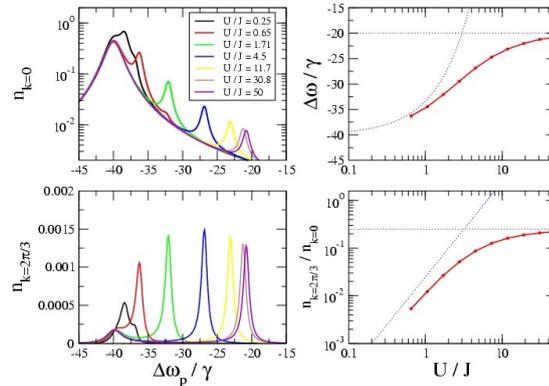
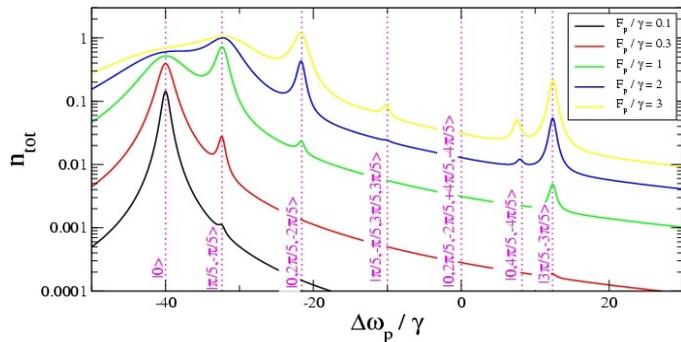


# Emitted light: spatial intensity correlations



- For each  $U/J$ , tune the pump laser on two-photon resonance
- intensity correlation between the emission from cavities  $i_1, i_2$
  - larger probability of having either  $N=0$  or  $2$  particles than  $N=1$
  - low  $U \ll J$ : bunched emission for all pairs of  $i_1, i_2$
  - large  $U \gg J$ : antibunched emission from a single site  
positive correlations between different sites.

# Theoretical conclusions



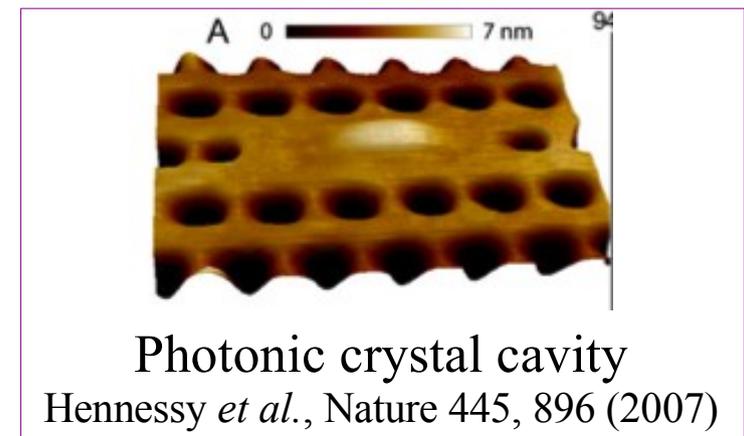
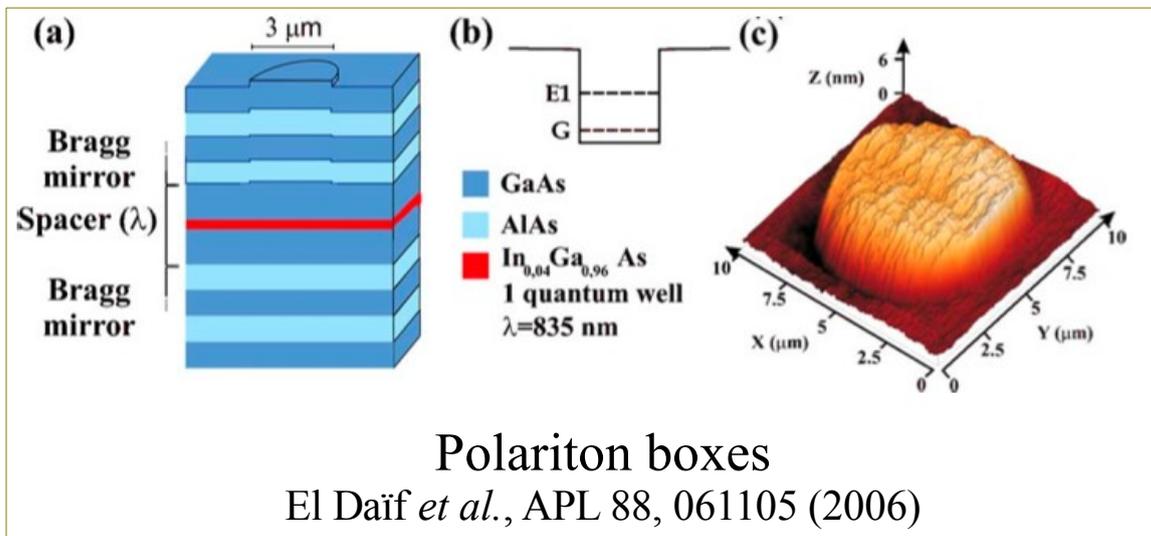
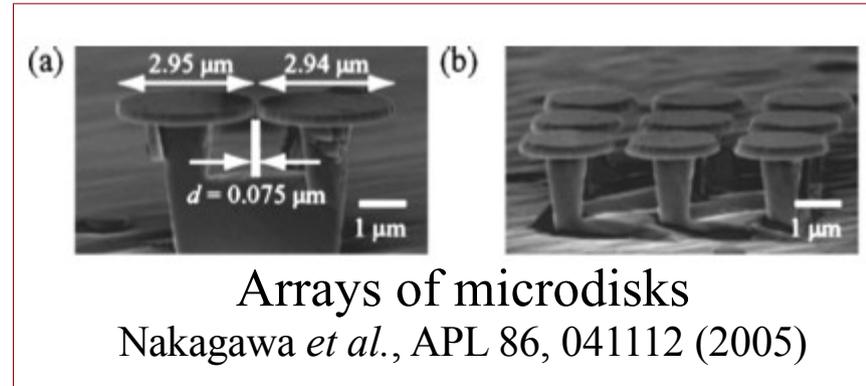
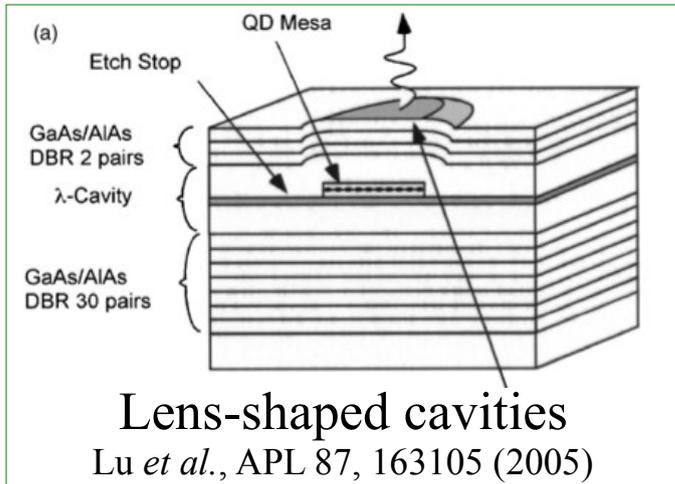
## Many-body structure of eigenstates characterized from optical spectra

- Position of peaks gives energy of many-body eigenstates
- Peaks spectrally well resolved in mesoscopic systems
- Spectral selection via pump and/or detection identifies single state
- Intensity of peaks as a function of pump intensity gives number of particles
- Spatially-resolved intensity correlation functions of emission provide info on microscopic structure of state
- Far-field correlations provide info on momentum distribution and momentum correlations

# Experimental requirements

- **Nonlinearity at single quantum level**
  - strong coupling with quantum dot or quantum well exciton
  - small cavity-mode volume
- **Scalability**
  - growth of many identical cavities
  - deterministic control of spectral position
  - significant inter-cavity tunnel coupling
- **Optical access and collection efficiency**
  - single-site addressability
  - momentum-space analysis of emission
  - high collection efficiency

# (Some) possible experimental routes



# Conclusions & future steps

## Non-equilibrium statistical mechanics of interacting photon gases

- a variety of **correlated many-body states** obtainable by spectral selection
- full **microscopic characterization** from **intensity** and **statistics of emission**

## Phase transitions

- transition **BEC** (“**superfluid**”) to **thermal** observed in weakly interacting systems
- some questions still open in **spatially extended geometries** (VCSELs, polariton BECs).
- what happens for large  $U/J$  ? **Mott Insulator/superfluid transition** ?

## Non-equilibrium: dissipation introduces finite effective “temperature”

- predicted by several models of **non-equilibrium quasi-BECs in low-D**  
M. Wouters, IC, PR B **74**, 245316 (2006); M. Szymanska, J. Keeling, P. Littlewood, PRL **96**, 230602(2006).
- **how general** is this result ? Analytical calculations in progress using laser theory

## What is meaning of superfluidity for non-equilibrium systems?

- **Landau superfluidity** of polaritons experimentally observed under resonant pump  
J.Lefrère, A.Amo, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, submitted (2008)
- No general consensus with other criteria, e.g. **metastability of supercurrents**