Interference of Fermi gases and atom-interferometrical detection of the superfluid order parameter in an ultracold Fermi gas

Iacopo Carusotto and **Yvan Castin**

- Experimental challenge: creating and detecting superfluid state of ultracold fermionic atoms
- The BCS order parameter: general issues, Monte Carlo simulations ...
- ... and its atom-interferometrical detection:
  - method 1: atom-number correlations in a two-particle interference setup
  - method 2: light scattering on a matter-wave grating

The current state of experiments

- Atomic Fermi gases cooled at temperatures well below the Fermi degeneracy $T \leq 0.1 T_F$.
- Control of atom-atom interactions by static magnetic field around a Feshbach resonance

- Actual challenge: to create and detect a superfluid state. BEC-BCS crossover:
  - $a_0 > 0$: BEC of strongly bound molecules $\longrightarrow$ Done.
    - Condensation apparent in TOF images like in atomic BEC

      S. Jochim *et al.*, Science **302**, 2101 (2003); M. Greiner *et al.*, Nature **426**, 537 (2003); M. W. Zwierlein *et al.*, PRL **91**, 250401 (2003); T. Bourdel *et al.*, PRL **93**, 050401 (2004).
  - $a_0 < 0$: BCS condensation of delocalized Cooper pairs $\longrightarrow$ indirect evidence available
    - Pairwise projection of fermionic atoms onto molecules by rapid scan of B: momentum distribution of molecules reproduces that of pairs.

      C. A. Regal *et al.*, PRL 92, 040403 (2004); M. W. Zwierlein *et al.*, PRL **92**, 120403 (2004)
    - Frequencies and damping rates of collective modes

      J. Kinast *et al.*, PRL **92**, 150402 (2004); M. Bartenstein *et al.*, PRL **92**, 203201 (2004)
    - Observation of the pairing gap in radio-frequency excitation spectra

      C. Chin *et al.*, Science **305**, 1128, (2004)

Some other theoretical proposals

- Light scattering. Measures density-density correlation function.

  J. Ruostekoski, Phys. Rev. A **60**, 1775 (1999); F. Weig and W. Zwerger, Europhys. Lett. **49**, 282 (2000)

- Superfluidity properties: braking of test particle, anomalous moment of inertia

  A. Minguzzi *et al.*, EPJD **17**, 49, (2001); M. Urban and P. Schuck, PRA **67**, 033611 (2003);
  M. Cozzini and S. Stringari, PRL **91** 070401, (2003)

- Aspect ratio of the cloud during ballistic expansion

  C. Menotti *et al.*, PRL **89**, 250402 (2002)

- Two-particle correlation function after ballistic expansion: momentum space pairing

  E. Altman *et al.*, PRA **70**, 013603 (2004)

The order parameter of the BCS transition

- Order parameter of BCS theory of superfluid transition in Fermi gas with weak attractive ($a_0 < 0$) s-wave interactions:

$$\Delta(\mathbf{x}) = -g_0 \left\langle \hat{\Psi}_\uparrow(\mathbf{x})\, \hat{\Psi}_\downarrow(\mathbf{x}) \right\rangle.$$

- The arbitrary phase of Δ corresponds to the spontaneously broken symmetry.

- Number-conserving approach: first-order coherence function of Cooper pairs

$$G^{(1)}_{\rm pair}(\mathbf{x}) = \left\langle \hat{\Psi}^\dagger_\downarrow(\mathbf{x})\, \hat{\Psi}^\dagger_\uparrow(\mathbf{x}) \hat{\Psi}_\uparrow(0)\, \hat{\Psi}_\downarrow(0) \right\rangle$$

  - broken symmetry $\Longrightarrow$ long-range coherence, finite long-distance limit $\lim_{\mathbf{x}\to\infty} G^{(1)}_{\rm pair}(\mathbf{x}) \neq 0$
  - condensate of pairs, i.e. macroscopic value of $\tilde{G}^{(1)}_{\rm pair}(\mathbf{k}=0)$

- Same order parameter everywhere along BCS-BEC crossover
  - on BEC side: momentum distribution of molecules

Quantum Monte Carlo simulations

Stochastic Hartree-Fock ansatz

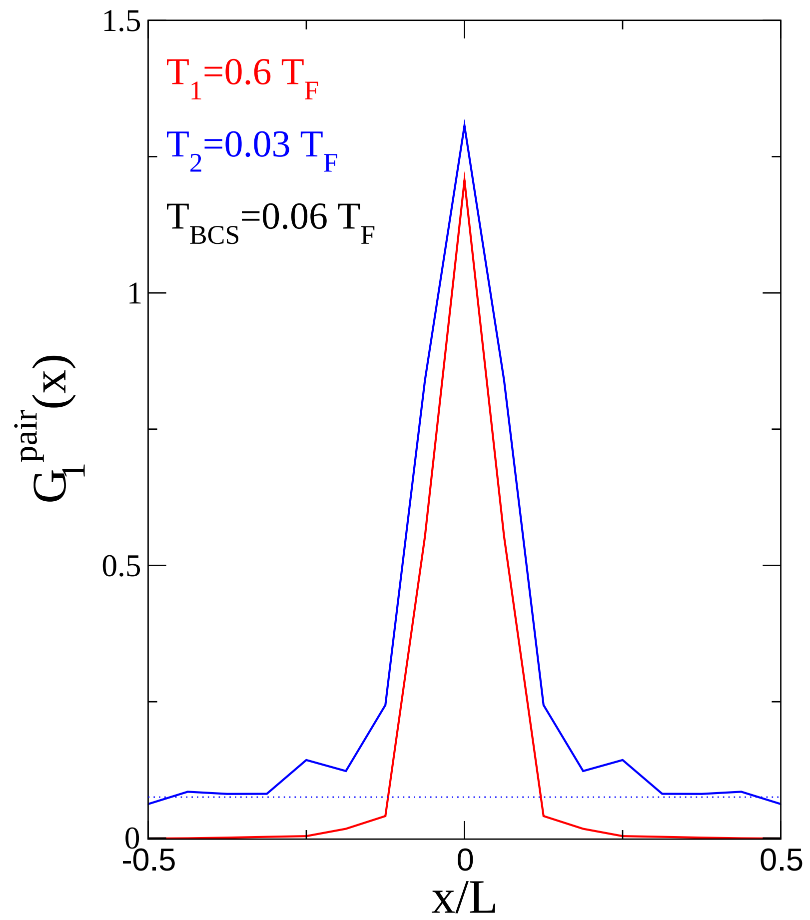
(O. Juillet and Ph. Chomaz, PRL **88**, 142503, 2002)

1D system, $N = 12$ atoms

$\mathcal{N} = 16$ points lattice

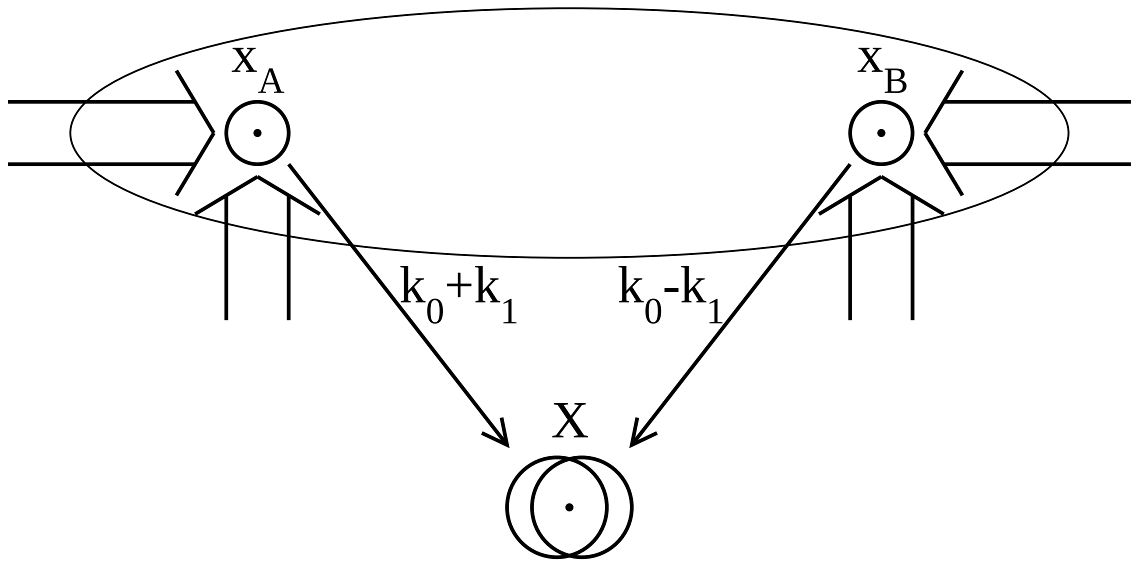
Comparison with mean-field BCS results

Finite system size $\Longrightarrow$ BCS order in 1D



I. Carusotto and Y. Castin, physics/0404025

Interferometric detection of $G^{(1)}_{\text{pair}}$

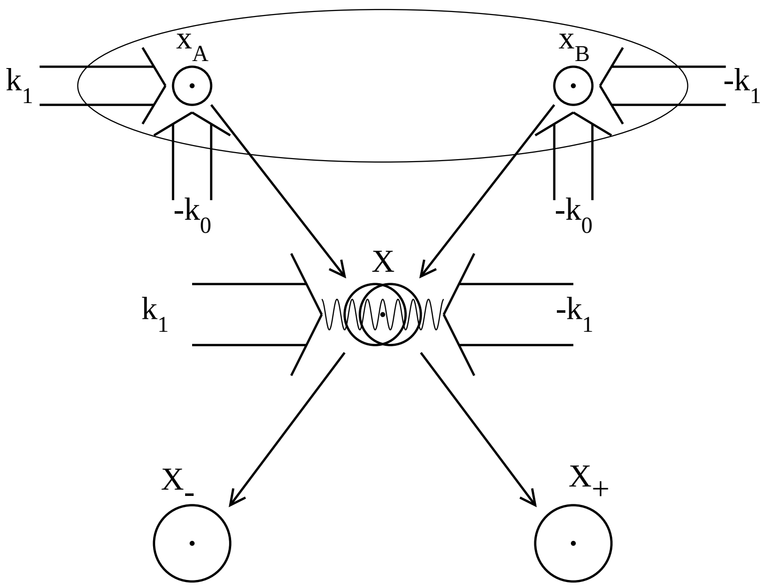


- At $t = 0$: atoms in both spin states are coherently extracted at points $\mathbf{x}_A$ and $\mathbf{x}_B$.
- Extraction region $\ell_u \gg \ell_F, \ell_{BCS}$. Gaussian amplitude $u(\mathbf{x} - \mathbf{x}_{A,B})$
- Momentum kick $\mathbf{k}_0 \pm \mathbf{k}_1$ by Bragg processes
- Simultaneously, trap potential and atom-atom interactions switched off
- At $t_1 \approx mL/2\hbar\,|\mathbf{k}_1|$, two wavepackets overlap
- Different sort of manipulations to measure $G^{(1)}_{\text{pair}}(\mathbf{x}_A, \mathbf{x}_B)$

Method (a): Atom-number correlations (I)

$G^{(1)}_{\rm pair}$ is two-body correlation function $\Longrightarrow$ two-atom interferometry scheme

Optical analogs
- Hanbury-Brown and Twiss interferometer. $\longrightarrow$ star radius.
- Ou-Mandel interferometer $\longrightarrow$ subpicosecond time intervals



- Coherent mixing by spin-insensitive 50/50 beam splitter with mixing phase ϕ (Bragg diffraction)
- Detect atom-number difference between the two exit arms $D_\sigma = N_\sigma^+ - N_\sigma^-$
- if $|\mathbf{x}_A - \mathbf{x}_B| \gg \ell_F$, no phase coherence between $\mathbf{x}_{A,B}$, vanishing average: $\langle D_\sigma \rangle = 0$
- If pairing is present:
  nontrivial cross-correlations of fluctuations even at large distances $|\mathbf{x}_A - \mathbf{x}_B| \gg \ell_F, \ell_{BCS}$

$$C_{\uparrow\downarrow} = \langle D_\uparrow \, D_\downarrow \rangle = \int d\boldsymbol{\xi} \, d\boldsymbol{\xi}' \, |u(\boldsymbol{\xi})|^2 \, |u(\boldsymbol{\xi}')|^2 \Big[e^{2i\phi} \Big\langle \hat{\Phi}^\dagger_\uparrow(\mathbf{x}_A + \boldsymbol{\xi}) \, \hat{\Phi}^\dagger_\downarrow(\mathbf{x}_A + \boldsymbol{\xi}') \, \hat{\Phi}_\downarrow(\mathbf{x}_B + \boldsymbol{\xi}') \, \hat{\Phi}_\uparrow(\mathbf{x}_B + \boldsymbol{\xi}) \Big\rangle + \\ + \Big\langle \hat{\Phi}^\dagger_\uparrow(\mathbf{x}_A + \boldsymbol{\xi}) \, \hat{\Phi}^\dagger_\downarrow(\mathbf{x}_B + \boldsymbol{\xi}') \, \hat{\Phi}_\downarrow(\mathbf{x}_A + \boldsymbol{\xi}') \, \hat{\Phi}_\uparrow(\mathbf{x}_B + \boldsymbol{\xi}) \Big\rangle + \text{h.c.} \Big].$$

Atom-number correlations (II)

Analytical calculation in the weak-coupling BCS limit:

$$C_{\uparrow\downarrow} = \langle D_\uparrow D_\downarrow \rangle = \frac{3\pi}{8\sqrt{2}} \cos(2\phi)\, |u_0|^2 \frac{\Delta}{E_F} N_\sigma$$

Throughout all the BEC-BCS crossover:

- $T > T_c$: $C_{\downarrow\uparrow} \to 0$ for large distances $|\mathbf{x}_A - \mathbf{x}_B| \gg \ell_F$.
- $T < T_c$: long-range order, $C_{\downarrow\uparrow} \neq 0$ for all distances $|\mathbf{x}_A - \mathbf{x}_B|$.

  Within BCS theory: $C_{\downarrow\uparrow}$ proportional to pairing gap Δ

Experimentally: $C_{\uparrow\downarrow}$ obtained as average over many realizations of the experiment starting from the same initial state

Noise on $C_{\uparrow\downarrow}$: estimated by $\langle D_\sigma^2 \rangle \approx 2N_\sigma$ Principally due to shot-noise in extraction

Method (b): the matter-wave grating

At $t = t_1$: wavepackets of momentum $\mathbf{k}_0 \pm \mathbf{k}_1$ spatially overlapping

Statistical properties of distribution of atomic positions:

- No phase coherence between $\mathbf{x}_{A,B}$: smooth mean-density profile.

$$n_\sigma(\boldsymbol{\xi}) = \left\langle \hat{\Psi}^\dagger_\sigma(\boldsymbol{\xi}, t_1)\hat{\Psi}_\sigma(\boldsymbol{\xi}, t_1)\right\rangle = 2\,|u(\boldsymbol{\xi})|^2\,\rho$$

- Density-density correlations show fringes:

$$\mathcal{G}^{(2)}_{\sigma\sigma'}(\boldsymbol{\xi}, \boldsymbol{\xi}') = \left\langle \hat{\Psi}^\dagger_\sigma(\mathbf{X}+\boldsymbol{\xi}, t_1)\,\hat{\Psi}^\dagger_{\sigma'}(\mathbf{X}+\boldsymbol{\xi}', t_1)\,\hat{\Psi}^\dagger_{\sigma'}(\mathbf{X}+\boldsymbol{\xi}', t_1)\,\hat{\Psi}_\sigma(\mathbf{X}+\boldsymbol{\xi}, t_1)\right\rangle$$

  – same-spin: $\mathcal{G}^{(2)}_{\sigma\sigma}(\boldsymbol{\xi}, \boldsymbol{\xi}') = 4\,|u(\boldsymbol{\xi})|^2\,|u(\boldsymbol{\xi}')|^2\left[\rho^2 - \cos^2\left[\mathbf{k}_1(\boldsymbol{\xi}-\boldsymbol{\xi}')\right]\left|G^{(1)}(\boldsymbol{\xi}-\boldsymbol{\xi}')\right|^2\right]$

  – opposite-spin $\mathcal{G}^{(2)}_{\uparrow\downarrow}(\boldsymbol{\xi}, \boldsymbol{\xi}') = |u(\boldsymbol{\xi})|^2\,|u(\boldsymbol{\xi}')|^2\left[4\rho^2 + \cos^2\left[\mathbf{k}_1(\boldsymbol{\xi}+\boldsymbol{\xi}')\right]\left|\mathcal{A}(\boldsymbol{\xi}'-\boldsymbol{\xi})\right|^2\right]$

  (in trap: $G^{(1)}_\sigma(\boldsymbol{\xi}) = \left\langle \hat{\Psi}^\dagger_\sigma(\mathbf{X}+\boldsymbol{\xi})\hat{\Psi}_\sigma(\boldsymbol{\xi})\right\rangle$ and anomalous average $A(\boldsymbol{\xi}) = \left\langle \hat{\Psi}_\downarrow(\mathbf{X})\,\hat{\Psi}_\uparrow(\mathbf{X}+\boldsymbol{\xi})\right\rangle$)

- Fringes in $\mathcal{G}^{(2)}_{\sigma\sigma'}$ can be detected by elastic light scattering

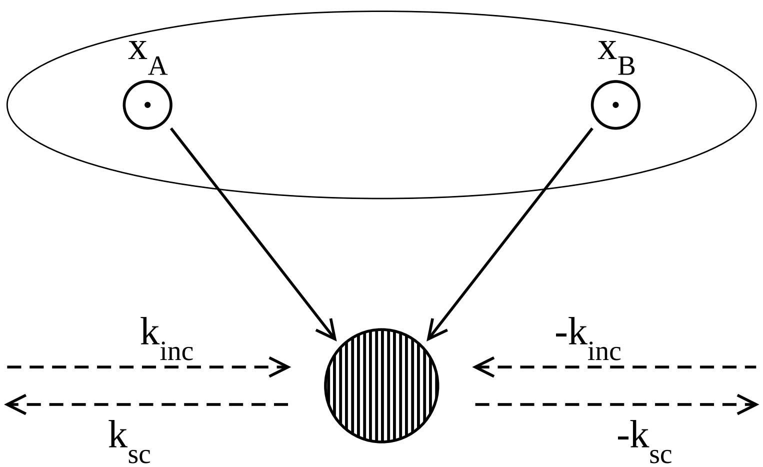
Elastic light scattering (I)

- Weak incident light intensity $\longrightarrow$ no saturation of atomic transition
- Atomic polarizability for $F_g = 1/2 \longrightarrow F_e = 1/2$ transition:

$$\chi_{ij}(\omega) = \frac{f}{\omega_0 - \omega - i\,\Gamma/2}\left[\delta_{ij} + i\sum_k \epsilon_{ijk}\,\mathrm{Tr}[\hat{\rho}_a \hat{\sigma}_k]\right]$$

- $N_{scatt}/N_{atoms} \ll 1 \;\longrightarrow\;$ NO optical pumping
- Incident light almost parallel to $\hat{z}$ axis. Polarizations defined with respect to $+\hat{z}$ axis
- Atoms in $\uparrow\downarrow$ spin state diffuse $\sigma_\pm$ light. Light polarization preserved

- Short imaging time $\longrightarrow$ fixed atomic positions
- Scattered light is fully coherent
- Each realization of the atomic positions
  $\longrightarrow$ different angular pattern.
- Average to be taken over many realizations



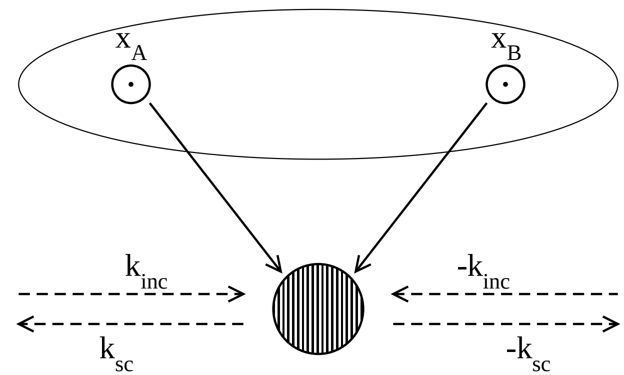
Elastic light scattering (II)

- Scattered amplitude

$$E_\sigma(\mathbf{k}_{sc}) = C\,\hat{n}_{-\sigma}(\mathbf{q})\,E_\sigma(\mathbf{k}_{inc})$$

- Nearly back-scattering geometry:

$$\mathbf{q} = \mathbf{k}_{sc} - \mathbf{k}_{inc} \approx 2\mathbf{k}_1$$



Average over many realizations of atomic positions:

- Average amplitude: $\langle E_\sigma(\mathbf{k}_{sc})\rangle = 0$
- Average intensity (F is F.T. of $|G^{(1)}(\boldsymbol{\xi})|^2$):

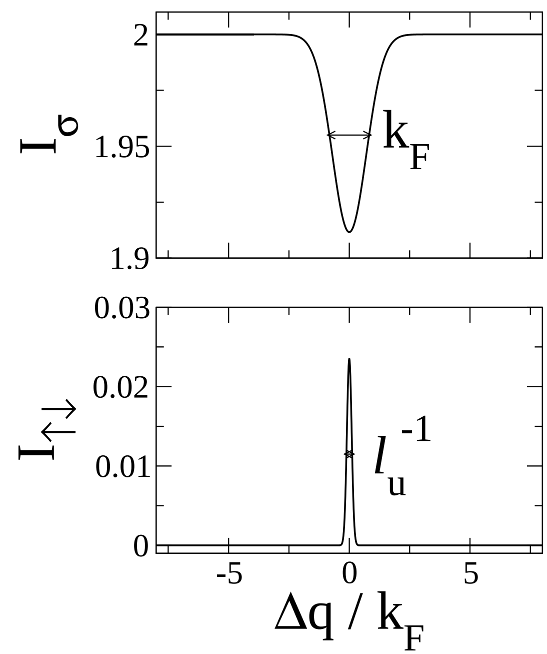
$$I_\sigma(\mathbf{k}_{sc}) = |C|^2\,N_\sigma\left[2 - \frac{|u_0|^2}{2^{3/2}}\,F(\mathbf{q}-2\mathbf{k}_1)\right]I_{inc}$$

- Pair of mutually coherent incident beams $\mathbf{k}_{inc}^{(1,2)}$:

$$I_{\downarrow\uparrow} = \langle E^\dagger_\sigma(\mathbf{k}^{(1)}_{sc})\,E_{-\sigma}(\mathbf{k}^{(2)}_{sc})\rangle =$$

$$= \frac{3\sqrt{\pi}}{2^{5/2}}\,|u_0|^2\,\frac{\Delta}{E_F}\,e^{-\ell_u^2(\mathbf{q}-2\mathbf{k}_1)^2/2}\,|C|^2\,N_\sigma\,I_{inc}$$

Relative phase of scatt. beams $\longrightarrow$ info on pairing



Mixing scattered beams: $I_{out}(\theta_{mix}) \longrightarrow$ fringes, contrast $I_{\downarrow\uparrow}/I_\sigma \propto \Delta$

Summary

- Transition to superfluid state: long-range order in the Cooper-pair coherence function:

$$G^{(1)}_{\rm pair}(\mathbf{x}) = \left\langle \hat{\Psi}^\dagger_\downarrow(\mathbf{x})\, \hat{\Psi}^\dagger_\uparrow(\mathbf{x}) \hat{\Psi}_\uparrow(0)\, \hat{\Psi}_\downarrow(0) \right\rangle$$

- Measured observable directly connected to static value of order parameter at thermal equilibrium for which theoretical data are available.
- No requirement of *a priori* knowledge of system dynamics and of molecular formation
- Clear and unambiguous signature of long-range order. NO spurious signal from pseudo-gap.

- Example of quantum optics with a Fermi field:

  a- two-atom interferometry *à la* Mandel: atom-number correlations are sensitive to pairing.

  b- interference fringes formed by two overlapping Fermi wavepackets and detected by elastic light scattering: coherence of scattered light sensitive to pairing.