Nonlinear atom optics and bright gap soliton generation in finite optical lattices

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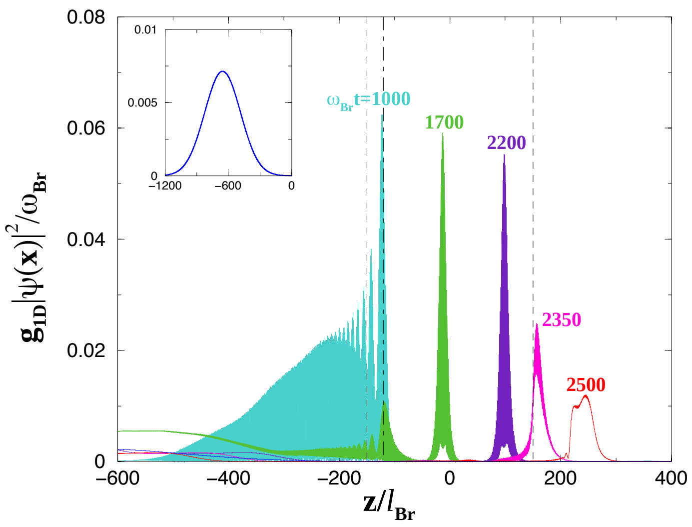

A Paris–Pisa collaboration:

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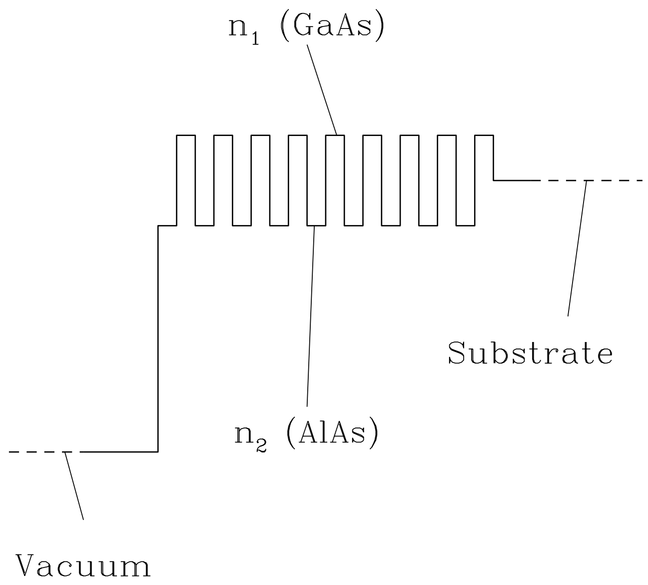
Background concepts

- Photonic Band Gap crystals and Bragg fibers
  - periodic stack of $\lambda/4$ layers with different refraction indices $n_{1,2}$

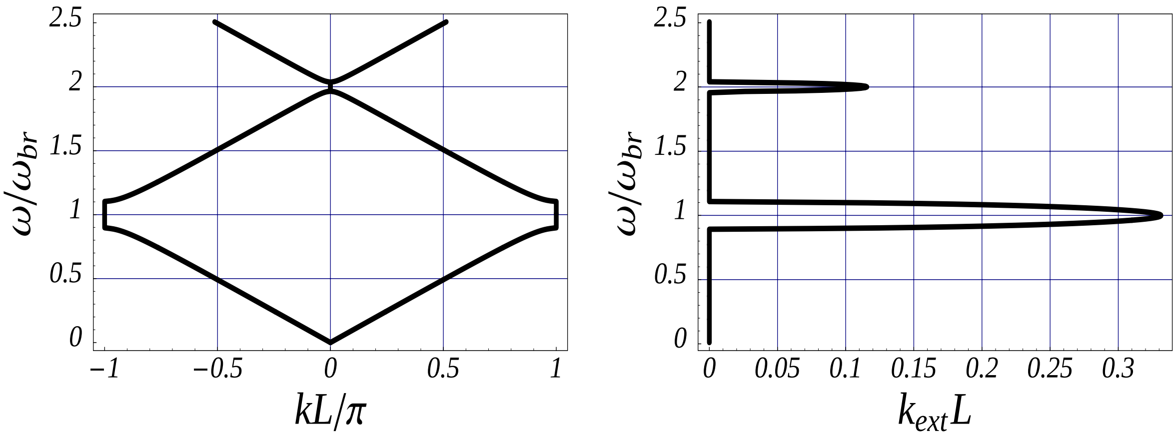


  - periodically corrugated optical fiber

- Bragg diffraction processes for $\omega_{Br} = \frac{c\pi}{L\bar{n}}$
- discrete translational symmetry with periodicity L
- physical analogy with electrons in crystalline solid
- dispersion with allowed bands and forbidden gaps

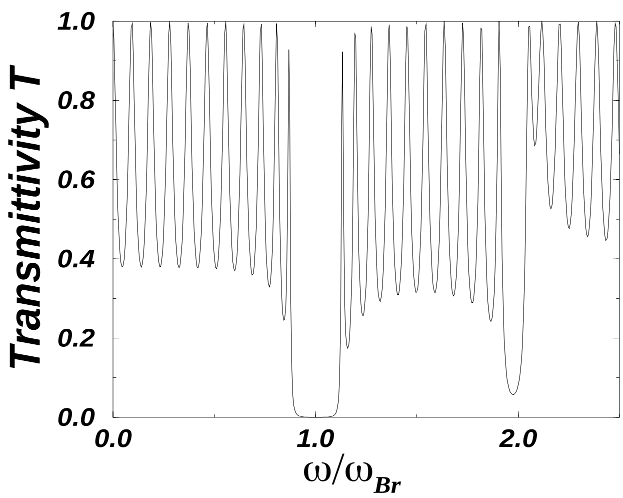
- **photonic dispersion**:
  - allowed **bands** and forbidden **gaps**
  - effective mass at band edge $m_{\rm eff} = \left(\frac{1}{\hbar}\frac{\partial^2 \omega}{\partial k^2}\right)^{-1}$

    Top of valence band $\longrightarrow$ $m_{\rm eff} < 0$



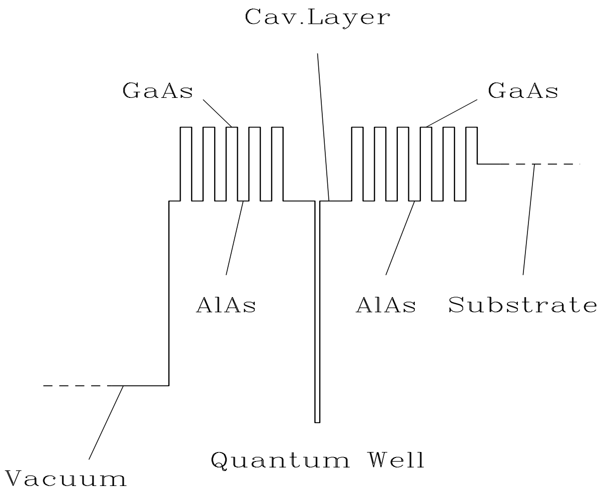
- (weak) laser beam at ω_{inc}:
  - if ω_{inc} corresponds to allowed band $\rightarrow$ transmitted
  - if ω_{inc} corresponds to forbidden gap $\rightarrow$ reflected

    (**Distributed Bragg Reflector**)

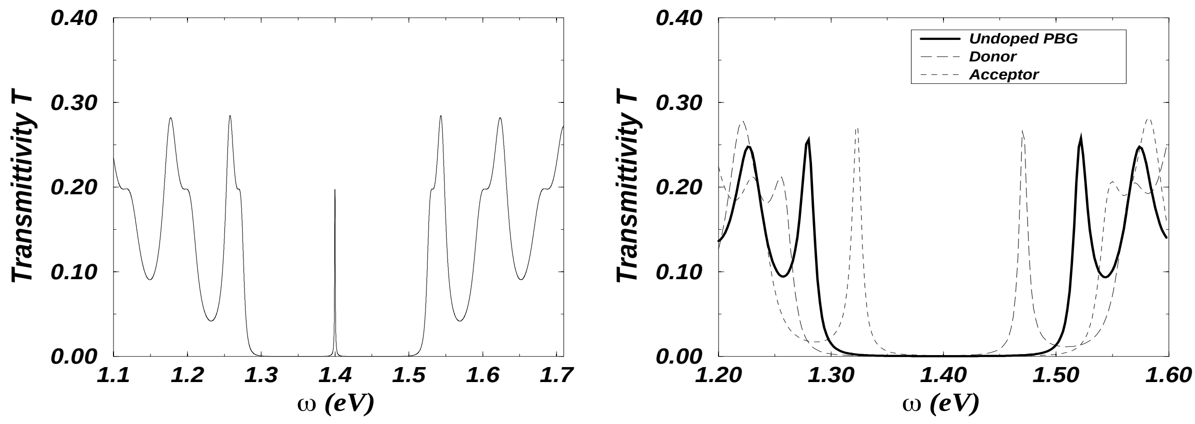


DBR microcavities

- a pair of DBR mirrors separated by a cavity layer



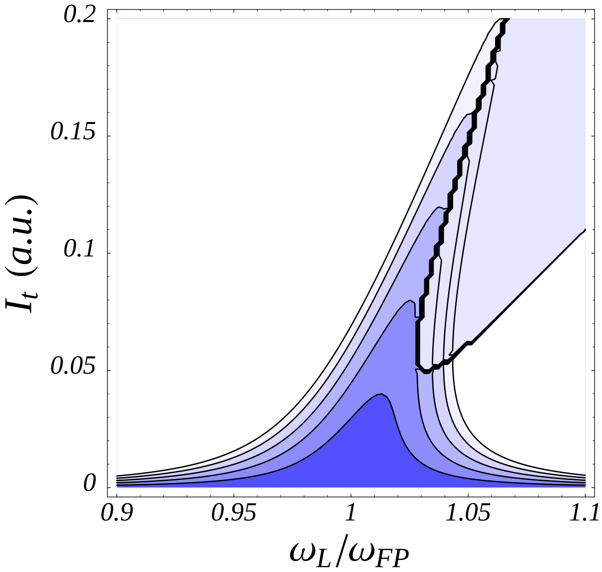
- localized cavity mode: resonant peak in transmission



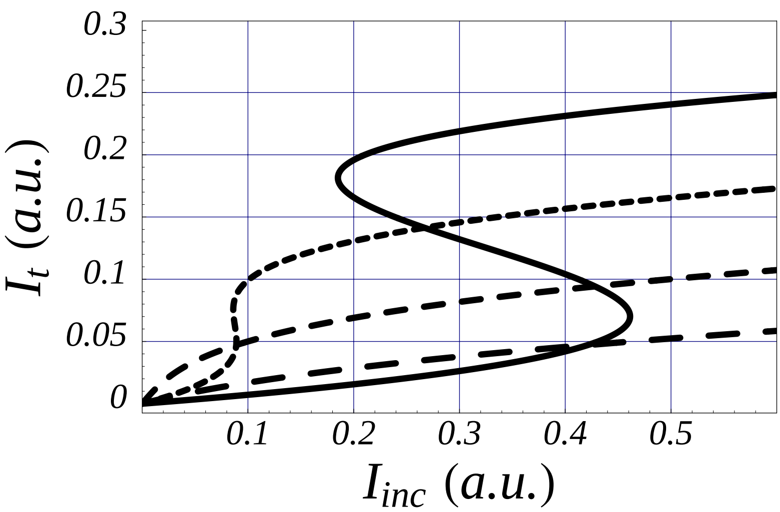
- alternative interpretation: impurity in PBG crystal

Nonlinear effects

- growing intensity $\longrightarrow$ refractive index modification
  frequency shift of cavity mode



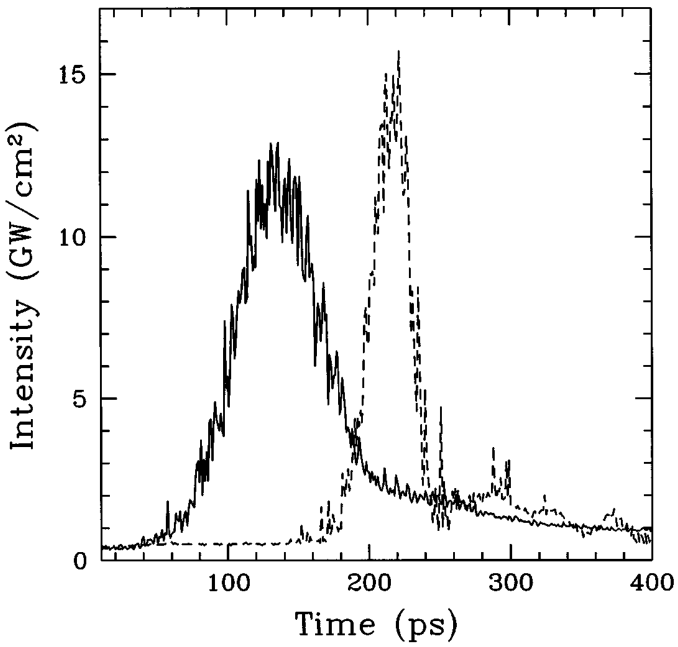
- characteristic I_t vs. I_{inc} curves at given ω_L:
  - positive feedback $\rightarrow$ optical bistability
  - negative feedback $\rightarrow$ optical limiting



Gap solitons

Group velocity dispersion $\longrightarrow$ spreading of wavepacket

Nonlinearity compensates $\longrightarrow$ stationary pulses



(from: B. J. Eggleton *et al.*, Phys. Rev. Lett. **76**, pagg. 1627-1630, 1996)

Properties of gap solitons:

- Slow group velocity
- Robustness against perturbations and collisions
- Depending on sign of nonlinearity:
  - $\chi^{(3)} > 0 \longrightarrow$ bottom of conduction band
  - $\chi^{(3)} < 0 \longrightarrow$ top of valence band

Light–matter wave analogy

Matter waves in optical lattices	$\longleftrightarrow$	**Light** waves in dielectric structures
Optical Potential $V^{\rm opt}$	$\longleftrightarrow$	Linear Refraction Index $n_{\rm lin}$
Atomic spin	$\longleftrightarrow$	Light Polarization
Collisions	$\longleftrightarrow$	Kerr Nonlinear Suscept. $\chi^{(3)}$
Gross-Pitaevskii eq.	$\longleftrightarrow$	Maxwell's eqs.

$$i\hbar\frac{\partial\psi(z,t)}{\partial t}=\left(-\frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial z^2}+V_{\rm opt}(z)+g_{\rm 1D}\left|\psi(z,t)\right|^2\right)\psi(z,t)$$

Laser $\longrightarrow$ nonlinear photon optics

BEC $\longrightarrow$ nonlinear atom optics

- Analogous coherence properties
- (Finite) optical lattice $\longrightarrow$ PBG crystal, DBR mirror
- Modulated optical lattice $\longrightarrow$ DBR microcavity

(I.Carusotto and G.C.La Rocca, Proceed. of 27^{th} Intern. School of Quantum Electronics (Erice, Oct.1999), Kluwer Academic / Plenum Publishers, New York, 2000.)

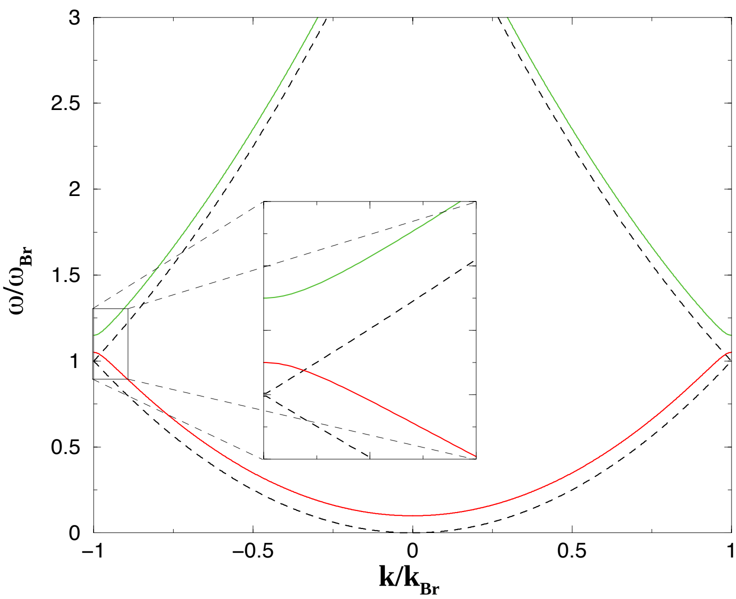
Linear regime: infinite lattice

- Optical potential $V_{\rm opt}(z) = V_0 \cos^2 k_{\rm Br} z$

- Weak lattice $V_0 \le \hbar\omega_{\rm Br} = \frac{\hbar^2 k_{\rm Br}^2}{2m_0}$

- Nearly-free atom approximation:

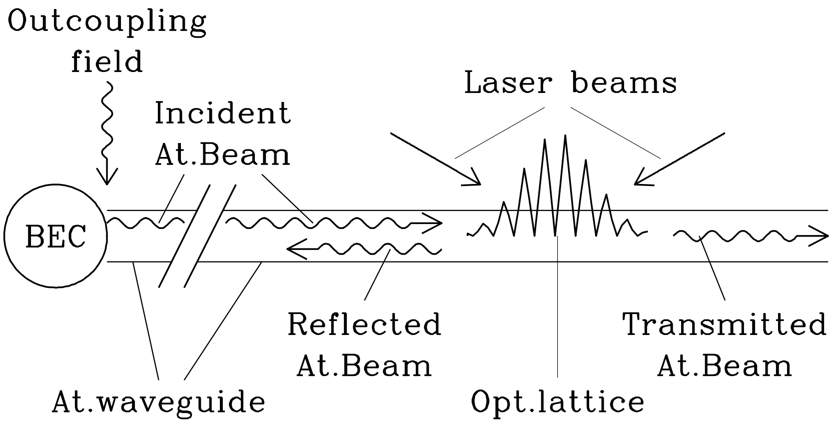
  - $\hbar\omega_{c,v}(k) = \hbar\omega_{\rm Br} + \frac{V_0}{2} + \hbar\omega_{\rm Br}\left[\left(\frac{\Delta k}{k_{\rm Br}}\right)^2 \pm 2\sqrt{\left(\frac{\Delta k}{k_{\rm Br}}\right)^2 + \left(\frac{V_0}{8\hbar\omega_{\rm Br}}\right)^2}\right]$
  - $\frac{1}{m_{\rm eff}} = \frac{1}{\hbar}\frac{\partial^2 \omega_{c,v}}{\partial k^2} = \frac{1}{m_0}\left(1 \pm \frac{8\hbar\omega_{\rm Br}}{V_0}\right) \simeq \pm\frac{8\hbar\omega_{\rm Br}}{V_0}$



- At band edge $|m_{\rm eff}| \ll m_0$:

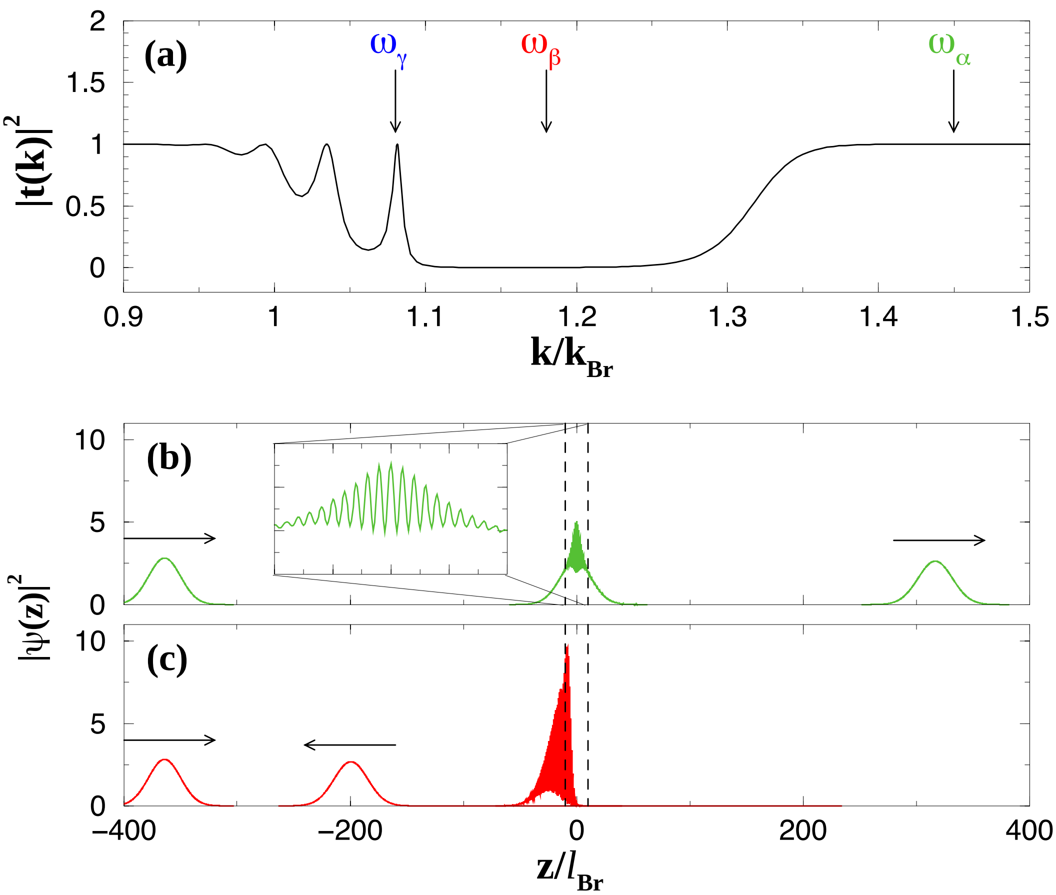
  - conduction band $\longrightarrow \; m_{\rm eff} > 0$
  - valence band $\longrightarrow \; m_{\rm eff} < 0$

Finite lattice



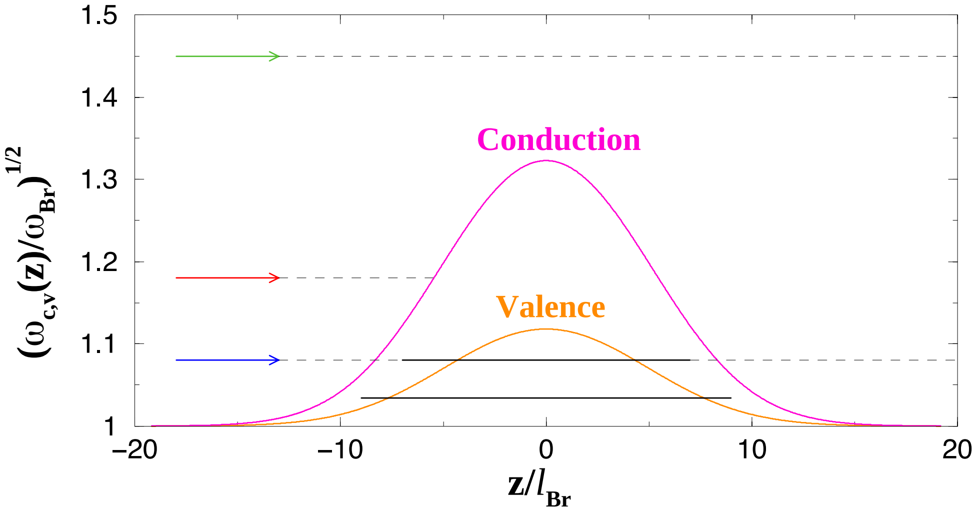
- Linearity $\longrightarrow \quad \tilde{\psi}_{\rm t}(k) = t(k)\,\tilde{\psi}_{\rm inc}(k)$

- Pulse transmission determined by amplitude $t(k)$



Fabry-Perot modes

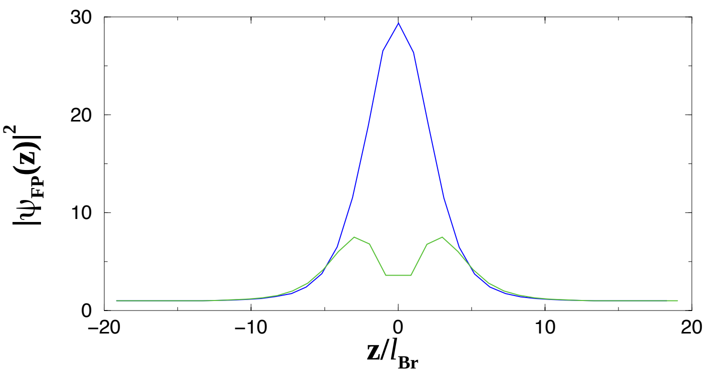
Local band edge position $\hbar\omega_{c,v}(k_{\mathrm{Br}})$



Local maximum of valence band edge

Valence band has negative effective mass $m_{\mathrm{eff}} < 0$

$\Longrightarrow$ localized bound state



Resonant transmission $\longrightarrow$ density enhanced

possibility of nonlinear effects

(I. Carusotto and G. C. La Rocca, Phys. Rev. Lett. **84**, 399, 2000)

Resonant transmission

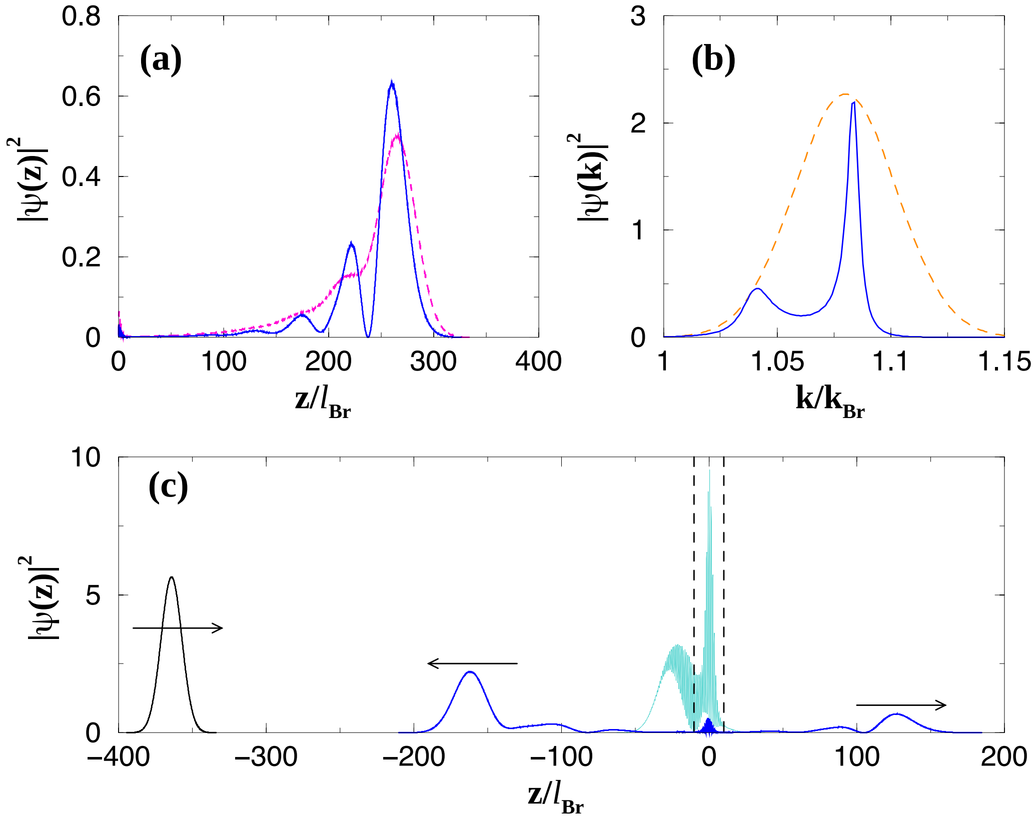
Long pulse $\longrightarrow$ single mode excited

$\Longrightarrow$ transm. pulse has exponential tail

Short pulse $\longrightarrow$ several modes within lineshape

$\Longrightarrow$ oscillations inside cavity

beatings in transmitted pulse



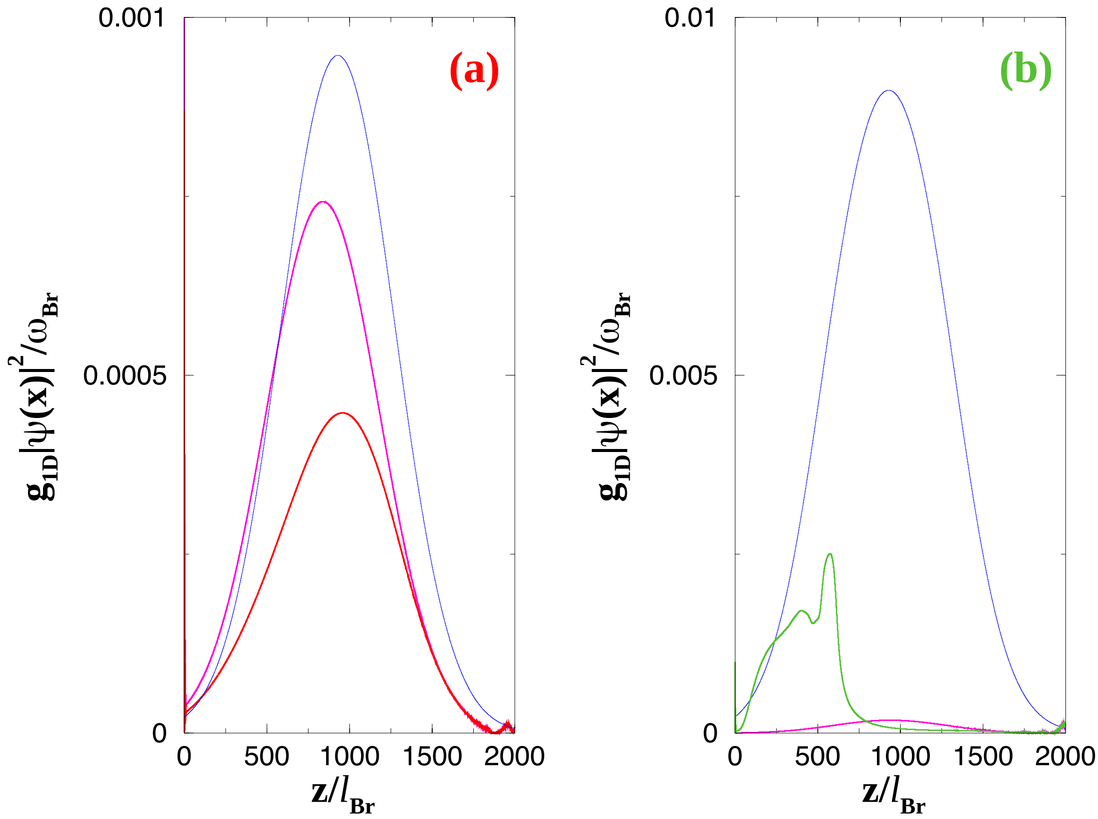
(I. Carusotto, D. Embriaco, G. C. La Rocca, PRA, in the press, 2002)

Nonlinear atom optics

Repulsive interactions $\longrightarrow$ blue shift cavity mode

- $\omega = \omega_{\mathrm{FP}} \longrightarrow$ negative feedback, optical limiting
- $\omega > \omega_{\mathrm{FP}} \longrightarrow$ positive feedback, optical bistability

CW features reflected into pulse transmission



Nonlinearity $\ll$ mode spacing $\Longrightarrow$ no mode mixing

(I. Carusotto, D. Embriaco, G. C. La Rocca, PRA, in the press, 2002)

Modulational instability

Free particles with $m_{\rm eff} < 0$ (no confinement)

Interaction potential $V_{\rm int} > 0$

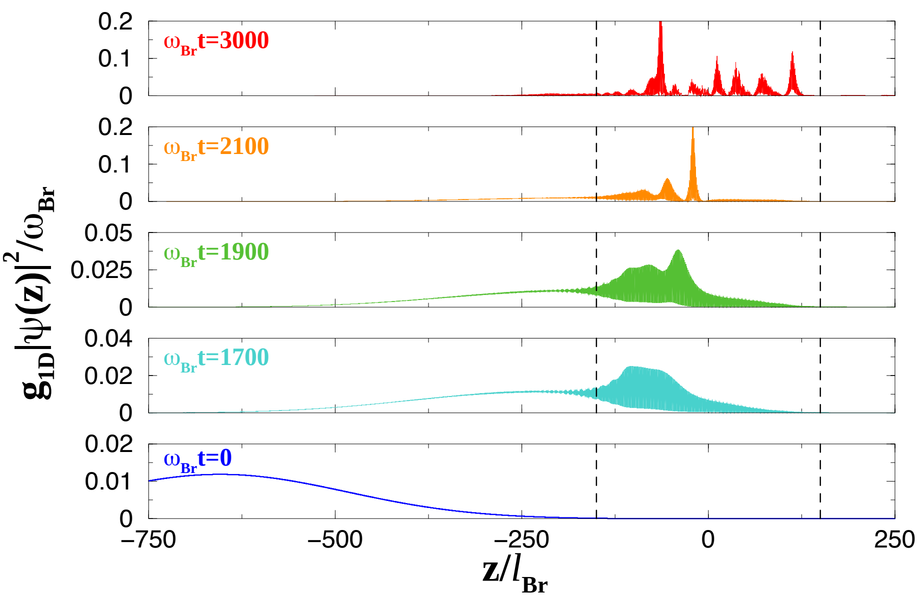
$\Longrightarrow$ effective attractive interaction

Red-detuned lattice $\longrightarrow$ no valence band bound states

Incident pulse within band $\longrightarrow$ pulse freely penetrates

$m_{\rm eff} < 0 \quad \Longrightarrow$ Modulational instability

Collapse into train of pulses



Not only thermodynamical Instability (Landau)

But also dynamical instability:

$\longrightarrow$ phonon amplitude exponentially growing,

$\longrightarrow$ small (quantum) fluctuations amplified

Bright gap solitons

Nonlinear Schrödinger Equation is integrable:

$\longrightarrow$ solitonic solutions for attractive interactions

$\longrightarrow$ robust wave packets propagating without spreading

$\longrightarrow$ collisions give phase shift, pulses unchanged

Envelope function approach $\psi(z,t) = \bar{\psi}(z,t)\, u_{k_{\rm sol}}(z)$:

$\longrightarrow$ $\bar{\psi}(z,t)$ slowly varying envelope

$\longrightarrow$ $u_{k_{\rm sol}}(z)$ oscillating Bloch eigenfunction

Slowly varying envelope approx. $\Longrightarrow$ NLSE for $\bar{\psi}(z,t)$

$$i\hbar \frac{\partial \bar{\psi}(z,t)}{\partial t} = \left(-\frac{\hbar^2}{2m_{\rm eff}} \frac{\partial^2}{\partial z^2} + g_{\rm eff} |\bar{\psi}(z,t)|^2 \right) \bar{\psi}(z,t)$$

$\longrightarrow$ $m_{\rm eff}$ effective mass from band structure

$\longrightarrow$ $g_{\rm eff}$ effective interaction (same sign as g_{1D})

For $m_{\rm eff} < 0$, $g_{\rm eff} > 0$ (or $m_{\rm eff} > 0$, $g_{\rm eff} < 0$):

$\longrightarrow$ solitonic solutions: gap solitons

Mathematically: not exactly integrable, rather solitary waves

Controlled soliton generation

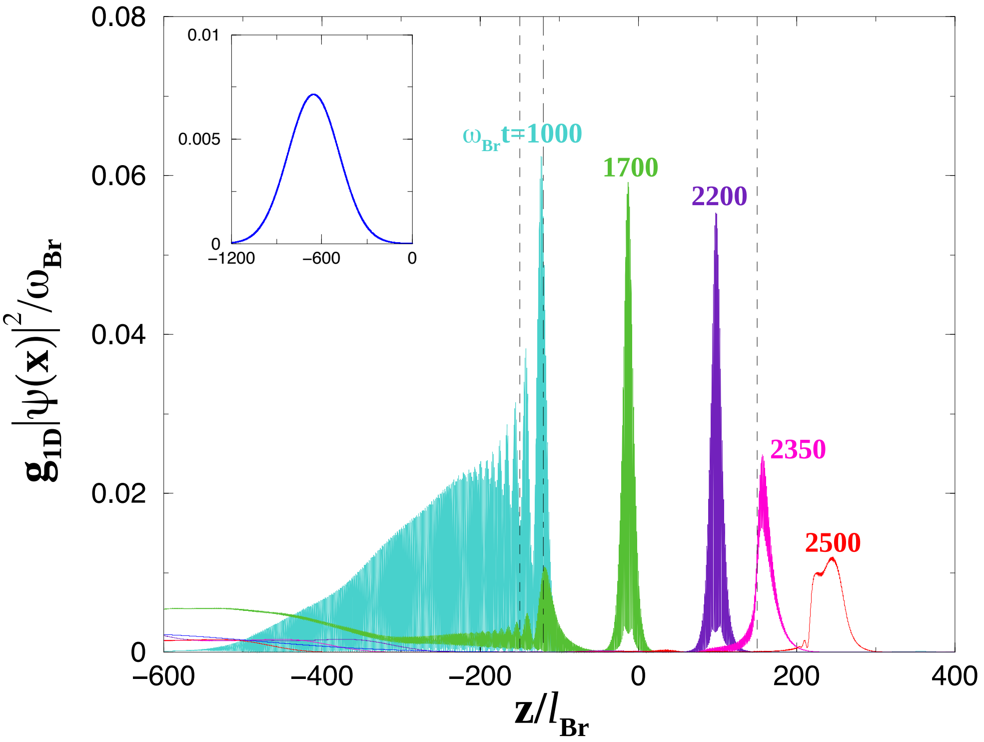
Matter wave incident on lattice, frequency within gap:

$\Longrightarrow$ nearly complete reflection at linear regime

standing matter wave created in front of lattice

At the antinodes, enhanced density shifts local band edge

$\Longrightarrow$ wave packets propagate as stable objects



Large solitonic width of the order of $10\,\mu\mathrm{m}$.

Incident density n of the order of $10^{-2}\,n_{\mathrm{Br}}$

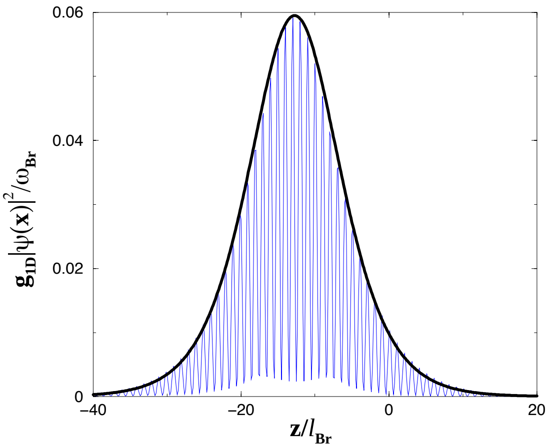
($n_{\mathrm{Br}} = \hbar\omega_{\mathrm{Br}}/g$; ^{87}Rb: $n_{\mathrm{Br}} \approx 4\;10^{14}\,\mathrm{cm}^{-3}$).

(I. Carusotto, D. Embriaco, G. C. La Rocca, PRA, in the press, 2002)

Solitonic nature of the pulse

Comparison with analytics $\bar{\psi}_{\rm sol}(z) = \bar{\psi}_{\rm max}\,{\rm sech}\left(\frac{z - v_g t}{\xi_{\rm sol}}\right)$

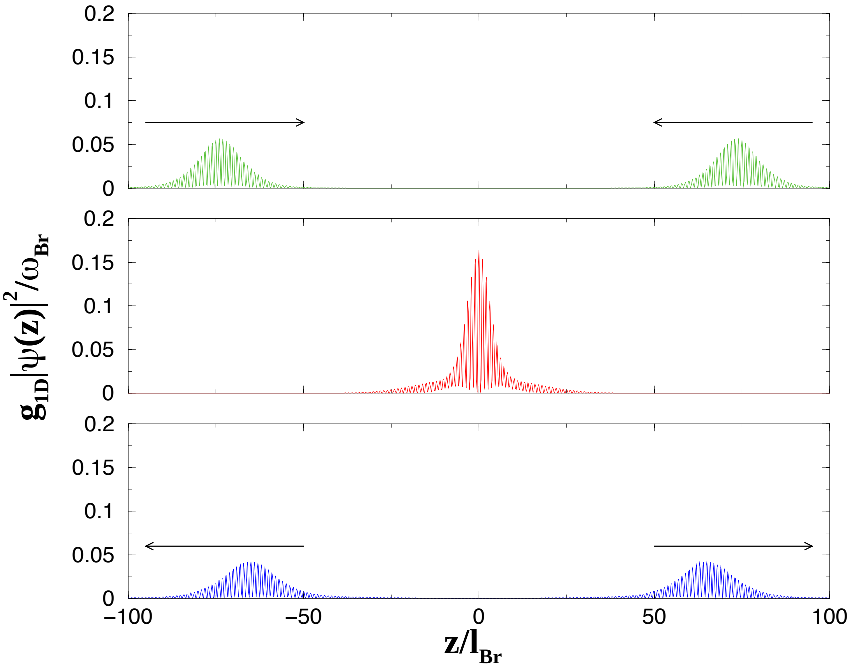
with effective healing length $\xi_{\rm sol} = \sqrt{\frac{\hbar^2}{|m_{\rm eff}|\, g_{\rm eff}\, |\bar{\psi}_{\rm max}|^2}}$



- At end of lattice, $m_{\rm eff}$ again positive

  $\Longrightarrow$ pulse no longer bound

- Collisions nearly preserve shape



Conclusions and perspectives

Bright gap solitons:

- positive scattering length
- small and negative effective mass
- effectively attractive interactions
- increased healing length and soliton size

A new scheme for **generating solitons**:

- not from the whole BEC cloud (risk of large heating, uncontrolled collapse)...
- ... but from a much shorter density bump of standing wave antinode
- modulational instability seeded from strongly modulated standing wave density profile

Perspectives:

- seeded modulational instability in different geometries
- gap soliton lifetime in presence of thermal cloud (calculations via stochastic field techniques ???)
- the **experiment**