Lecture 3 Static properties of BEC

Standard BEC "at equilibrium", e.g. ultracold atoms
(and liquid He, to some extent)

Order parameter $\psi(r,t) = \langle \hat{\psi}(r,t) \rangle$ of Spont. Symmetry Breaking.

Gross-Pitaevskii eq. (GPE)

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{ext}\psi + \frac{4\pi\hbar^2 a}{m}|\psi|^2\psi$$

equivalent of Maxwell's eqs for atomic Bose field.

Ground state: $$\mu\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{ext}\psi + \frac{4\pi\hbar^2 a}{m}|\psi|^2\psi$$

which minimizes μ at given $N = \int d^3r\, |\psi|^2$

* homogeneous system $V_{ext} = 0$: $\psi(r) = \sqrt{n}$
  $\mu = g n$ "equation of state"

* slowly varying external potential $\psi(r) = \sqrt{\dfrac{\mu - V_{ext}(r)}{g}}$

  Thomas-Fermi approx valid if:
  $\rightarrow \dfrac{\hbar^2}{2m\ell_V^2} \ll \mu$ (ℓ_V = length of variation of V)

  i.e. $\ell_V \gg \sqrt{\dfrac{\hbar^2}{m\mu}} = \xi$ "healing length"

* hard wall $V_{ext}(z) = V_0\,\Theta(-z)$ with $V_0 \gg \mu$
  $\psi(z) = \sqrt{n_\infty}\,\tanh(z/\xi)$ gives physical meaning to ξ

What happens at non-equilibrium, i.e. in the photon case?

external coherent drive and/or amplification is needed to compensate radiative (and non radiative) losses.

→ steady-state is not THERMODYNAMICAL equil but rather DRIVEN-DISSIPATIVE steady state.

a) coherent drive at $k_{inc} = 0$: $E_{inc}(r_\parallel, t) = E_{inc}\, e^{-i\omega_{inc} t}$

$$i\partial_t E = \left[\omega_0 - i\Gamma/2 - \frac{\hbar}{2m^*}\nabla^2 + \delta\omega(r_\parallel) + g_{NL}|E|^2\right]E + i\,K\,E_{inc}(r,t)$$

homogeneous case $= 0$

ansatz : $E(r,t) = E\, e^{-i\omega_{inc} t}$

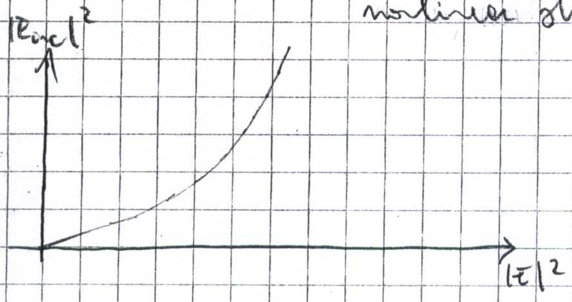
$$\omega_{inc} E = \left[\omega_0 - i\Gamma/2 + g_{NL}|E|^2\right]E + i\,K\,E_{inc}$$

$$\left[\omega_0 + g_{NL}|E|^2 - \omega_{inc} - i\Gamma/2\right]E = -i\,K\,E_{inc}$$

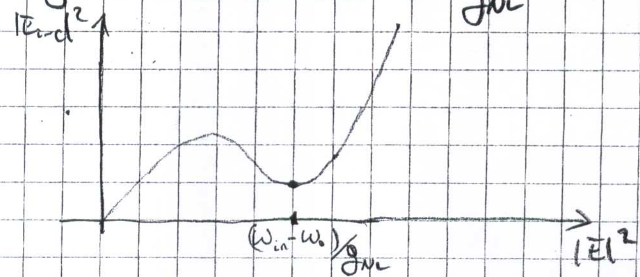
"equation of state"

$$|E|^2\left[\left(\omega_0 + g_{NL}|E|^2 - \omega_{inc}\right)^2 + \Gamma^2/4\right] = K^2\,|E_{inc}|^2$$

nonlinear shift of cavity resonance. Assume $g_{NL} > 0$



$(\omega_{inc} < \omega_0)$



$(\omega_{inc} > \omega_0)$

$\omega_{inc} < \omega_0$: $|E|^2$ is monotonic, sublinear function of $|E_{inc}|^2$

↳ OPTICAL LIMITER

$\omega_{inc} > \omega_0$: non-monotonic behavior

hysteresis : several $|E|^2$ possible for one $|E_{inc}|^2$

↳ OPTICAL BISTABILITY

Slowly varying inhomogeneous system $\delta\omega(r_\parallel)$, $E_{inc}(r_\parallel)$

- "local density approximation" $|E(r_\parallel)|^2$ depends on $|E_{inc}(r)|^2$

+ some prescription in case of bistability to choose which branch

(previous temporal dynamics, spatial profile ...)

Coherent pump at finite k_{inc} :

- ansatz $E_{cav}(r,t) = E\, e^{ik_{inc}r}\, e^{-i\omega_{inc}t}$
- cavity resonance shifted at $\omega_0 + \frac{\hbar}{2m^*} k_{inc}^2$

↳ generate moving photon gas

Is gas a BEC ?

(Penrose - Onsager)

- $\langle E^+(r,t)\, E(r',t) \rangle = E^*(r)\, E(r') \neq 0$ yes!

- but $\langle E(r,t) \rangle \neq 0 \longrightarrow$ no spontaneous symmetry breaking, phase is locked to pump.

More complex mass profiles can generate different configurations : vortices, non-uniform flows, acoustic black holes ...

b) (saturable) amplifier in cavity

$$\delta\omega = + \frac{i}{2} \frac{\Gamma_A}{1 + \underbrace{\eta |E|^2}_{\text{saturation of gain}}}$$

Γ_A ← max gain rate, proportional to (incoherent) external pumping

$$i\partial_t E = \left[\omega_0 - i\frac{\Gamma}{2} - \frac{\hbar}{2m^*}\nabla^2 + \frac{i}{2}\frac{\Gamma_A}{1+\eta|E|^2} + g_{NL}|E|^2\right] E$$

* without external pumping $\Gamma_A = 0$:

  system relaxes back to $E=0$ within time Γ^{-1}

* external pumping $\Gamma_A < \Gamma$ (below threshold) :

  $E=0$ is only steady state, all solutions relax there in time $(\Gamma - \Gamma_A)^{-1}$. For $\Gamma \to \Gamma_A^{-1}$ slow down of relaxation

* above threshold $\Gamma_A > \Gamma$:

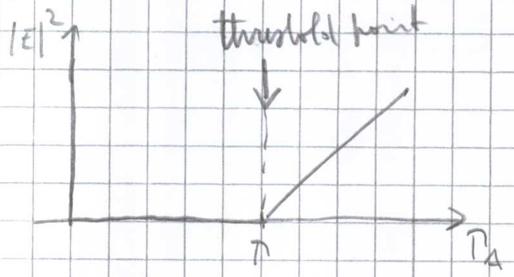
  ansatz $E_{cw}(r,t) = E\, e^{-i\omega t}$ with $\omega \in \mathbb{R}$

$$\omega E = \left[\omega_0 - i\frac{\Gamma}{2} + \frac{i}{2}\frac{P_A}{1+\eta|E|^2} + g_{NL}|E|^2 \right] E$$

Real part $\qquad \omega = \omega_0 + g_{NL}|E|^2$

Imag part $\qquad \Gamma = \dfrac{P_A}{1+\eta|E|^2}$

$$\Rightarrow |E|^2 = \frac{1}{\eta}\left[\frac{P_A}{\Gamma} - 1\right]$$



Is this BEC ?

- Penrose Onsager $\langle E^\dagger(r,t)\, E(r',t)\rangle = |E|^2 > 0$ yes!
- phase of E free : if E solution, $E e^{i\phi}$ also solution
  ↳ one phase randomly chosen, spontaneous symmetry breaking phenomenon

This theory is simple model for :

- VCSEL operation (vertical cavity surface emitting laser)
- "polariton BEC" (or polariton lasing), when E is dressed photon field (lower polariton)

Differently from standard BEC → No thermodynamical equil. !

Time-reversal and shape of BEC

Standard GPE : invariant under T :

if $\psi(r,t)$ solution $\Longrightarrow$ $\psi^*(r,-t)$ is solution

↳ as ground state unique $\Longrightarrow$ ψ has constant phase

↳ no flow

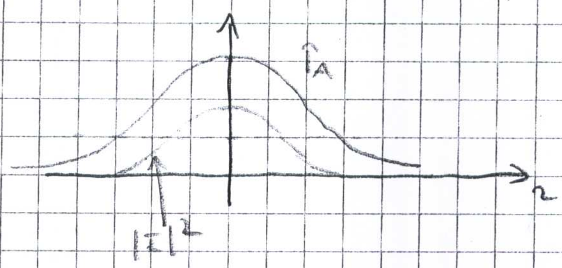
$$j = \frac{1}{2i}\left[\psi^* \nabla\psi - \nabla\psi^* \psi\right] = 0$$

No longer true in the presence of losses/driving/absorption

↳ flow in steady state signature of non-equilibrium

Experiment : $\Gamma_A(r)$ slowly varying in space, e.g. Gaussian shape

locally $|E(r)|^2 = \frac{1}{\eta}\left(\Gamma_A(r) - \Gamma\right)$



we require BEC oscillate coherently at single ω :

$$\omega = \omega_0 + \frac{\hbar}{2m^*}\nabla^2 E + g_{NL}|E(r)|^2$$

spatial variation of $|E(r)|^2$ has to be compensated

by kinetic energy term.

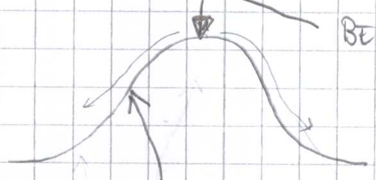
$$E(r,t) = e^{i\omega t}|E(r)|e^{i\phi(r)}$$

with $k = \nabla\phi(r)$ such that

$$\frac{\hbar}{2m}k(r)^2 + g_{NL}|E|^2 = \omega - \omega_0 = c^{te}$$

↳ radial flow of photons in steady state

Image of volcano:



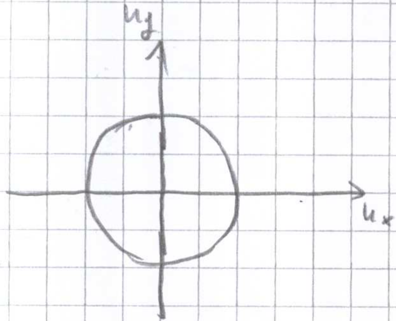
then photons accelerate while sliding down potential hill

At each point r: $\frac{\hbar}{2m^*}k^2 + V = c^{te}$ $\quad g_{NL}|E|^2$

from conservation of energy

In k-space $n(k) = |\mathcal{F}[E(k)]|^2$:

Fourier transform of radial flow is centred on a ring at k (provided radial speed is uniform)



to be contrasted with standard BEC $n(k) \simeq \delta(k)$

Bibliography

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