Optical Non-Reciprocity and Optical Diodes

Consider an externally driven emitter oscillating at frequency ω with complex-valued amplitude $\mathbf{d}$

1. Calculate the electric field in the far-field region for the two cases of a linearly polarized $\mathbf{d} = d_0\,\hat{e}_z$ dipole and of a circularly polarized $\mathbf{d} = \frac{d_0}{\sqrt{2}}\,[\hat{e}_x + i\,\hat{e}_y]$ one.
2. Compare the geometrical shapes of phase-fronts of the emitted wave in the two cases and identify the main differences.

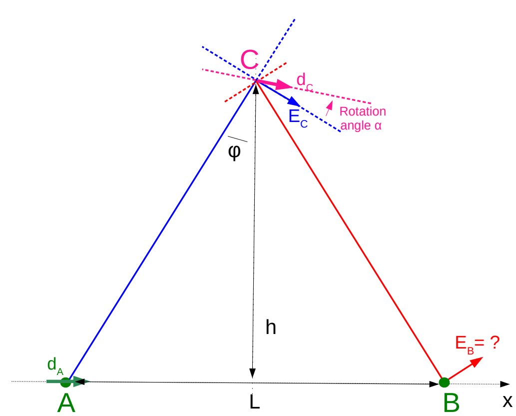
Consider a polarizable object displaying different susceptibilities in the circular $\sigma_\pm$ polarizations defined as $(\hat{e}_x \pm i\hat{e}_y)/\sqrt{2}$,

$$\chi = (\chi_0 + \chi_1)\,|\sigma_+\rangle\langle\sigma_+| + (\chi_0 + \chi_1)\,|\sigma_-\rangle\langle\sigma_-|\,. \tag{1}$$

3. Determine the amplitude and orientation of the dipole induced by a linearly polarized electric field directed at an angle ϑ with respect to the x axis, $\hat{e}_\vartheta = \cos\vartheta\,\hat{e}_x + \sin\vartheta\,\hat{e}_y$.
4. Propose some simple atomic configuration which displays the *circular birefringent* behaviour encoded in Eq.1.

Take an emitter and an antenna located at points A and B separated by a distance L along the x direction as in the sketch. The emitter has an oscillating dipole oriented along x, $\mathbf{d} = d_0\,\hat{e}_x$.

5. Show that no field is detected by the antenna.



In order to obtain effective electromagnetic transmission from the emitter to the antenna, we introduce a third polarizable object of susceptibility χ as defined in Eq.1 at the point C located a distance h from the mid-point of the emitter-antenna segment. In doing this, it is useful to define the oriented angle φ between the y axis and the AC segment and to consider the rotation angle α between the induced dipole and the applied electric field.

6. Evaluate the electric field generated by the emitter at the location C of the polarizable object.

7. Evaluate the electric dipole induced in the object via its susceptibility χ as in Eq.1.

8. Evaluate the amplitude of the electric field that is scattered to the antenna's position B.

Let us now assess the performance of this configuration as an *optical diode*.

9. Repeat the previous calculation for a configuration with exchanged positions of the emitter and the antenna. Is the amplitude of the field reaching the antenna the same as before? Interpret the result as the non-reciprocal behaviour of an optical diode.

10. Identify the angle φ for which the optical diode behaviour is the strongest.

Let us now unravel the necessary conditions to observe the optical diode behaviour. Consider a linearly birefringent medium displaying a real-valued and symmetric susceptibility.

11. Repeat the previous calculation for a linearly birefringent object and show that the transmission is in this case fully *reciprocal*, with the same transmission amplitude in the two directions of propagation.

12. Show that this result directly generalizes to arbitrary numbers of scattering objects and to arbitrary numbers of scattering events within perturbation theory. Interpret this general result as an example of *reciprocity theorem*.

13. Discuss the connection between the presence of a finite circular birefringence and time-reversal-symmetry breaking.