Light emission from an emitter located in front of a mirror

Consider a one-dimensional model of an emitter located at a distance L from a perfect mirror. The emitter is modeled as a charged harmonic oscillator of charge q, mass m and natural frequency ω_0. As it was discussed in Lecture 3, this models a planar layer of charges (a so-called dipole sheet) located at a distance L from a planar mirror oriented along the xy plane and oscillating along the transverse x direction.

In the first part of the exercise, assume that the emitter is directly driven by a monochromatic drive of amplitude F and frequency ω.

1. Imposing the boundary condition that the field has to vanish at the mirror position $E(z = 0) = 0$, write the amplitude of the radiated field as a function of the current in the emitter. To this purpose, it is useful to write the field in the $0 < z < L$ region as the superposition of left- and right-going waves propagating in opposite directions with opposite amplitudes $E_{\mathrm{R}} = -E_{\mathrm{L}}$, and the field in the $z > L$ region as a purely right-going wave of amplitude E_{rad}; the current in the emitter sets the boundary condition between the field in the two regions. Interpret the result in terms of Feynman diagrams for the different emission processes allowed.

2. Taking inspiration from the Drude-Lorentz model with an additional force and keeping in mind that all fields oscillate at the drive frequency ω, write the motion equation for the current in the emitter and determine its steady-state under the combined effect of the external force of amplitude F and frequency ω and the local value $E(z = L)$ of the radiated electric field.

3. Combine the two equations to determine the radiated field amplitude as a function of the driving amplitude and frequency. Determine the form of the radiative broadening and relate it to the result of point (1) for the radiation amplitude.

4. What is the physical meaning of the imaginary part of the radiative broadening term?

In the second part of the exercise, consider the case where at $t = 0$ the emitter is initially set into motion with a current amplitude J_0 and is then let free to evolve in the absence of any driving force.

5. Assuming that the radiative damping occurs on a time-scale τ_{rad} that is very long as compared to the distance from the mirror and to the emitter frequency, write an effective equation of motion for the emitter dynamics. Interpret this equation in terms of radiative damping. Write a quantitative condition for the validity of the effective equation of motion and give a physical interpretation to the "non-Markovian" corrections stemming from the frequency-dependence of the radiative broadening.