Optical bistability and optical limiting

Consider an optical cavity formed by a pair of plane-parallel mirrors of reflectivity R placed at a distance L and enclosing an optically nonlinear material of intensity-dependent refractive index

$$n = n^{(1)} + n^{(3)}\,|E|^2.$$

We wish to characterize the transmission of light through the cavity as a function of the intensity and frequency of the incident light. For simplicity, assume that the nonlinear refractive index $n^{(3)}$ is negative and restrict yourself to a single-mode description.

1. Justify why the evolution of the in-cavity field amplitude can be described by the ordinary nonlinear differential equation

$$i\frac{d\alpha}{dt} = \omega_0\,\alpha + \omega_{nl}|\alpha|^2\,\alpha - i\frac{\gamma}{2}\alpha + F_{inc}\,e^{-i\omega t} \tag{1}$$

   and identify the main physical assumptions underlying this model. Relate the coefficients appearing in this equation to the cavity parameters. Relate the amplitude of the transmitted field to the in-cavity field amplitude α, and the amplitude of the incident field to the driving term F_{inc}.

2. Assuming the system has reached a steady-state with $\alpha(t)$ oscillating at the incident frequency ω, write an expression for the incident intensity $I_{inc} = |F_{inc}|^2$ as a function of the transmitted intensity I_{tr} (proportional to the internal one $I_{int} = |\alpha|^2$) and of the incident frequency ω.

3. Make a schematic plot of the I_{inc} vs. I_{tr} dependence in the two cases of $\omega \gtrless \omega_0$.

   (a) For $\omega < \omega_0$, interpret this result in terms of optical limiting.

   (b) For $\omega > \omega_0$, identify the regions where this dependence is not monotonic. Interpret the result in terms of optical bistability.

4. How does the system behave under a slow ramp of the incident intensity at a fixed frequency? What happens when the ramp is reversed?

5. Using some plotting software, plot the fixed-I_{inc} curves in the (ω, I_{tr}) plane. Identify the optical bistability regions. How does the system behave under a slow ramp of the incident frequency ω?