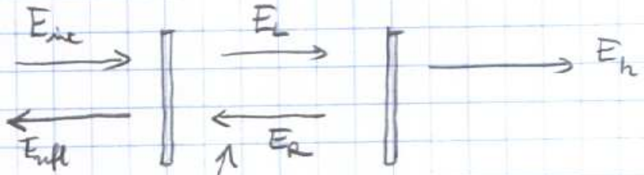
exercise 2 : the nonlinear Fabry-Perot interferometer

Kerr optical nonlinearity $\chi^{(3)}$: $P = 1 + 4\pi\left(\chi^{(1)} E + \chi^{(3)} |E|^2 E\right)$

$\hookrightarrow$ refractive index $n = (1 + 4\pi\chi)^{1/2}$ depends on light intensity

$$\Downarrow \quad n = \left(1 + 4\pi\chi^{(1)} + 4\pi\chi^{(3)} |E|^2\right)^{1/2}$$



refractive index depends on E.

* normally one should have spatially dependent $n(x)$

$$n(x) = \left[\chi^{(1)} + \chi^{(3)} \cdot |E(x)|^2\right]^{1/2}$$

$\hookrightarrow$ sinusoidal profile of cavity mode

* full solution of NL. equations can be written in terms of elliptic functions

[see e.g. Marburger and Felber PRA $\underline{17}$, 335 (1978)]

* simplifying assumption $n = n^{(1)} + n^{(3)}\left[|E_L|^2 + |E_R|^2\right]$

spatially homogeneous

for $r \simeq 1$, $t \ll r$:

$$E_R^{(2)} \simeq E_L^{(2)} = \frac{1}{t} E_{tr}$$

$$n = n^{(1)} + n^{(3)} \left[|E_L|^2 + |E_R|^2 \right] \simeq n^{(1)} + \frac{2}{|t|^2} n^{(3)} |E_{tr}|^2$$

$$|E_{inc}|^2 = \frac{1}{T_{tot}} |E_{tr}|^2 = \frac{1}{T^2} \left| 1 - r^2 e^{2i n \omega L/c} \right|^2 \cdot |E_{tr}|^2$$

depends on n, thus on $|E_{tr}|^2$

New resonance condition $\quad 2 \frac{\omega L}{c} \cdot \left[n^{(1)} + n^{(3)} \left[|E_L|^2 + |E_R|^2 \right] \right] = 2\pi M$

Develop around single resonance $\quad 2 n^{(1)} \omega_0 L/c = 2\pi M$

$$2 n \omega L/c \simeq 2 n^{(1)} \omega_0 L/c + 2 n^{(1)} \omega_0 L/c \, \frac{\omega - \omega_0}{\omega_0} + $$
$$+ 2 n^{(1)} \omega_0 L/c \, \frac{n^{(3)}}{n^{(1)}} \cdot \left[|E_L|^2 + |E_R|^2 \right]$$

$$\frac{|E_{inc}|^2}{|E_{tr}|^2} = \frac{1}{T^2} \left| 1 - R \exp\left\{ 2\pi M \, i \left(\frac{\omega - \omega_0}{\omega_0} + \frac{n^{(3)}}{n^{(1)}} \left[|E_L|^2 + |E_R|^2 \right] \right) \right\} \right|^2$$

$$\simeq \frac{1}{T^2} \left[(1-R)^2 + R^2 (2\pi M)^2 \left(\frac{\omega - \omega_0}{\omega_0} + \frac{n^{(3)}}{n^{(1)}} \left[|E_L|^2 + |E_R|^2 \right] \right)^2 \right]$$

$$= \frac{1}{T^2} \left[(1-R)^2 + \left(\frac{2\pi M R}{\omega_0} \right)^2 \left(\omega - \omega_0 + \frac{n^{(3)}}{n^{(1)}} \omega_0 \left[|E_L|^2 + |E_R|^2 \right] \right)^2 \right\}$$

→ nonlinearity shifts resonance by

$$\Delta\omega_{nl} = -\frac{n^{(2)}}{n^{(1)}}\,\omega_0\left(|E_L|^2+|E_R|^2\right) = -\frac{2n^{(2)}\omega_0}{n^{(1)}T}\,|E_{tr}|^2$$

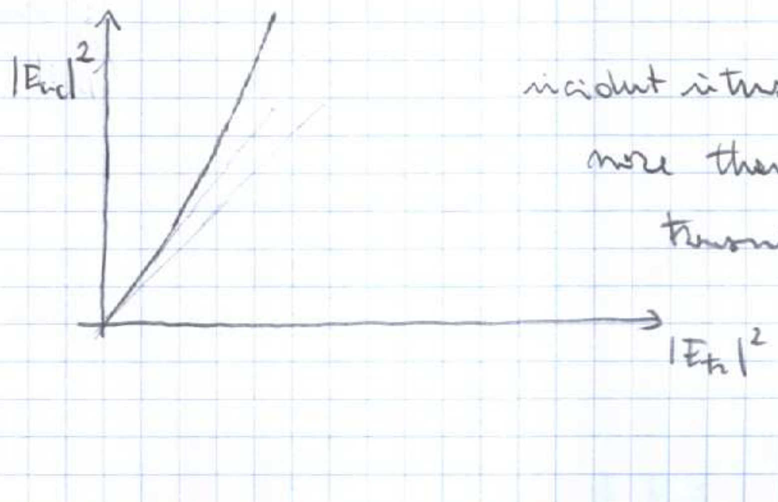
Assume $n^{(2)} < 0$ → resonance BLUE-shifts.

* if $\omega < \omega_0$ → NL shifts resonance further away
NEGATIVE feedback on transmiss.

* if $\omega > \omega_0$ → NL approaches resonance
POSITIVE feed back.

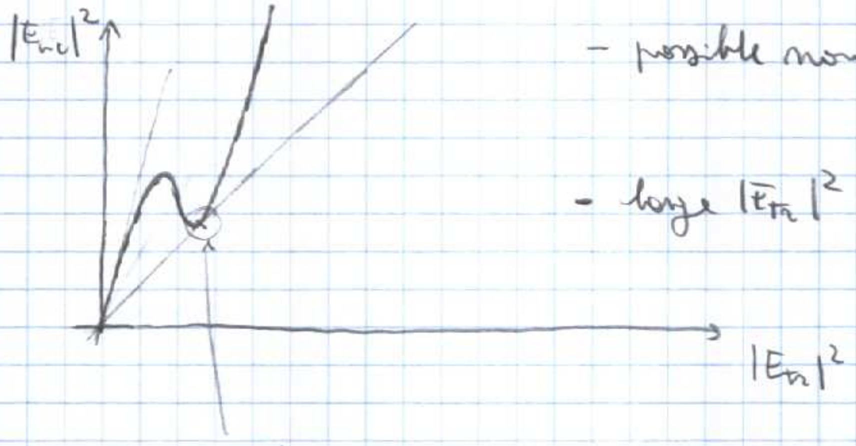
$$|E_{inc}|^2 = \left(\frac{2\pi M}{\omega_0 T}\right)^2\left\{\left[\omega-\left(\omega_0+\frac{2n^{(2)}\omega_0}{n^{(1)}T}|E_{tr}|^2\right)\right]^2+\left[\frac{\omega_0}{2\pi M}(1-R)\right]^2\right\}\times |E_{tr}|^2$$

* <u>if $\omega-\omega_0<0$:</u>



incident intensity grows more than linearly with transmitted intensity

* if $\omega - \omega_0 > 0$:



- possible non-monotonic behaviour

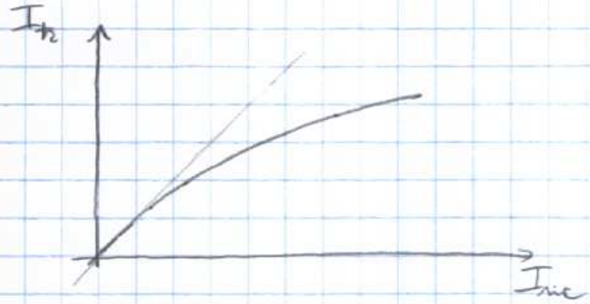
- large $|E_{tr}|^2$ limit same as before

resonant point

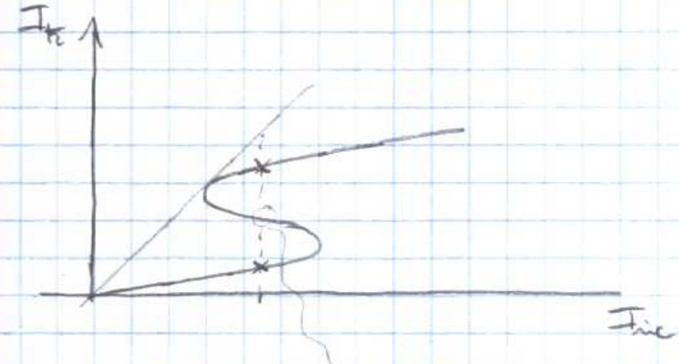
$$\omega = \omega_0 + \frac{2|n^{(3)}|\,\omega_0}{n^{(1)}\,T} \cdot |E_{tr}|^2$$

$\hookrightarrow |E_{inc}|^2 = |E_{tr}|^2$.

Physical discussion



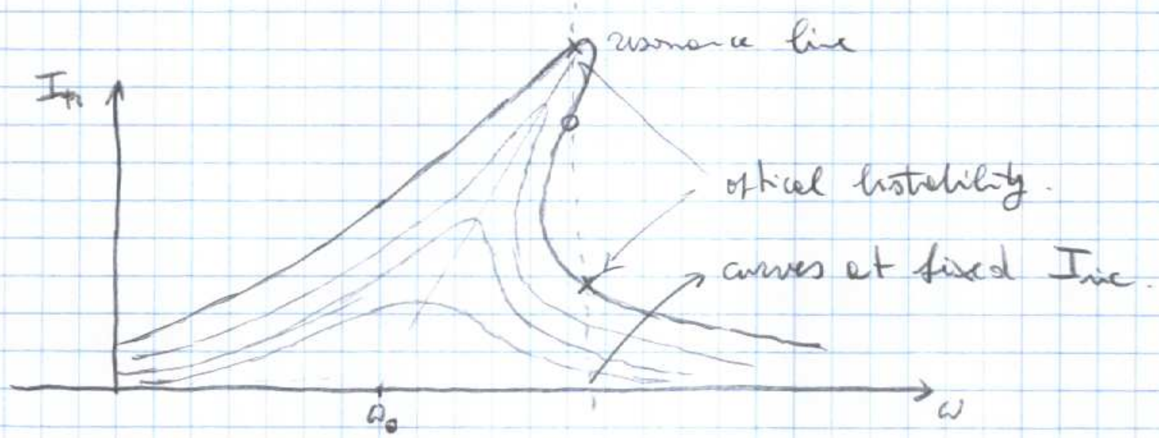
optical limiter



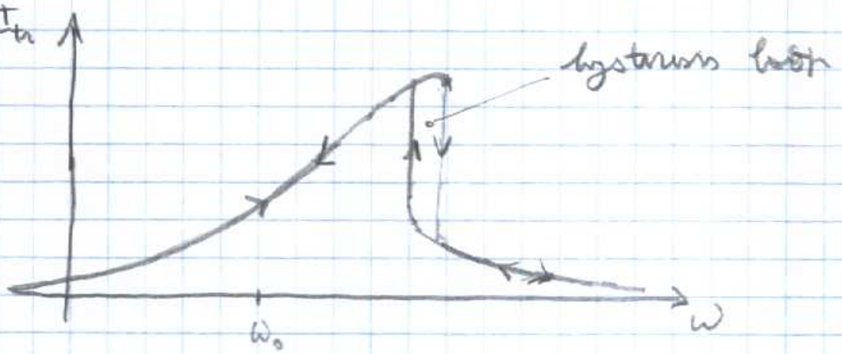
optical bistability (hysteresis loop)

unstable solution.

* In the (ω, I_{tr}) plane:



* Ramp of ω at given I_{inc}:



A modern application: Latta et al Nat. Phys 5, 758 (2009)

↳ nonlinearity arising from nuclear spin dynamics.

NOTE The same results can be obtained using the time-evolution approach to the cavity field dynamics of 2.19 - 2.20 with an additional non-linear frequency-shift term:

$$i\dot{\alpha} = \omega_0 \alpha - i\frac{\gamma}{2}\alpha + \omega_{nl}|\alpha|^2 \alpha + i f(t)$$