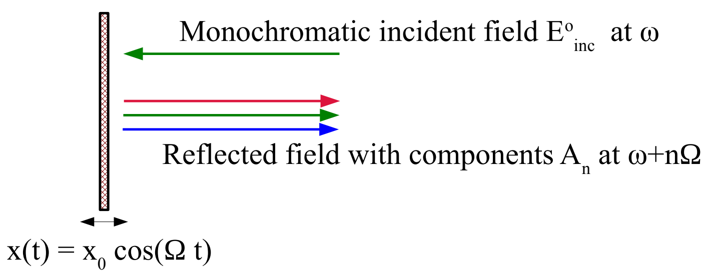
Phase conjugating mirror (and dynamical Casimir effect for the brave ones)



Consider a one-dimensional of a mechanically oscillating, perfectly reflecting metallic mirror, whose position follows a harmonic law $x_m(t) = x_0\,\cos(\Omega t)$. The mirror is illuminated by a (real-valued) monochromatic incident field

$$E_{\text{inc}}(x,t) = E^o_{inc}\,e^{-i\omega\left(\frac{x}{c}+t\right)} + E^{o\,*}_{inc}\,e^{+i\omega\left(\frac{x}{c}+t\right)}. \tag{1}$$

The goal of the exercise is to characterize the reflected field and describe the amplitude of its frequency components.

1. Justify the assumption of writing the reflected field as a superposition of many frequency components spaced by Ω,

$$E_{\text{refl}}(x,t) = \sum_{n=-\infty}^{\infty} A_n e^{i(\omega+n\Omega)\left(\frac{x}{c}-t\right)} + \sum_{n=-\infty}^{\infty} A_n^* e^{-i(\omega+n\Omega)\left(\frac{x}{c}-t\right)}\,. \tag{2}$$

2. Justify the assumption of imposing that the field vanishes at the mirror's position,

$$E(x,t)\big|_{x=x_m(t)} = 0 \tag{3}$$

and write explicitly this condition in terms of the incident and reflected fields.

3. Assuming a small amplitude x_0 for the mirror motion, express the amplitudes of the different components of the reflected field in terms of the incident field amplitude $E^o_{\rm inc}$: in particular, explain why the higher components $A_{|n|\geq 2}$ are of higher order in x_0 and give an explicit expression for $A_{0,\pm 1}$.

4. Give a physical interpretation to the sidebands at $\omega \pm \Omega$.

From now on, focus on the most exciting $\Omega > \omega$ case where the mirror oscillation frequency is larger than the incident frequency.

5. Identify the observable frequency of the different components in the expression Eq.(2) of the reflected field. In particular, offer a physical interpretation to the fact that the sideband at $\omega - \Omega$ falls at negative frequency.

6. Write the complex-valued amplitude of the observable reflected field at $\Omega - \omega$ in terms of the incident amplitude $E^o_{\rm inc}$ at ω.

7. Translate this relation to the time-domain and express it in terms of a *phase conjugation* effect.

8. Critically discuss the experimental feasibility of such a configuration in the different frequency windows, e.g. optical and microwaves.

More advanced questions (to be addressed at the end of the Quantum Optics course):

9. Interpreting E^o_{inc} and E^{o*}_{inc} as the classical counterpart of destruction and creation operators in the incident field, provide an analogous interpretation for the reflected field amplitudes.

10. Express the reflection conditions found in the previous part of the exercise as a relation between the operators for the reflected and the incident fields.

11. Discuss the quantum consequences of having the destruction operator of the reflected field proportional to the creation operator of the incident field. Estimate the zero-point emission from the moving mirror in the absence of any incident light, the so-called *Dynamical Casimir Effect*.