Extinction vs. Absorption

Consider a non-magnetic optical medium described by the Drude-Lorentz dielectric constant:

$$\epsilon(\omega) = 1 + \frac{4\pi n e^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega}. \tag{1}$$

The goal of the exercise is to characterize the plane-wave solutions for light propagation in such a medium with

$$\mathbf{E}(z,t) = \mathbf{E}_0\, e^{i(kz-\omega t)} \tag{2}$$

$$\mathbf{B}(z,t) = \mathbf{B}_0\, e^{i(kz-\omega t)} \tag{3}$$

with real frequency ω and possibly complex wavevector k.

Consider first the lossless medium case with $\gamma = 0^+$.

1. For a frequency ω within the polariton gap, calculate the Poynting vector.
2. (Optional) For a frequency ω within the polariton gap, calculate the reflection and transmission coefficients through a slab of thickness L surrounded by vacuum. Interpret the result in terms of tunnel effect.
3. For a frequency ω belonging to either the upper or the lower polariton band, calculate the Poynting vector, the energy density and the group velocity. Identify some simple relation relating these three quantities and provide a physical interpretation.

Move on to the weakly lossy case with small γ.

1. At lowest order in γ, calculate the absorption length (inverse of the imaginary part of the wavevector) and discuss its behaviour as a function of frequency. Try to relate it to the group velocity calculated above.

Useful formulas (from Jackson, chap.6 and Landau-Lifshits, vol.8 chap IX):

Poynting vector: $$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\mathbf{E}(\mathbf{r},t) \times \mathbf{B}(\mathbf{r},t) \tag{4}$$

Energy density: $$u = \frac{1}{8\pi}\left[\frac{d[\omega\,\epsilon(\omega)]}{d\omega}\mathbf{E}(\mathbf{r},t)\cdot\mathbf{E}(\mathbf{r},t) + \mathbf{B}(\mathbf{r},t)\cdot\mathbf{B}(\mathbf{r},t)\right] \tag{5}$$

Note that the physically observable quantities are the averaged ones over the optical period.