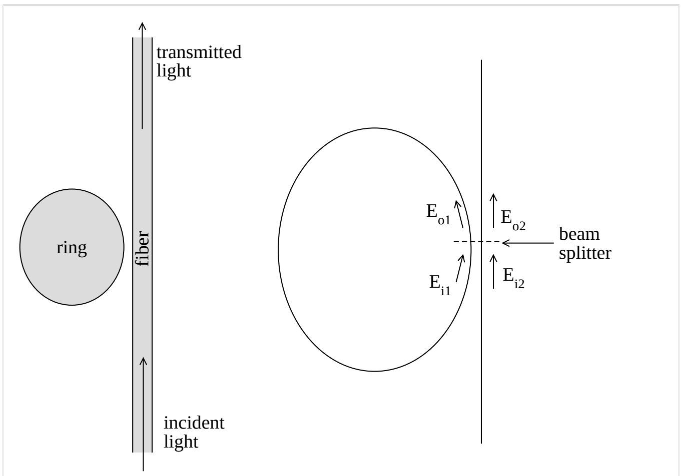
Fibers, Beam-splitters & Rings

Consider the system sketched in the figure: a rectilinear fiber coupled to a ring cavity of radius R. The goal of the exercise is to characterize the amplitude of the transmitted light through the fiber as a function of the incident light frequency ω. We assume that propagation along both the fiber and the disk edge can be described in terms of a one-dimensional model with a frequency-dependent wavevector ω/c.



1. The fiber-ring coupling can be modelled as a two-port beam splitter. A beam splitter is characterized by a two-by-two matrix $\mathbf{S}$ connecting the out-going fields to the in-going ones:
$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = \mathbf{S} \begin{pmatrix} E_{i1} \\ E_{i2} \end{pmatrix} \tag{1}$$
Discuss the constraint imposed on the matrix $\mathbf{S}$ by energy conservation in the case of a non-absorbing beam splitter. Write the most general form of $\mathbf{S}$ for a symmetric, non-absorbing beam-splitter.

2. Assuming the material of the disk to be perfectly transparent, propagation around the disk simply induces a phase shift $2\pi R\omega/c$ to the field. Eliminating the fields in the disk, derive an equation relating the out-going field in the fiber E_{o2} to the in-going one E_{i2}. The ratio $\tau(\omega) = E_{o2}/E_{i2}$ is generally called transmission amplitude.

3. Expand the denominator in the formula for $\tau(\omega)$ in powers and give a physical interpretation of the resulting series in terms of "diagrams".

4. Discuss the frequency-dependence of the transmittivity $T(\omega) = |\tau(\omega)|^2$ and of the transmission phase $\phi(\omega) = \mathrm{Arg}[\tau(\omega)]$. Give a physical interpretation of the resulting delay time $t_{del} = \frac{d\phi(\omega)}{d\omega}$ in terms of radiative linewidth of the ring mode.

5. Generalize the discussion to the case of a weakly absorbing ring (i.e. weak losses per round trip). In particular, study the dependence of the resonant transmission on the relative value of the radiative and non-radiative cavity damping rates. Identify and characterize the so-called *under-coupling* and *over-coupling* regimes.