Optomechanics

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Outline

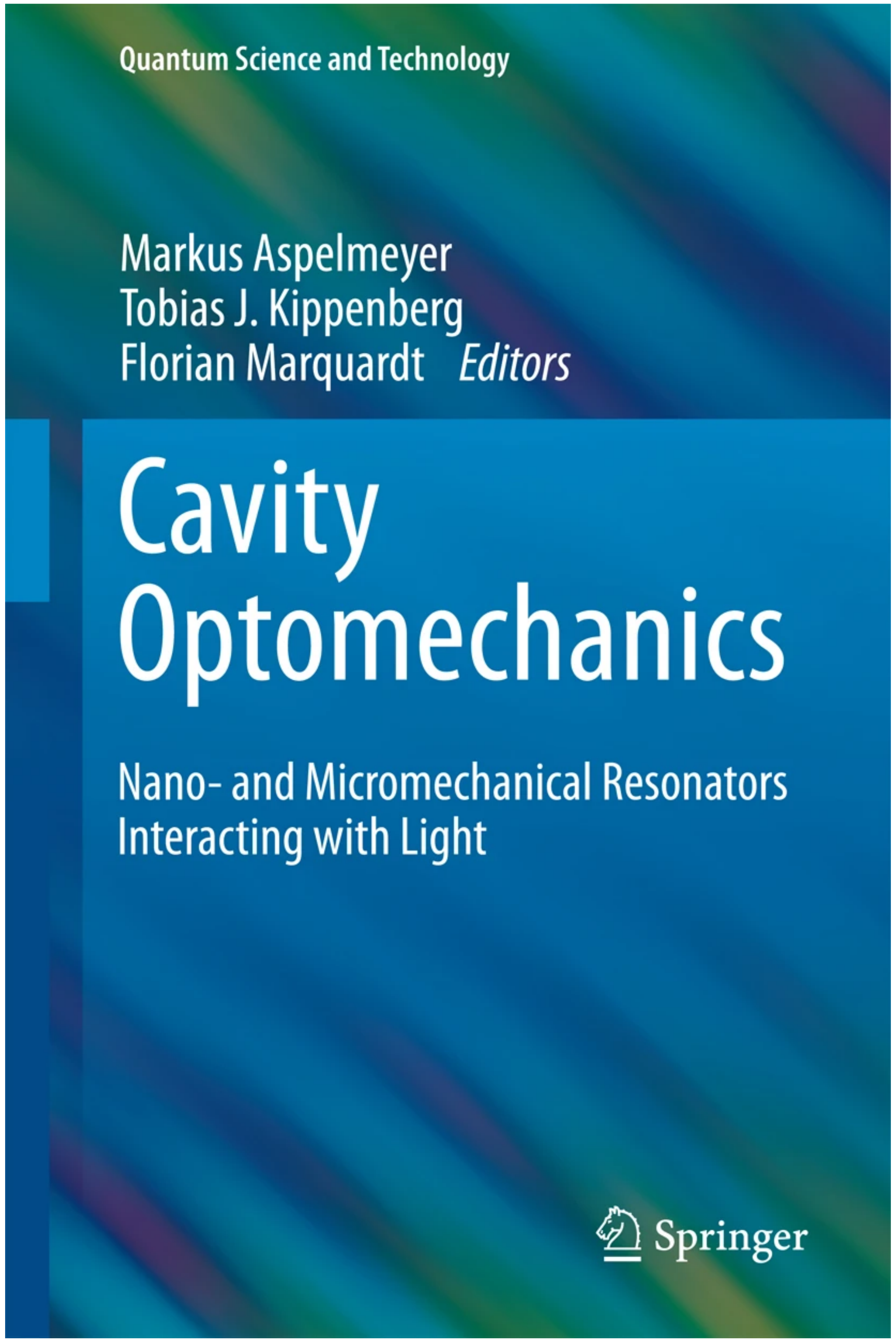
1. Open quantum systems: Caldeira-Leggett model and quantum Langevin equations

2. Optomechanical systems

3. Theory of the ground state cooling

References

Cavity optomechanics (review article)
M. Aspelmeyer, T. J. Kippenberg, F. Marquard, Rev. Mod. Phys. **86**, 1391 (2014)



Other references

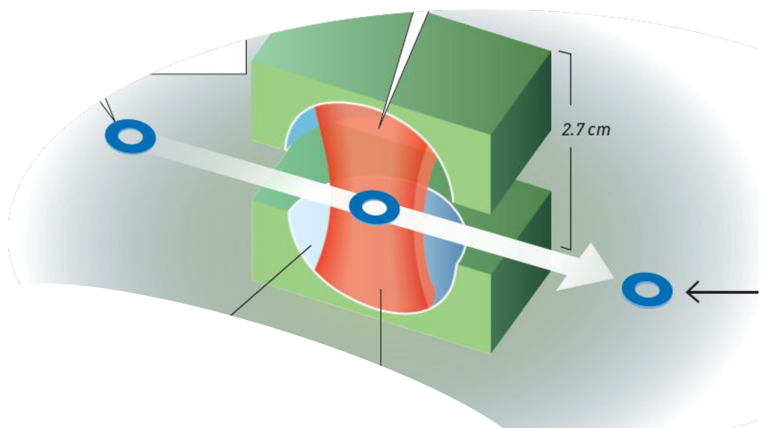
Quantum dissipative systems (book) [Caldeira-Leggett model]
U. Weiss, P. Zoller, World Scientific Ed. (2012)

Quantum noise (book) [Input-Output theory]
C.W. Gardiner, P. Zoller, Springer Ed. (2000)

Quantum Squeezing (book) [Input-Output theory]
P.D. Drummond, Z. Ficek, Springer Ed.
-> **Chapter 3**, pp 53 *Input-Output Theory*, B. Yurke

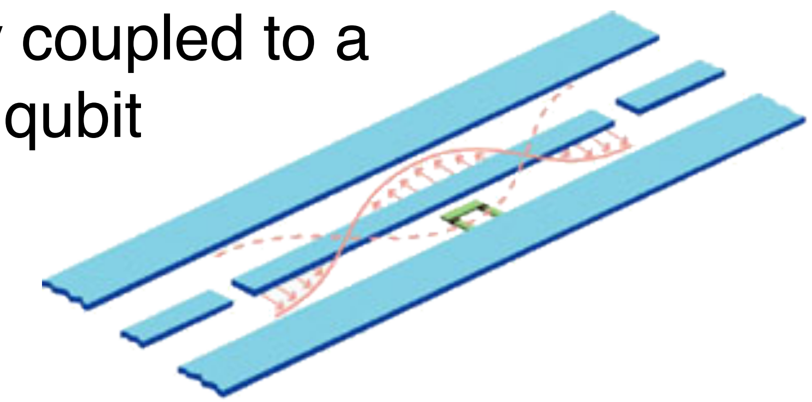
A world of oscillators and spins

Cavity QED
optical cavity coupled to a single atom



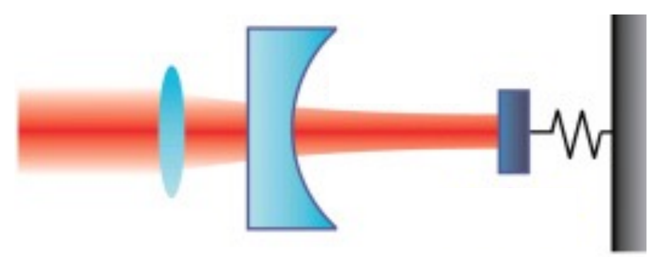
S. Haroche, M. Brune, J. M. Raimond,
Nature Physics vol. **16**, p. 243 (2020)

Circuit QED
microwave cavity coupled to a superconducting qubit



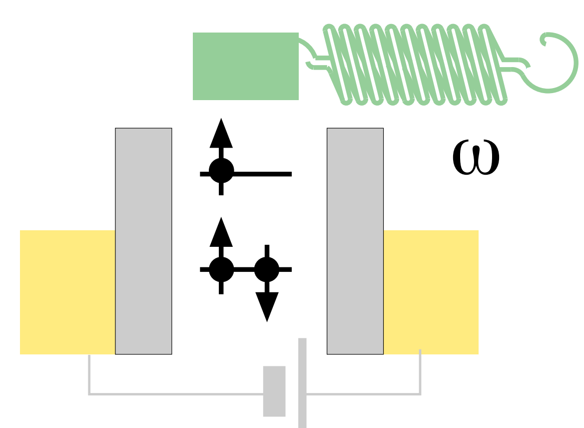
A. Blais, A. L. Grimsmo, S. M. Girvin, A. Wallraff,
Review Modern Physics vol. **92**, p. 025005 (2021)

Optomechanical systems
photon cavity coupled to a mechanical oscillator



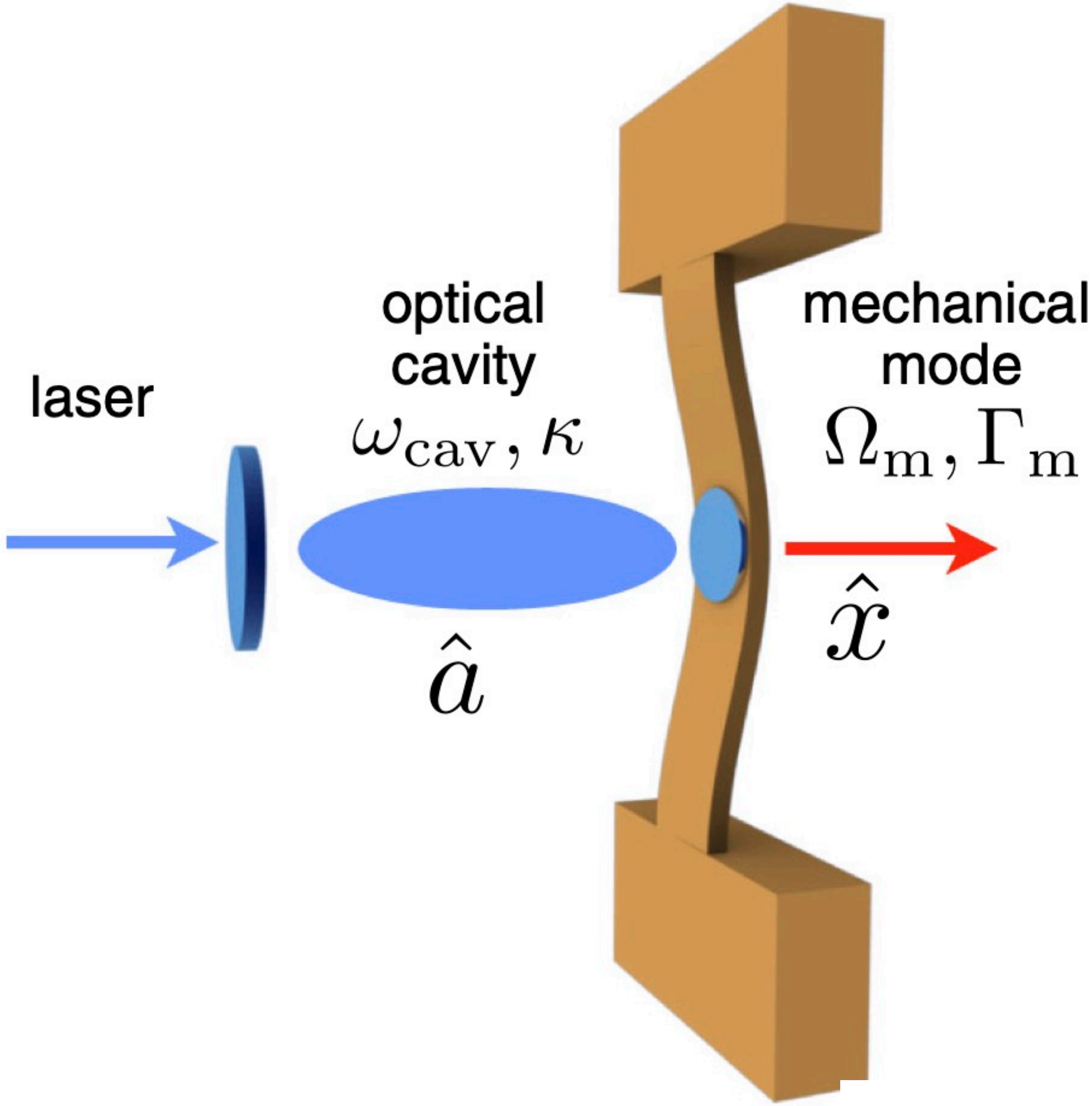
M. Aspelmeyer,
T. J. Kippenberg,
F. Marquard,
Rev. Mod. Phys. vol. **86**,
p. 1391 (2014)

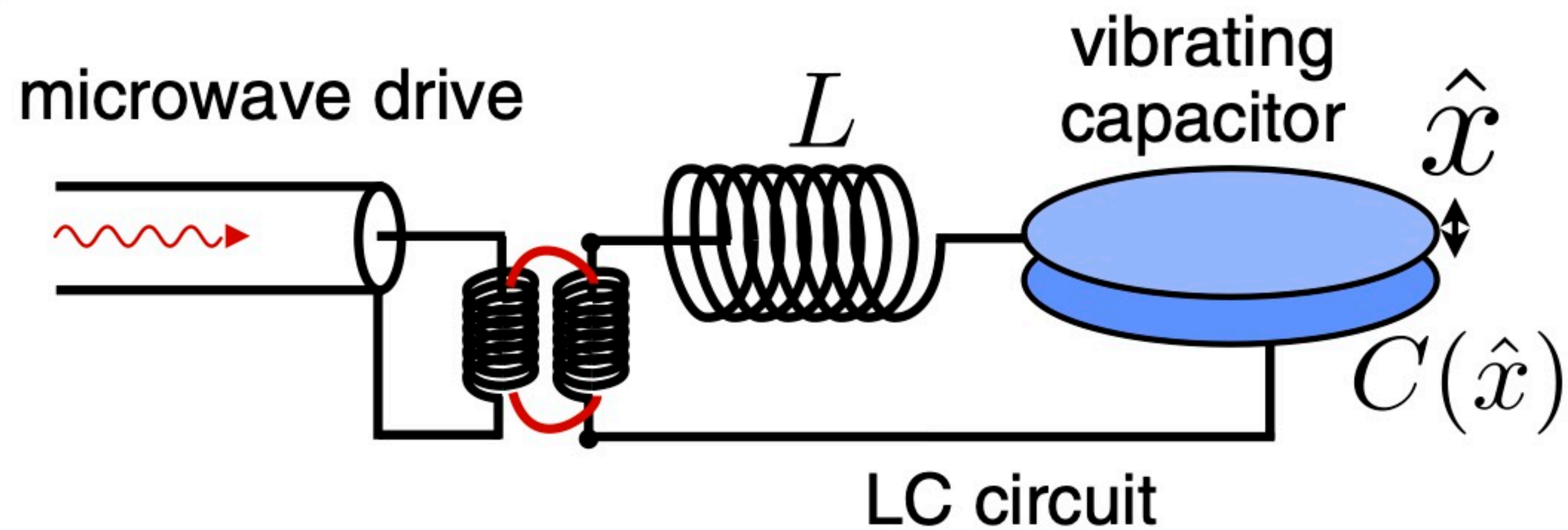
Electromechanical systems
mechanical resonators coupled to quantum conductors

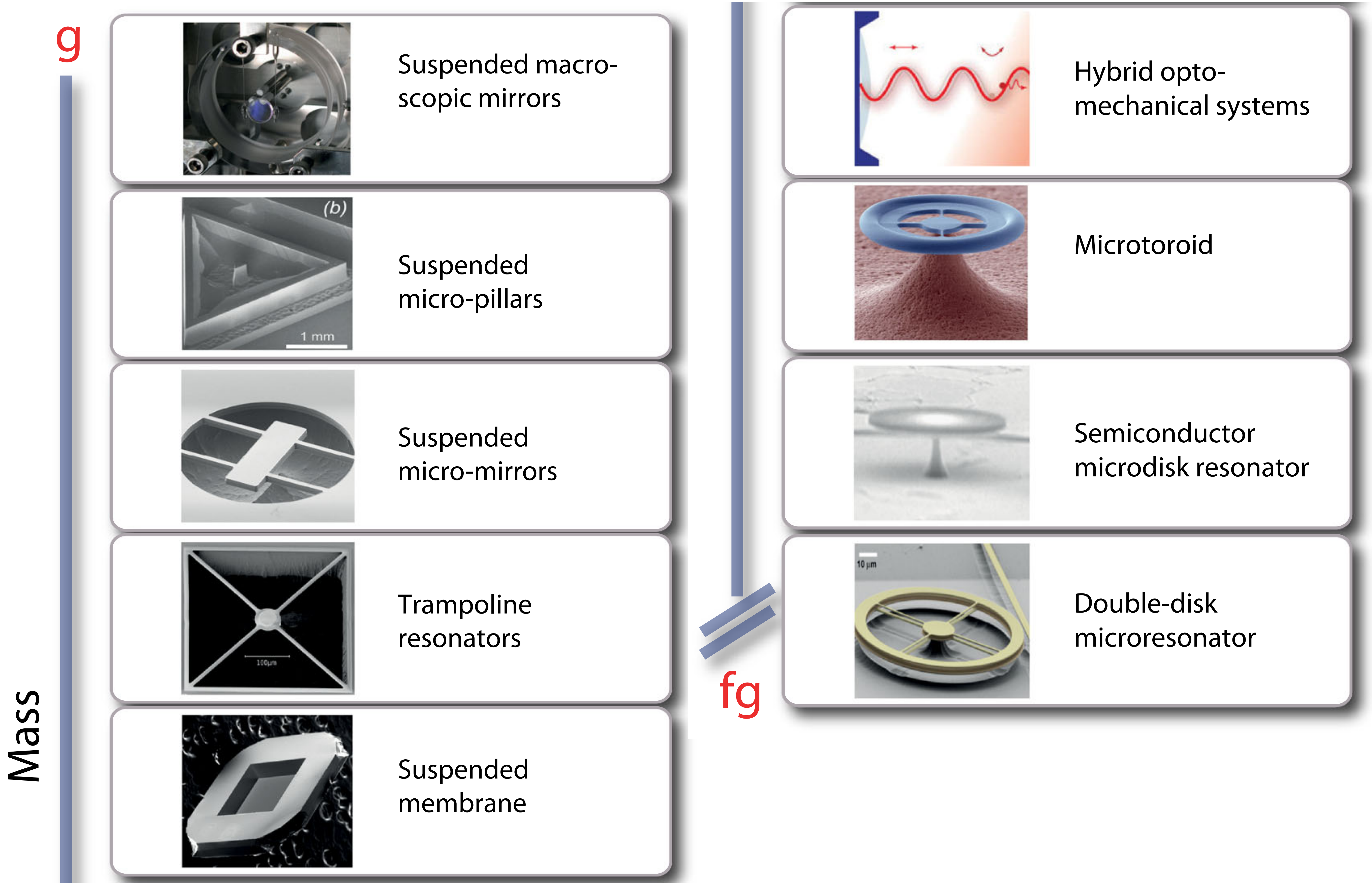


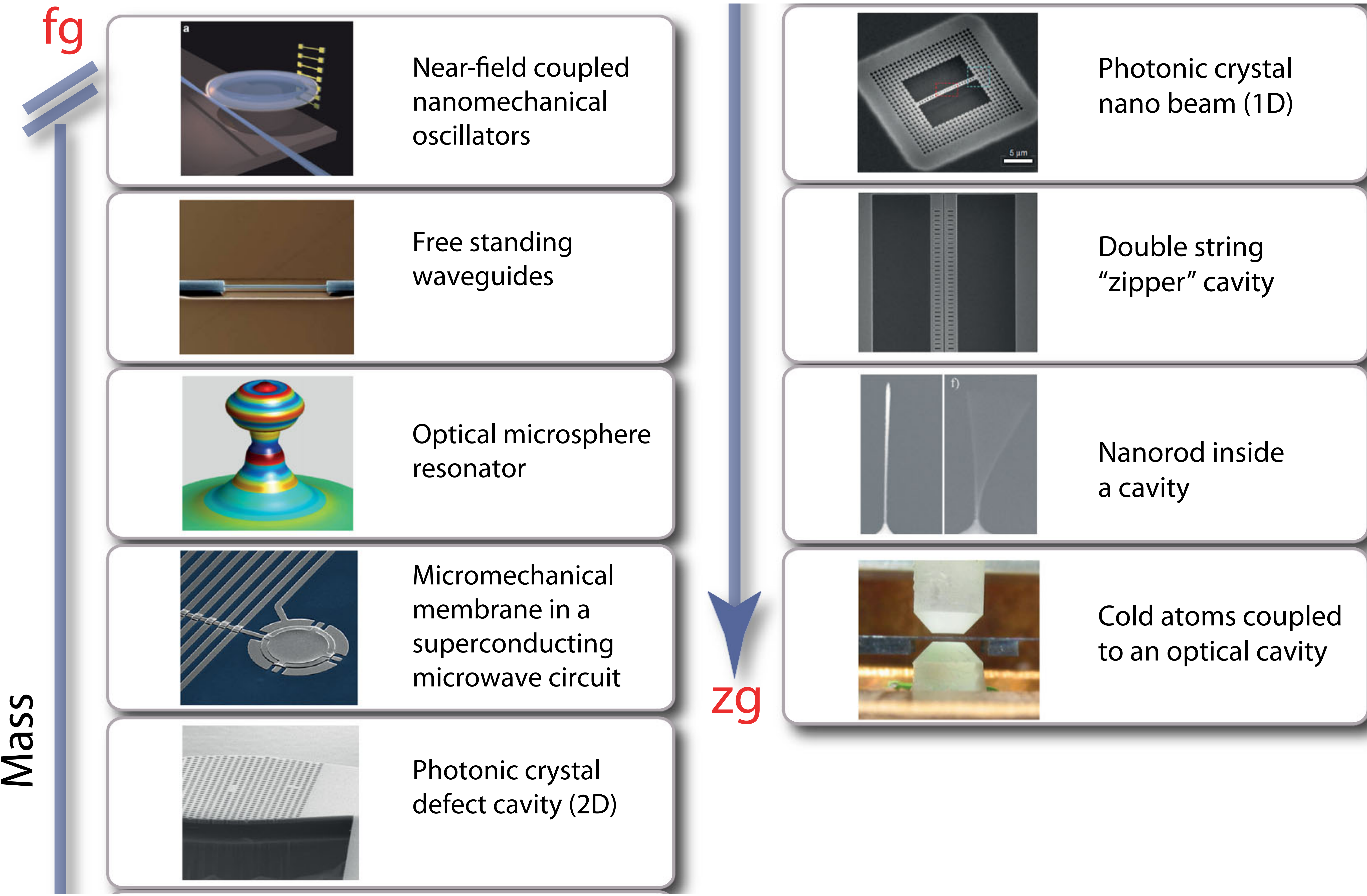
Poot, van der Zant, Phys. Rep. **511** (2012).

Other systems
cold atoms, trapped ions, etc.

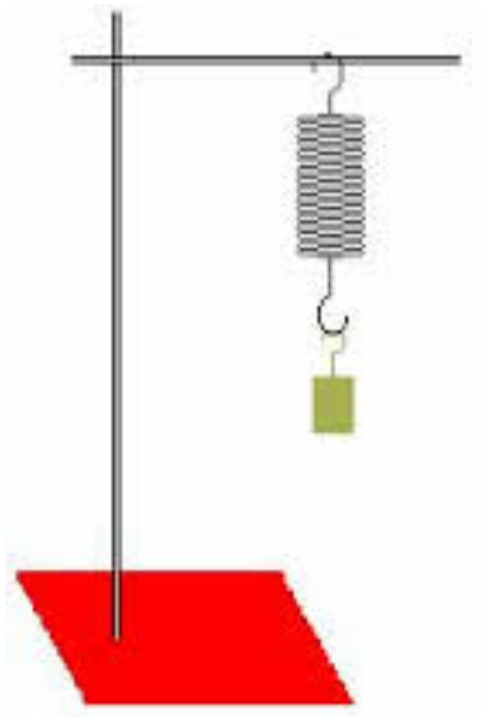

laser
optical
cavity
$\omega_{\rm cav}, \kappa$
$\hat{a}$
mechanical
mode
$\Omega_{\rm m}, \Gamma_{\rm m}$
$\hat{x}$


microwave drive
L
vibrating
capacitor
$\hat{x}$
$C(\hat{x})$
LC circuit

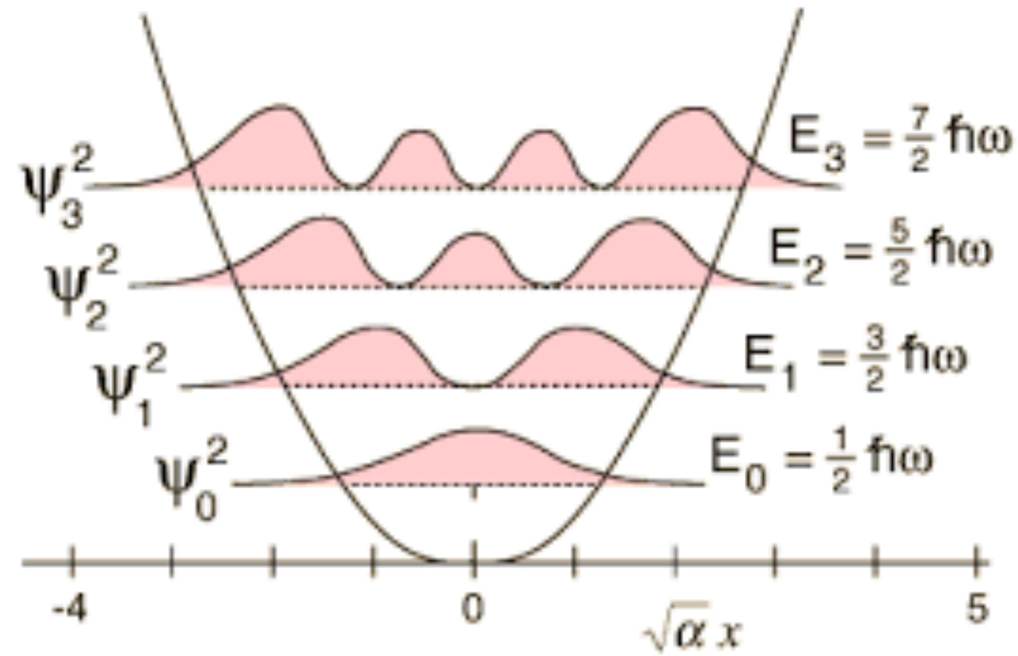

g
Mass
Suspended macro-scopic mirrors
(b)
1 mm
Suspended micro-pillars
Suspended micro-mirrors
Trampoline resonators
Suspended membrane
Hybrid opto-mechanical systems
Microtoroid
Semiconductor microdisk resonator
Double-disk microresonator
fg


fg
Mass
Near-field coupled nanomechanical oscillators
Free standing waveguides
Optical microsphere resonator
Micromechanical membrane in a superconducting microwave circuit
Photonic crystal defect cavity (2D)
zg
Photonic crystal nano beam (1D)
Double string "zipper" cavity
Nanorod inside a cavity
Cold atoms coupled to an optical cavity

Quantum mechanics in macroscopic systems?



?



Classical harmonic oscillator

Quantum harmonic oscillator

Interaction with the external world sets the reality!

Haroche and Wineland

(Nobel Prize 2012 for *"measuring and manipulation of individual quantum systems"*)

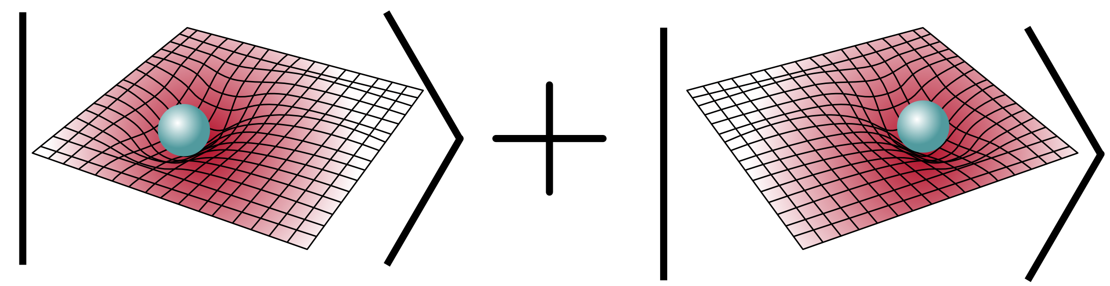
Application: quantum sensors, quantum memories, quantum computers

Optomechanics: quantum effect and gravity?

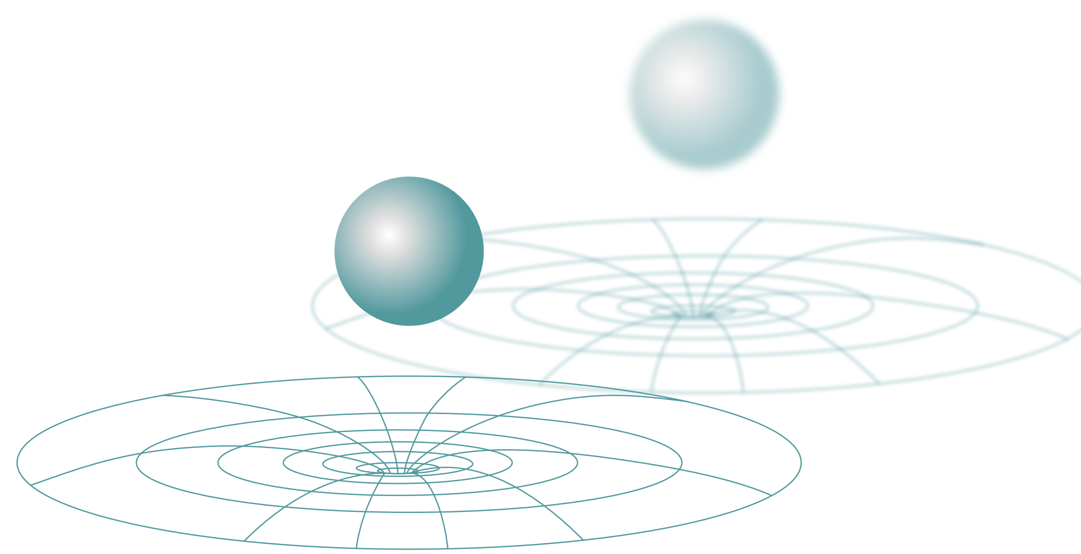
Ground State Cooling of massive object

(energy quantization, vacuum squeezing, etc.)

Quantum Superposition and Mass Interference



Entanglement between Massive Particles



Caldeira-Leggett model and quantum Langevin equations

Caldeira-Leggett model

$$\hat{H}_{CL} = \overbrace{\frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{2}\hat{q}^2}^{\text{system}} + \overbrace{\sum_{n=1}^{N}\left[\frac{\hat{P}_n^2}{2} + \frac{\omega_n^2}{2}\left(\hat{Q}_n - \frac{\lambda_n}{\omega_n^2}\hat{q}\right)^2\right]}^{\text{environment / thermal bath}}$$

$$[\hat{q}, \hat{p}] = i\hbar \qquad \left[\hat{Q}_n, \hat{P}_{n'}\right] = \delta_{nn'}\, i\hbar$$

[all other commutators are zero]

Second quantisation

$$\hat{Q}_n = \sqrt{\frac{\hbar}{2\omega_n}}\,(\hat{a}_n + \hat{a}_n^\dagger) \qquad \hat{P}_n = \frac{1}{i}\sqrt{\frac{\hbar\omega_n}{2}}\,(\hat{a}_n - \hat{a}_n^\dagger) \qquad \left[\hat{a}_n, \hat{a}_{n'}^\dagger\right] = \delta_{nn'}$$

$$\hat{H}_{CL} = \frac{\hat{p}^2}{2m} + \frac{1}{2}\left(m\omega_0^2 + \sum_{n=1}^{N}\frac{2\tilde{\lambda}_n^2}{\hbar\omega_n}\right)\hat{q}^2 + \sum_{n=1}^{N}\hbar\omega_n\hat{a}_n^\dagger\hat{a}_n - \hat{q}\sum_{n=1}^{n}\tilde{\lambda}_n\,(\hat{a}_n + \hat{a}_n^\dagger)$$

$$\tilde{\lambda}_n = \sqrt{\frac{\hbar}{2\omega_n}}\lambda_n$$

Caldeira-Leggett model

Heisenberg's picture: Equations of motion

$$\hat{O}(t) = \hat{U}^\dagger(t)\hat{O}\hat{U}(t) \qquad \hat{U}(t) = e^{-\frac{i}{\hbar}\hat{H}_{CL}(t-t_0)} \qquad \frac{d\hat{O}(t)}{dt} = -\frac{i}{\hbar}\hat{U}^\dagger(t)\left[\hat{O}, \hat{H}_{CL}\right]\hat{U}(t)$$

Equations and solution for the Bath's operators

$$\frac{d\hat{a}_n(t)}{dt} = -i\omega_n\hat{a}_n(t) + \frac{i\tilde{\lambda}_n}{\hbar}\hat{q}(t)\,,$$

$$\hat{a}_n(t) = \hat{a}_n(t_0)e^{-i\omega_n(t-t_0)} + \frac{i\tilde{\lambda}_n}{\hbar}\int_{t_0}^{+\infty} dt'\,\theta(t-t')\underbrace{e^{-i\omega_n(t-t')}q(t')}_{\frac{1}{i\omega_n}\left(\frac{de^{-i\omega_n(t-t')}\hat{q}(t')}{dt'} - e^{-i\omega_n(t-t')}\frac{d\hat{q}(t')}{dt'}\right)}$$

Equations for the harmonic oscillator

$$\frac{d\hat{q}}{dt} = \frac{\hat{p}(t)}{m}$$

$$\frac{d\hat{p}}{dt} = -\left(\omega_0^2 + \sum_{n=1}^{N}\frac{2\tilde{\lambda}_n^2}{\hbar\omega_n}\right)\hat{q}(t) + \sum_{n=1}^{N}\tilde{\lambda}_n\left(\hat{a}_n(t) + \hat{a}_n^\dagger(t)\right)$$

Caldeira-Leggett model

Close equation for the harmonic oscillator

$$\eta(t) = \frac{\theta(t)}{m} \sum_{n=1}^{N} \frac{2\tilde{\lambda}_n^2}{\hbar\omega_n} \cos(\omega_n t)$$

response function

$$\delta\hat{F}(t) = \frac{1}{m} \sum_{n=1}^{N} \tilde{\lambda}_n \left[\left(\hat{a}_n(t_0) - \frac{\tilde{\lambda}_n}{\hbar\omega_n} \hat{q}(t_0) \right) e^{-i\omega_n (t-t_0)} + \text{h.c.} \right]$$

fluctuating force

Thermodynamic limit $N \to \infty$

$$\frac{d^2\hat{q}(t)}{dt^2} = -\omega_0^2 \hat{q} - \int_{t_0}^{+\infty} dt'\, \eta(t-t') \frac{d\hat{q}(t')}{dt'} + \delta\hat{F}(t)$$

Quantum Langevin Equation

Caldeira-Leggett model

Response function

$$\eta(t) = \frac{\theta(t)}{m} \sum_{n=1}^{N} \frac{2\tilde{\lambda}_n^2}{\hbar\omega_n} \cos(\omega_n t)$$

$$\eta(\omega) = \int_{-\infty}^{+\infty} dt\ e^{i\omega t} \eta(t) = \frac{1}{m} \sum_{n=1}^{N} \frac{\tilde{\lambda}_n^2}{\hbar\omega_n} \left[\pi \left(\delta(\omega - \omega_n) + \delta(\omega + \omega_n) \right) + i \left(P(\frac{1}{\omega + \omega_n} + P(\frac{1}{\omega - \omega_n}) \right) \right]$$

Thermodynamic limit $N \to \infty$

$\eta(\omega) =$ regular, continuous function

$$\eta(\omega) = \frac{\Gamma_m}{1 - i\omega/\omega_c}$$

Ohmic damping $\eta(t) = \theta(t)\Gamma_m \,\omega_c\, e^{-\omega_c t}$

(high frequency cutoff)

Markovian regime

$$\int_{-\infty}^{t} dt' \eta(t - t') \frac{d\hat{q}(t')}{dt'} \simeq \Gamma_m \frac{d\hat{q}(t)}{dt}$$

$1/\omega_c \ll$ typical time scale for the evolution of $\hat{q}(t)$

Caldeira-Leggett model

Quantum noise

$$\delta \hat{F}(t) = \frac{1}{m} \sum_{n=1}^{N} \tilde{\lambda}_n \left(\hat{a}_n(t_0) e^{-i\omega_n (t-t_0)} + \text{h.c.} \right) - \eta(t - t_0)\hat{q}(t_0)$$

$$t - t_0 \gg \tau_c$$

$$\left\langle \delta \hat{F}(t_1) \delta \hat{F}(t_2) \right\rangle_{t_0} = \frac{1}{m^2} \sum_{1}^{N} \tilde{\lambda}_n^2 \left(\underbrace{\left\langle \hat{a}_n(t_0) \hat{a}_n^\dagger(t_0) \right\rangle}_{n_B(\omega_n)+1} e^{-i\omega_n (t_1 - t_2)} + \underbrace{\left\langle \hat{a}_n^\dagger(t_0) \hat{a}_n^{(} t_0) \right\rangle}_{n_B(\omega_n)} e^{i\omega_n (t_1 - t_2)} \right)$$

Thermal equilibrium at t_0

$$n_B(\omega) = \left(e^{\frac{\hbar\omega}{k_B T}} - 1 \right)^{-1}$$

Fluctuation-Dissipation theorem

$$S(\omega) = \int_{-\infty}^{+\infty} dt \, e^{i\omega t} \left\langle \delta \hat{F}(t) \delta \hat{F}(0) \right\rangle = 2\pi \left(1 + n_B(\omega)\right) \frac{\hbar\omega}{\pi} \mathrm{Re}\left[\eta(\omega)\right]$$

Caldeira-Leggett model

Quantum Langevin Equation (Markovian regime)

$$\frac{d^2\hat{q}(t)}{dt^2} = -\omega_0^2\hat{q} - \Gamma_m\frac{d\hat{q}(t)}{dt} + \delta\hat{F}(t)$$

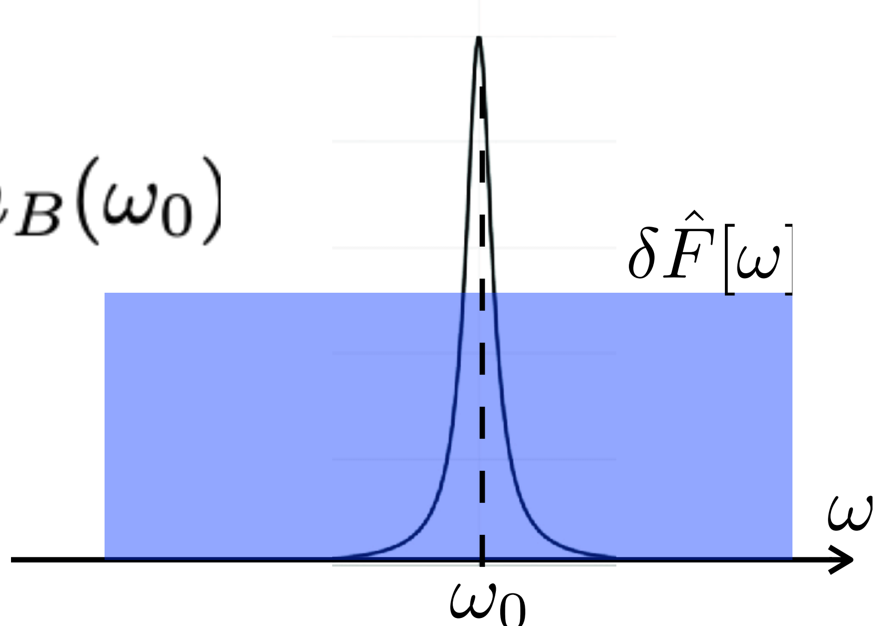
Solution

$$\hat{q}[\omega] = \chi(\omega)\ \ \delta\hat{F}[\omega] \qquad \chi(\omega) = \frac{1}{\omega_0^2 - \omega^2 - i\omega\Gamma_m}$$ susceptibility

Rotating Wave Approximation (R.W.A.) $\omega_0 \gg \Gamma_m$ + **white noise regime**

$$\hat{b}(t) = \frac{1}{2}\left[\sqrt{\frac{2m\omega_0}{\hbar}}\hat{q}(t) + i\sqrt{\frac{2}{m\hbar\omega_0}}\hat{p}(t)\right] \simeq \sqrt{\frac{\hbar}{2m\omega_0}}\sum_n\left(\frac{\tilde{\lambda}_n}{\hbar}\right)\frac{e^{-i\omega_0(t-t_0)}}{\omega_0 - \omega_n - i\Gamma_m/2}\hat{a}_n(t_0)$$

$$\left\langle\hat{b}^\dagger\hat{b}\right\rangle \simeq n_B(\omega_0)\int_0^{+\infty}d\omega\frac{\Gamma_n/\pi}{(\omega-\omega_0)^2 + \Gamma_m^2/4} \simeq n_B(\omega_0)$$



Caldeira-Leggett model

Canonical transformation

$$\hat{b}(t) = \frac{1}{2}\left[\sqrt{\frac{2m\omega_0}{\hbar}}\hat{q}(t) + i\sqrt{\frac{2}{m\hbar\omega_0}}\hat{p}(t)\right]$$

Langevin equation for $\hat{b}(t)$

$$\frac{d\hat{b}(t)}{dt} = -i\omega_0\hat{b}(t) - \frac{\Gamma_m}{2}\left(\hat{b}(t) + \hat{b}^\dagger(t)\right) + i\sqrt{\frac{\hbar}{2m\omega_0}}\sum_n \frac{\tilde{\lambda}_n}{\hbar}\left[\hat{a}_n(t_0)\,{}^{-i\omega_n t} + \hat{a}_n^\dagger(t_0)\,e^{i\omega_n t}\right]$$

Rotating Wave Approximation (RWA)

$$\frac{d\hat{b}(t)}{dt} = -i\omega_0\hat{b}(t) - \frac{\Gamma_m}{2}\hat{b}(t) + i\sqrt{\frac{\hbar}{2m\omega_0}}\sum_n \frac{\tilde{\lambda}_n}{\hbar}\hat{a}_n(t_0)e^{-i\omega_n t}$$

Input noise operator

Effective equation (Markov regime + white noise + RWA)

$$\frac{d\hat{b}(t)}{dt} = -i\omega_0\hat{b}(t) - \frac{\Gamma_m}{2}\hat{b}(t) + \sqrt{\Gamma_m}\,\hat{b}_{in}(t)$$

$$\left[\hat{b}_{in}(t), \hat{b}_{in}^\dagger(t')\right] = \delta(t-t')$$

$$\left\langle \hat{b}_{in}(t)\hat{b}_{in}^\dagger(t')\right\rangle = \left[1 + n_B(\omega_0)\right]\delta(t-t')$$

$$\left\langle \hat{b}_{in}^\dagger(t)\hat{b}_{in}(t')\right\rangle = n_B(\omega_0)\delta(t-t')$$

Caldeira-Leggett model

Forced vibration $$\frac{d^2\hat{q}(t)}{dt^2} = -\omega_0^2\hat{q}(t) + F_d\cos\omega_d t - \Gamma_m\frac{d\hat{q}(t)}{dt} + \delta\hat{F}(t)$$

Unitary transformation $\hat{U}_d(t) = e^{-i\omega_d t\hat{b}^\dagger\hat{b}}$ (rotating frame of the drive)

$$\hat{H}_0^{'} = \hbar\left(\omega_0 - \omega_d\right)\hat{b}^\dagger\hat{b}$$

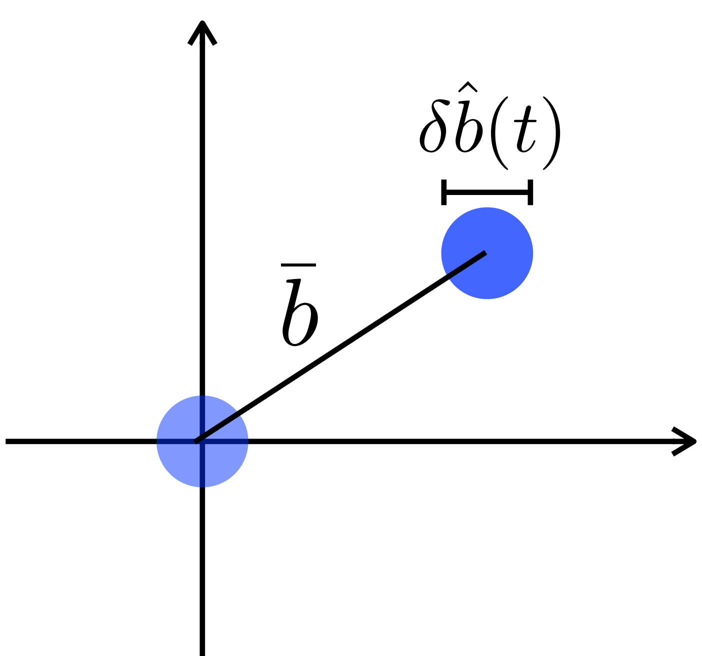
$$\hat{U}_d^\dagger(t)\hat{q}\hat{U}_d(t) = \sqrt{\frac{\hbar}{2m\omega_0}}\left[\hat{b}\,e^{-i\omega_d t} + \hat{b}^\dagger\,e^{i\omega_d t}\right]$$

$$\frac{d\hat{b}(t)}{dt} = i\left(\omega_d - \omega_0\right)\hat{b}(t) - \frac{\Gamma_m}{2}\hat{b}(t) + e^{i\omega_d t}\sqrt{\Gamma_m}\,\hat{b}_{in}(t) + i\left(\sqrt{\frac{2}{m\hbar\omega_0}}\right)\frac{F_d}{4}$$

Solution $\hat{b} = \bar{b} + \delta\hat{b}$

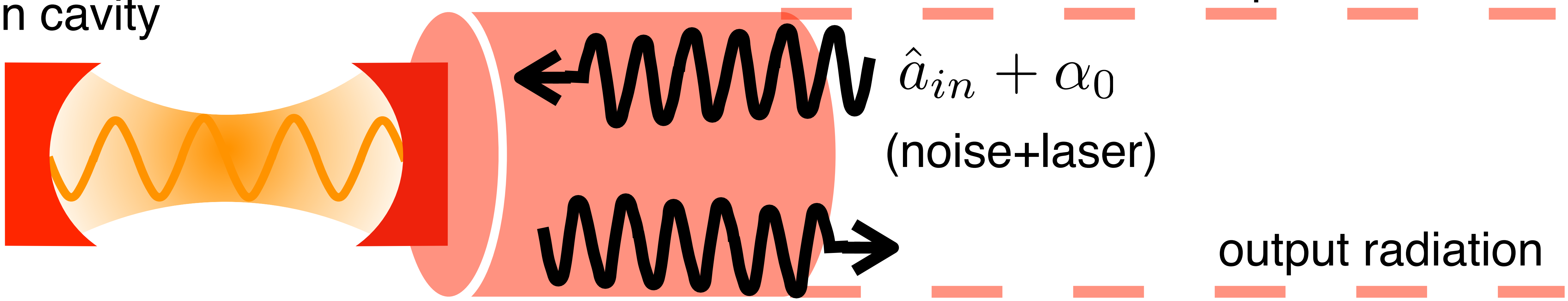
$$\frac{d\bar{b}}{dt} = 0 = i\left(\omega_d - \omega_0\right)\bar{b} - \frac{\Gamma_m}{2}\bar{b} + i\left(\sqrt{\frac{2}{m\hbar\omega_0}}\right)\frac{F_d}{4}$$

$$\frac{d\delta\hat{b}(t)}{dt} = i\left(\omega_d - \omega_0\right)\delta\hat{b}(t) - \frac{\Gamma_m}{2}\delta\hat{b}(t) + e^{i\omega_d t}\sqrt{\Gamma_m}\,\hat{b}_{in}(t)$$



Optomechanical systems

Input-output theory (in short)



$$\hat{H}_{opt} = \hbar\omega_c \hat{a}^\dagger \hat{a} - i\hbar\left(\bar{f}_0^* \quad e^{i\omega_L t}\hat{a} - \text{h.c.}\right) - i\hbar\sum_k \left(f_k \hat{a}_k^\dagger \hat{a} - \text{h.c.}\right) + \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k$$

single mode — injected laser (drive) — interaction — modes of the transmission line

Effective equation in rotating frame (Markov regime + white noise + RWA)

$$\frac{d\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2}\right)\hat{a} + \sqrt{\kappa}\hat{a}_{in} + \bar{f}_0$$

$\Delta = \omega_L - \omega_c$ detuning

$\kappa =$ damping (cavity losses)

$\hat{a}_{in} =$ input noise

$$\left[\hat{a}_{in}(t), \hat{a}_{in}^\dagger(t')\right] = \delta(t-t')$$

$$\left\langle \hat{a}_{in}(t)\hat{a}_{in}^\dagger(t')\right\rangle = \left(1 + n_B(\omega_c)\right)\delta(t-t')$$

$$\left\langle \hat{a}_{in}^\dagger(t)\hat{a}_{in}(t')\right\rangle = n_B(\omega_c)\delta(t-t')$$

Noise spectrum of the cavity

Average radiation + noise: $\hat{a} = \bar{a} + \delta\hat{a}$ $\qquad |\bar{a}|^2 = \bar{n}_{cav} \gg 1$

$$\left(i\Delta - \frac{\kappa}{2}\right)\bar{a} + \bar{f}_0\alpha_0 = 0 \qquad \frac{d\delta\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2}\right)\delta\hat{a} + \sqrt{\kappa}\hat{a}_{in}$$

Solution

$$\delta\hat{a}[\omega] = \frac{\sqrt{\kappa}\,\hat{a}_{in}[\omega]}{\frac{\kappa}{2} - i\omega - i\Delta} \qquad \int_{-\infty}^{+\infty} dt\ e^{i\omega t}\left\langle \delta\hat{a}(t)\delta\hat{a}^\dagger(0)\right\rangle = \frac{\kappa}{\frac{\kappa^2}{4} + (\omega+\Delta)^2}$$

$$\hat{N} = \hat{a}^\dagger\hat{a} - |\bar{a}|^2 \simeq \bar{a}^*\delta\hat{a} + \bar{a}\delta\hat{a}^\dagger$$ fluctuations of the photon number

$$\int_{-\infty}^{+\infty} dt\ e^{i\omega t}\left\langle \hat{N}(t)\hat{N}(0)\right\rangle = \bar{n}_{cav}\frac{\kappa}{\frac{\kappa^2}{4} + (\omega+\Delta)^2}$$

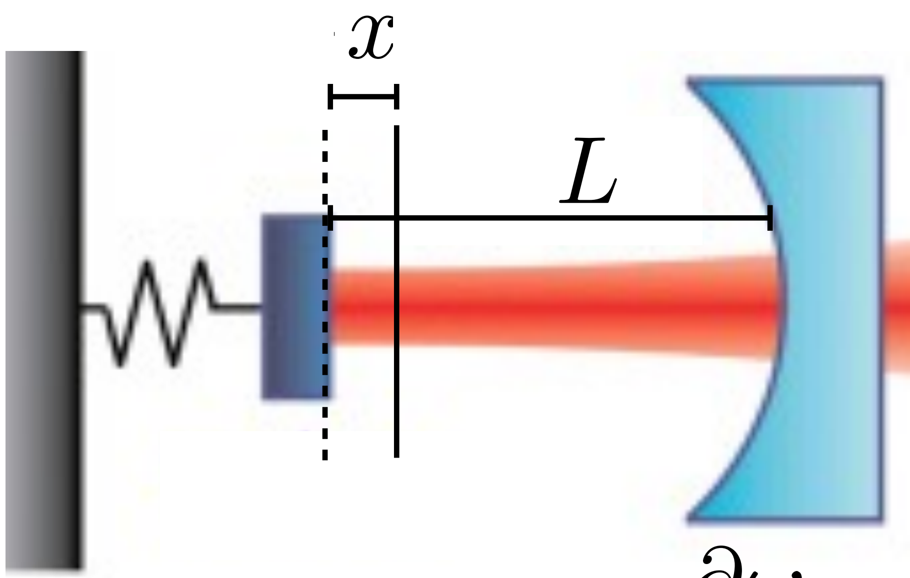
$$n_B(\omega_c) \simeq 0$$

(zero temperature limit)

Optomechanical systems

$$\hat{H}_s = \hbar \left[\omega_c \hat{a}^\dagger \hat{a} - g_0 \left(\hat{b} + \hat{b}^\dagger \right) \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} \right]$$

cavity mode — interaction — mechanical mode



$$\omega_c(x) = \omega_c + \frac{\partial \omega_c}{\partial x} x$$

$$g_0 = -\frac{\partial \omega_c}{\partial x} \sqrt{\frac{\hbar}{2 m \omega_m}}$$ coupling constant

Radiation pressure $$\left\langle \hat{F}_q \right\rangle = \frac{\hbar \omega_c}{L} \bar{n}_{cav}$$

(⟶ static shift of the mechanical resonator)

Optomechanical systems

Linerarization of the interaction $\qquad \hat{a} = \bar{a} + \delta\hat{a}$

$$\hat{H}_{int}^{(eff)} = -\hbar g_0 \left[\bar{a}^* \delta\hat{a} + \bar{a}\delta\hat{a}^\dagger\right] \left(\hat{b} + \hat{b}^\dagger\right)$$

$|g| = g_0\sqrt{\bar{n}_{cav}}$ renormalized interaction constant

Linearized quantum Langevin equations (in the frame of the laser)

$$\frac{d\delta\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2}\right)\delta\hat{a} + \sqrt{\kappa}\,\hat{a}_{in} + ig\left(\hat{b} + \hat{b}^\dagger\right)$$

cavity mode

$$\frac{d\hat{b}}{dt} = \left(-i\omega_m - \frac{\Gamma_m}{2}\right)\hat{b} + \sqrt{\Gamma_m}\,\hat{b}_{in} + ig\left(\delta\hat{a} + \delta\hat{a}^\dagger\right)$$

or

$$\frac{d^2\hat{q}}{dt} = -\omega_m^2\hat{q} - \Gamma_m\frac{d\hat{q}}{dt} + \delta\hat{F}_{in} + \frac{\hbar G\sqrt{\bar{n}_c}}{m}\left(\delta\hat{a} + \delta\hat{a}^\dagger\right)$$

mechanical mode

Optomechanical systems

Optical backaction

solution for the cavity mode

$$\delta\hat{a}[\omega] = \frac{\sqrt{\kappa}\,\hat{a}_{in}[\omega]}{\frac{\kappa}{2} - i\omega - i\Delta} - \frac{g}{\omega + \Delta + i\frac{\kappa}{2}}\left(\hat{b}[\omega] + \hat{b}^{\dagger}[\omega]\right)$$

solution for the mechanical mode

$$\hat{q}[\omega] = \chi^{eff}(\omega)\left[\delta\hat{F}_{in} + \hbar G\sqrt{\bar{n}_{cav}}\left(\frac{\sqrt{\kappa}\,\hat{a}_{in}[\omega]}{\frac{\kappa}{2} - i\left(\omega + \Delta\right)} + \frac{\sqrt{\kappa}\,\hat{a}^{\dagger}_{in}[\omega]}{\frac{\kappa}{2} + i\left(\Delta - \omega\right)}\right)\right.$$

noise injected and filter by the cavity

$$\chi^{eff}(\omega) = \frac{1}{\omega_m^2 + 2\omega_m\delta\omega_{opt} - \omega^2 - i\left(\Gamma + \Gamma_{opt}\right)\omega} \qquad \kappa \gg \Gamma_m$$

optical spring effect
(shift of the mechanical frequency)

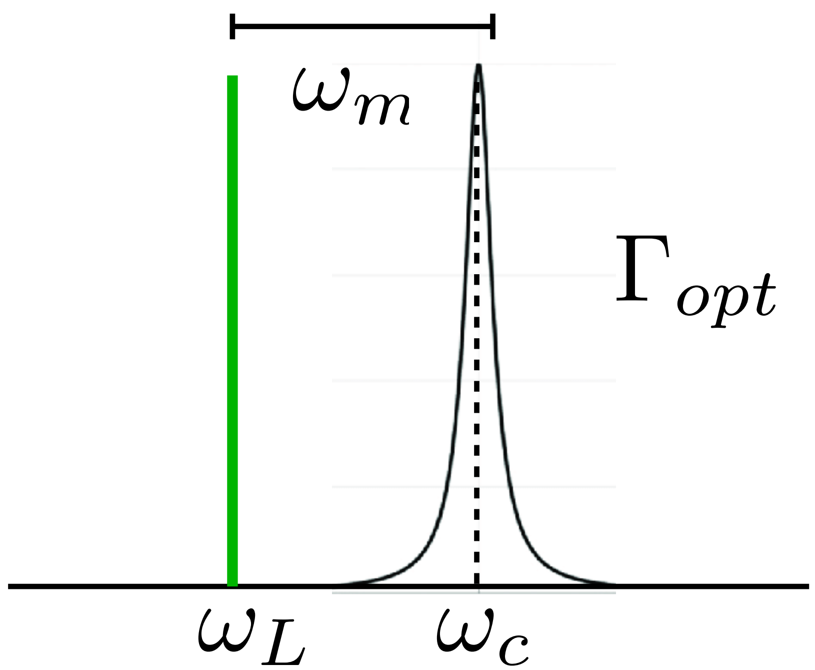
optomechanical damping

Optomechanical systems

Optomechanical damping

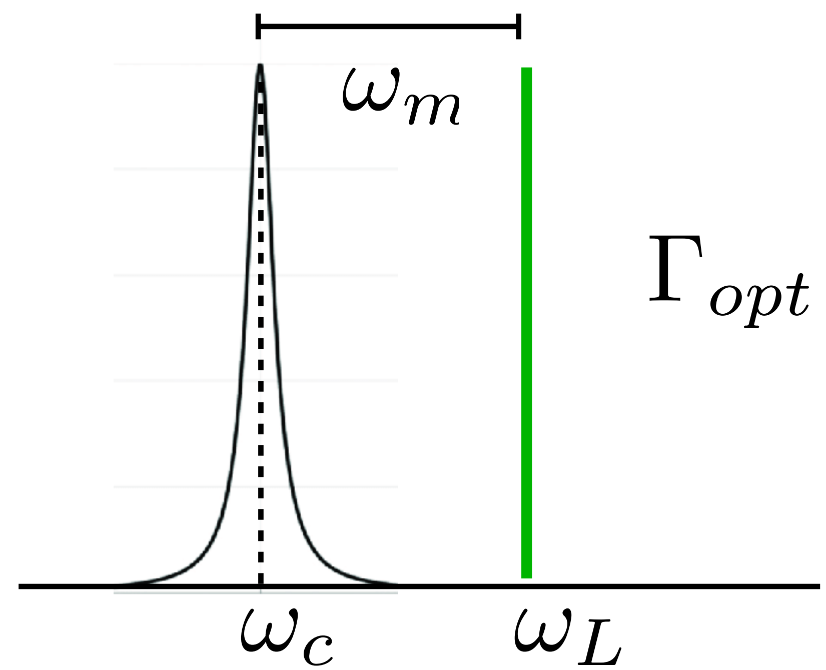
$$\Gamma_{opt} = g^2 \left(\frac{\kappa}{\frac{\kappa^2}{4} + (\Delta + \omega_m)^2} - \frac{\kappa}{\frac{\kappa^2}{4} + (\Delta - \omega_m)^2} \right)$$

red detuning $\omega_c \simeq \omega_L + \omega_m$



$\Gamma_{opt} > 0$

blue detuning $\omega_c \simeq \omega_L - \omega_m$



$\Gamma_{opt} < 0$

positive-damping: absorption of phonons are enhanced

(ground-state cooling)

negative damping: emission of phonons are enhanced

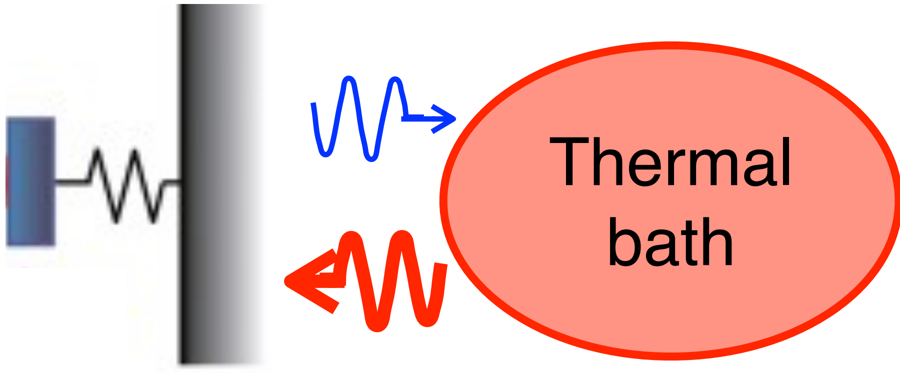
when $\Gamma_m + \Gamma_{opt} = 0$ → instability

(self-sustained oscillations)

(see → laser)

Ground state cooling

Ground state cooling



Mechanical resonator as open quantum system

$$\hat{H} = \hat{H}_{env} + \lambda\sqrt{\frac{\hbar}{2m\omega_m}}\left(\hat{b}+\hat{b}^\dagger\right)\hat{F}_e + \hbar\omega_m \hat{b}^\dagger \hat{b}$$

thermal bath — interaction with the bath — noise force

Emission and absorption rate (Fermi Golden rule)

$$\Gamma_{n\to n+1} = (n+1)\Gamma_\uparrow \qquad \Gamma_{n\to n-1} = n\Gamma_\downarrow$$

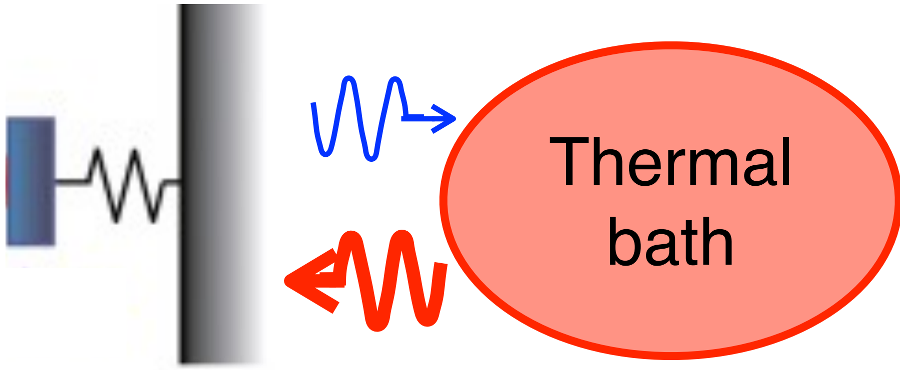
$$\Gamma_\uparrow = \frac{\lambda^2}{\hbar^2}\left(\frac{\hbar}{2m\omega_n}\right)S_{FF}(-\omega_m) \qquad \Gamma_\downarrow = \frac{\lambda^2}{\hbar^2}\left(\frac{\hbar}{2m\omega_n}\right)S_{FF}(+\omega_m)$$

absorption of 1 phonon — emission of 1 phonon

$$S_{FF}(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\left\langle \hat{F}_e(t)\hat{F}_e(0)\right\rangle$$

spectrum of the quantum noise

Ground state cooling



Mechanical resonator as open quantum system

$$\hat{H} = \hat{H}_{env} + \lambda\sqrt{\frac{\hbar}{2m\omega_m}}\left(\hat{b}+\hat{b}^\dagger\right)\hat{F}_e + \hbar\omega_m\hat{b}^\dagger\hat{b}$$

Rate equations: p_n = probability of occupation of the state n

$$\frac{dp_n}{dt} = -\left(\Gamma_{n\to n+1} + \Gamma_{n\to n-1}\right)p_n + \Gamma_{n+1\to n}p_{n+1} + \Gamma_{n-1\to n}p_{n-1}$$

Average occupation $\bar{n} = \sum_{n=1}^{+\infty} n\, p_n$

$$\frac{d\bar{n}}{dt} = -\underbrace{\left(\Gamma_\downarrow - \Gamma_\uparrow\right)}_{\Gamma_m \text{ damping}}\bar{n} + \Gamma_\uparrow$$

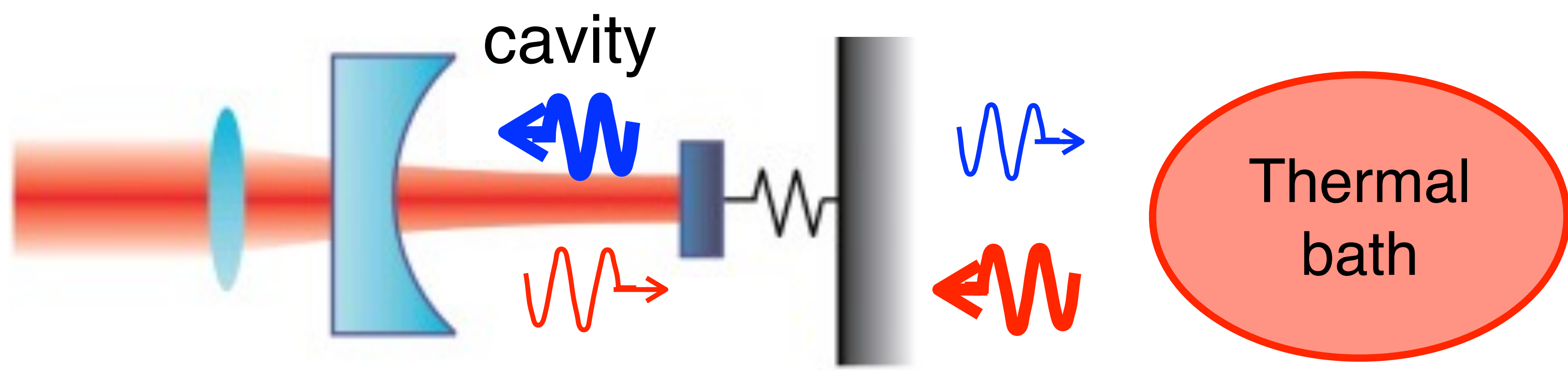
Stationary solution

$$\frac{d\bar{n}}{dt} = 0 \qquad \bar{n} = n_B(\omega_m) \qquad \frac{\Gamma_\uparrow}{\Gamma_\downarrow} = e^{-\frac{\hbar\omega_m}{k_B T}}$$

(detailed balance)

Ground state cooling

General model



$$\hat{H} = \hat{H}_{env} + \lambda\sqrt{\frac{\hbar}{2m\omega_m}}\left(\hat{b}+\hat{b}^\dagger\right)\hat{F}_e + \hbar\omega_m\hat{b}^\dagger\hat{b} - \hbar g\left(\hat{b}+\hat{b}^\dagger\right)\hat{N} + \hat{H}_{opt}$$

interaction with the cavity

driven and open cavity (single mode)

$$\hat{N} = \hat{a}^\dagger\hat{a} - \bar{n}_{cav}$$

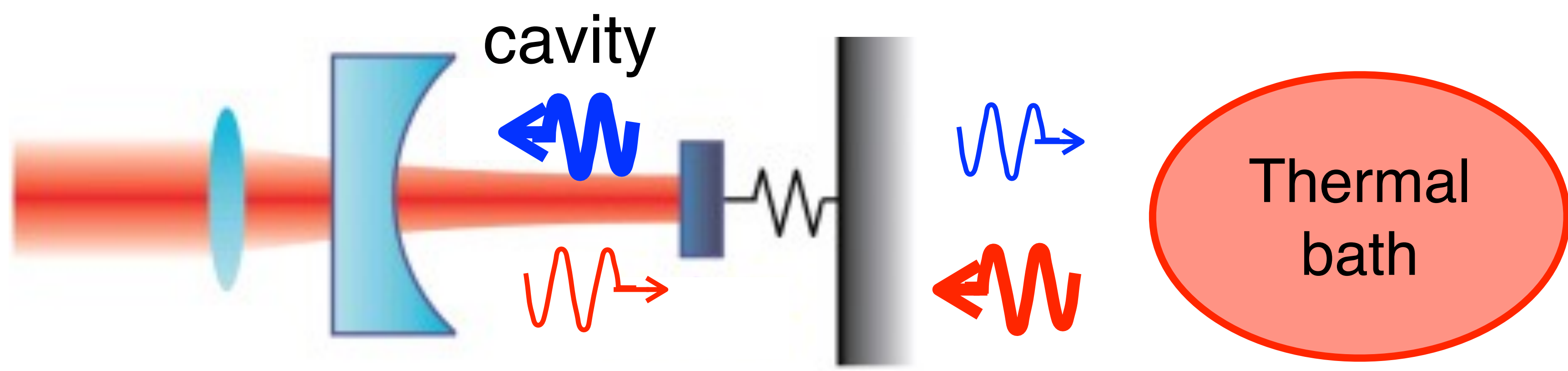
fluctuating radiation pressure (fluctuating number of photons)

$$\Gamma^{opt}_{n\to n+1} = (n+1)A_\uparrow \qquad A_\uparrow = g^2 S_{NN}(-\omega_F)$$

$$\Gamma^{opt}_{n\to n-1} = nA_\downarrow \qquad A_\downarrow = g^2 S_{NN}(+\omega_F)$$

$$A_\downarrow - A_\uparrow = \Gamma_{opt}$$ optomechanical damping

Ground state cooling



$$\hat{H} = \hat{H}_{env} + \lambda\sqrt{\frac{\hbar}{2m\omega_m}}\left(\hat{b}+\hat{b}^\dagger\right)\hat{F}_e + \hbar\omega_m \hat{b}^\dagger\hat{b} - \hbar g\left(\hat{b}+\hat{b}^\dagger\right)\hat{N} + \hat{H}_{opt}$$

General rate equation

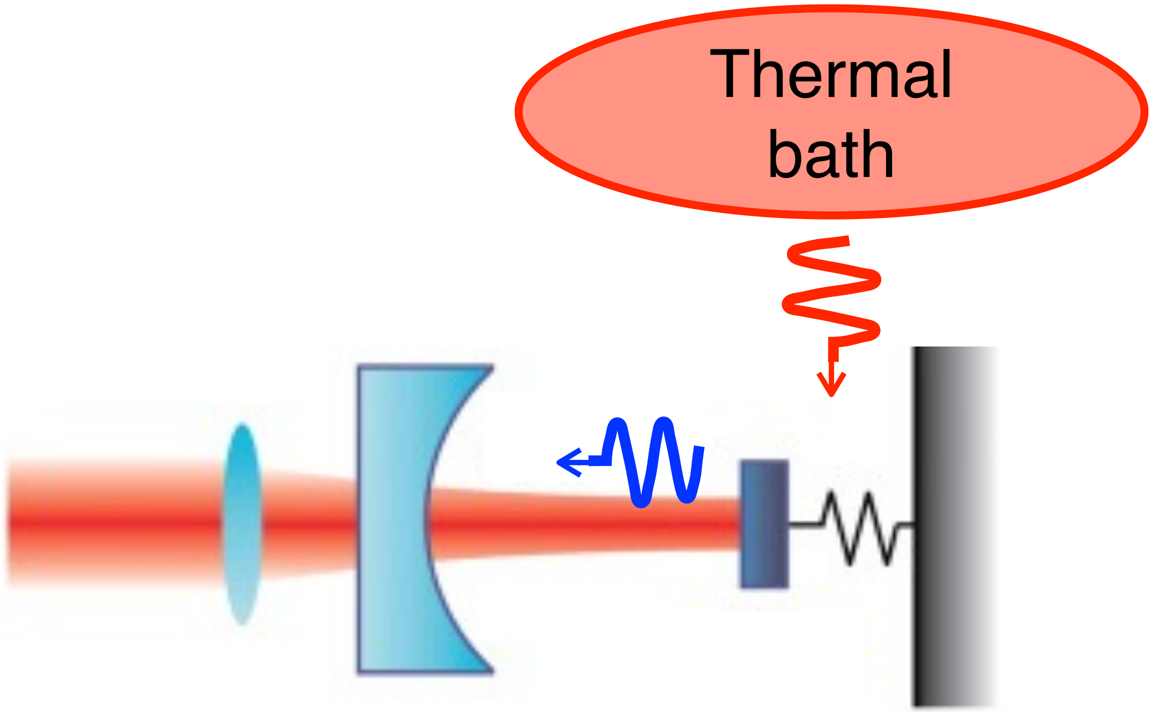
$$\frac{d\bar{n}}{dt} = -\left(\Gamma_m + \Gamma_{opt}\right)\bar{n} + \Gamma_\uparrow + A_\uparrow$$

Stationary solution

$$\frac{d\bar{n}}{dt} = 0 \qquad \bar{n} = \frac{\Gamma_\uparrow + A_\uparrow}{\Gamma_m + \Gamma_{opt}} = \frac{\Gamma_m n_B(\omega_m) + \Gamma_{opt}\left(\frac{A_\uparrow}{\Gamma_{opt}}\right)}{\Gamma_m + \Gamma_{opt}}$$

Ground state cooling

$$\bar{n} = \frac{\Gamma_m n_B + \Gamma_{opt} n_{opt}}{\Gamma_m + \Gamma_{opt}}$$



Γ_m = intrinsic damping (bath)

Γ_{opt} = optomechanical damping

$$n_B = 1/(e^{\frac{\hbar\omega_0}{k_B T}} - 1)$$ = thermal phonon occupation (Boltzmann)

$$n_{opt} = \frac{1}{A_\downarrow / A_\uparrow - 1}$$ = effective nonequilibrium phonon occupation

$$n_{opt} \simeq \left(\frac{\kappa}{4\omega_m}\right)^2 \ll 1$$

red detuning $\omega_c \simeq \omega_L + \omega_m$

(resolved sideband regime)

INFO

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MASTER - LECTURES (2nd semester)

Superconducting Quantum Nanosystems, Circuits and Devices

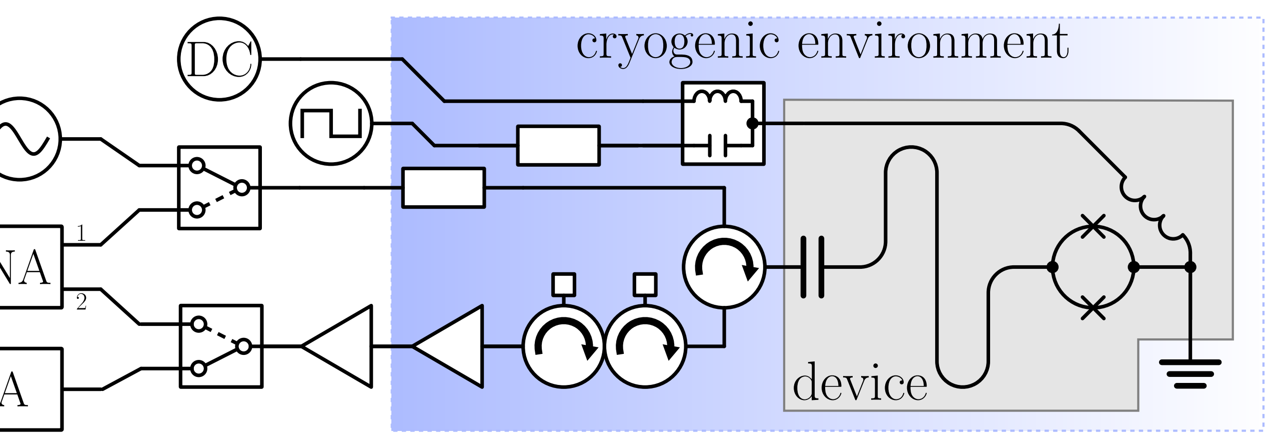
theoretical part (me)

experimental part (Federica Mantegazzini)

MASTER - THESIS

1) Superconducting systems

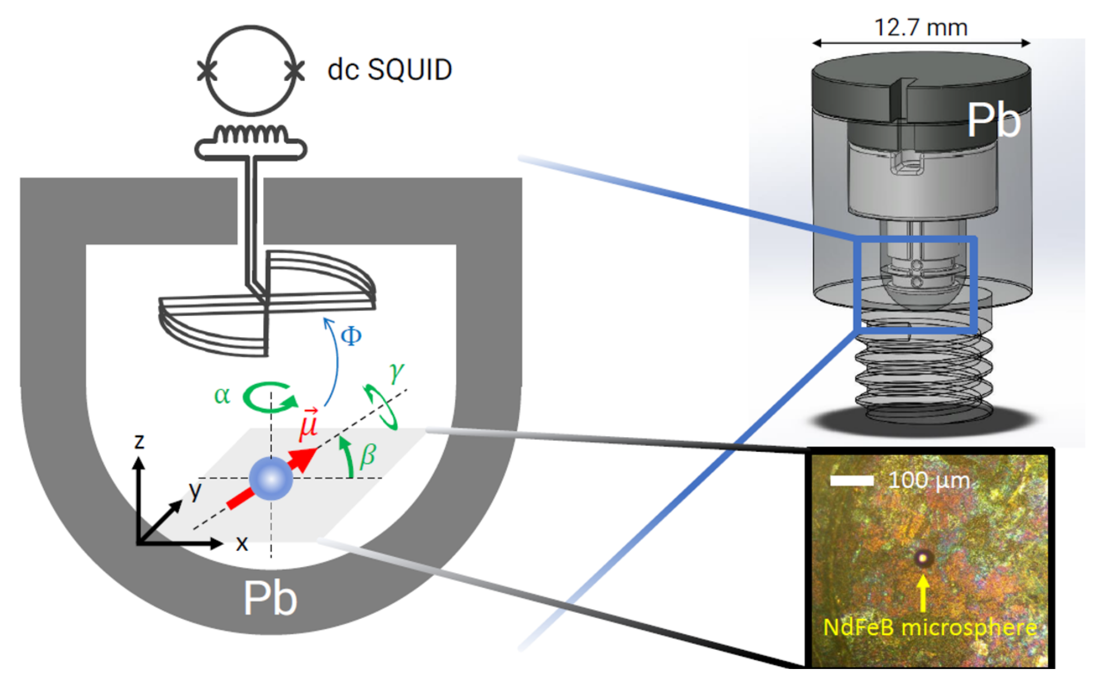
theory and experiments of quantum microwave optics with superconducting devices



in collaboration with the experimental group of Federica Mantegazzini

2) Levitated systems

cooling of suspended nano magnet, classical (and quantum) feedback)



in collaboration with the experimental group of Andrea Vinante