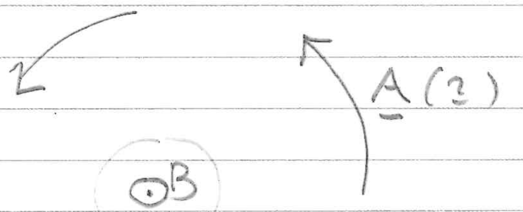
Aharonov - Bohm effect

2/5/26



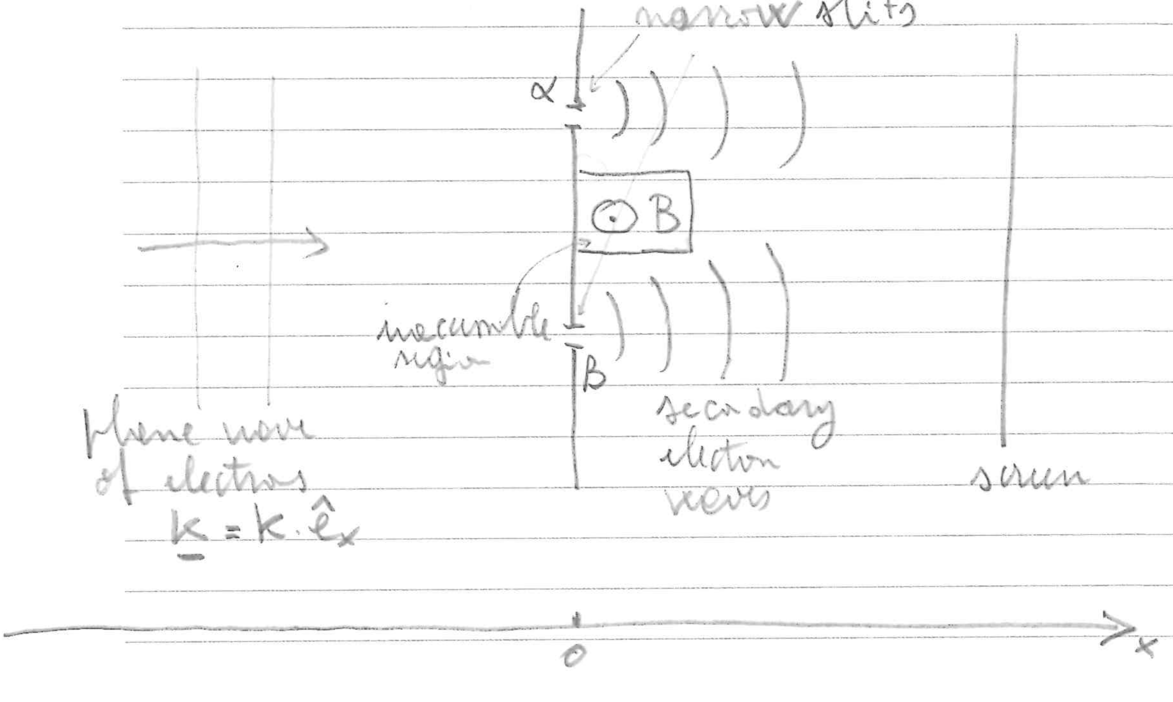
magnetic field B localized in region of area S

electrons can not penetrate this region

$$\underline{A}(r) = \frac{1}{2\pi r}\,\Phi \cdot \hat{e}_\Theta \quad , \quad \Phi = B \cdot S \text{ magnetic flux.}$$

[from Stokes' thm: $\oint d\underline{r} \cdot \underline{A}(r) = \Phi = \int_S d^2\sigma\, \underline{B}(r)$]

expt. configuration



gauge change

$$A' = A + \nabla\lambda \quad , \quad \phi' = \phi - \frac{1}{c}\frac{\partial\lambda}{\partial t}$$

$$\psi' = \psi\, e^{iq\lambda/\hbar c}$$

$$\lambda = -\frac{1}{2\pi}\Phi\cdot\theta \longrightarrow \nabla\lambda = -\frac{1}{2\pi}\Phi\frac{\hat{e}_\theta}{r}$$

$$A' = A + \nabla\lambda = \frac{1}{2\pi r}\Phi\,\hat{e}_\theta - \frac{\Phi}{2\pi r}\hat{e}_\theta = \emptyset$$

but : $e^{iq\lambda/\hbar c} = \exp\left(-\frac{iq}{\hbar c}\frac{\Phi}{2\pi}\theta\right) =$

$$= \exp\left(-i\frac{q}{2\pi\hbar c}\Phi\,\theta\right)$$

is not continuous @ $\theta = 0$ vs. 2π

$$\Phi/\Phi_0 = \Phi/\left(2\pi\hbar c/q\right) = \text{integer}$$

Φ_0 unit magnetic flux.

For $x \leq 0$: gauge change continuous as $\pi/2 < \theta < 3\pi/2$

For $x \geq 0$: discontinuity @ $\theta = 0, 2\pi$

$$\left.\begin{aligned} \text{define } \lambda' &= -\frac{1}{2\pi}\Phi\,\theta \text{ for } \frac{\pi}{2} > \theta > 0 \\ &= -\frac{1}{2\pi}\Phi(\theta - 2\pi) \text{ for } 2\pi > \theta > \frac{3\pi}{2} \end{aligned}\right\}$$

continuous @ $\theta = 0, 2\pi$

Discont. moved @ π

for both gauge changes $A' = 0$

$$\psi_1 = e^{iq\lambda/\hbar c}\,\psi \qquad \text{in} \quad x \le 0$$

in particular $\psi_1(\alpha) = \exp\left(-i\frac{q\Phi}{2\pi\hbar c}\frac{\pi}{2}\right)\psi(\alpha)$

$$\psi_1(\beta) = \exp\left(-\frac{iq\Phi}{2\pi\hbar c}\frac{3\pi}{2}\right)\psi(\beta)$$

at the slits located @ α, β

In $x \le 0$ $\quad \psi_1(2) = e^{ikx}\,\psi_0$

$$\psi_2(\alpha) = \exp\left(-i\frac{q\Phi}{2\pi\hbar c}\frac{\pi}{2}\right)\psi(\alpha) = \psi_1(\alpha)$$

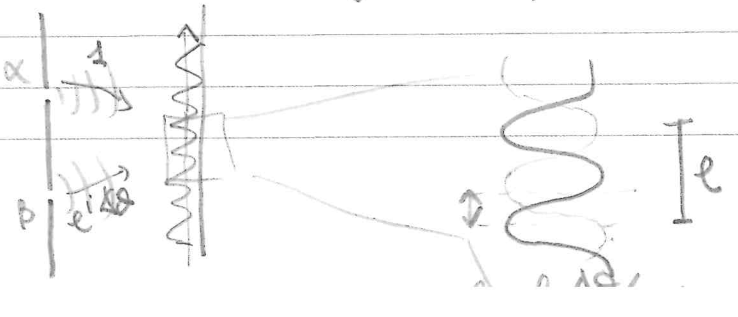
$$\psi_2(\beta) = \exp\left(-i\frac{q\Phi}{2\pi\hbar c}\left(\frac{\pi}{2}-2\pi\right)\right)\psi(\beta) = e^{iq\Phi/\hbar c}\,\psi_1(\beta)$$

In $x \ge 0$: secondary waves have relative phase

$$\Delta\theta = \frac{q\Phi}{\hbar c} = 2\pi\frac{q\Phi}{2\pi\hbar c} = 2\pi\,\Phi/\Phi_0$$

As $A' = 0 \longrightarrow$ standard two-slit interference

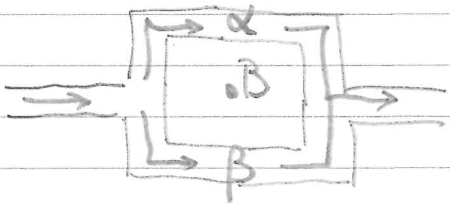
fringes shifted of $\Delta\theta = 2\pi\,\Phi/\Phi_0$



For superconductors :

elementary object "Cooper pair" → charge $2q$

SQUID device



interference of current along paths α, β with

relative phase $\Delta\vartheta = 2\pi \, \Phi / (2\pi\hbar c/2q) = 4\pi \, \Phi/\Phi_0$

Effective Josephson current :

$$J = J_\alpha \cdot \sin(\vartheta_\alpha) + J_\beta \sin(\vartheta_\beta)$$

$$\overset{J_\alpha = J_\beta}{=} J_0 \left(\sin\vartheta_\alpha + \sin\vartheta_\beta\right) = J_0\left(\sin\left(\vartheta_\alpha + \frac{\Delta\vartheta}{2}\right) + \sin\left(\vartheta_\alpha - \frac{\Delta\vartheta}{2}\right)\right)$$

$$= J_0 \cdot 2\sin\vartheta_\alpha \cos\frac{\Delta\vartheta}{2}$$

$$= \underbrace{J_0 \, 2\cos\left(\frac{\Delta\vartheta}{2}\right)}_{\text{effective Josephson current through SQUID}} \cdot \sin\vartheta_\alpha$$