More on the photon mass in cavities/waveguides

E.m. field in **planar cavity** (for simplicity):

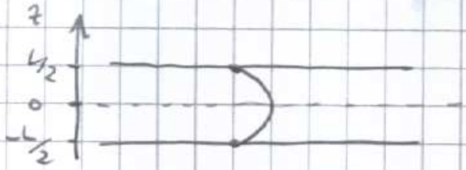
$$E(\underline{r},t) = \int \frac{d^2k}{(2\pi)^2}\, e_k \cdot \mathcal{E}_k(z)\, e^{i(k_x x + k_y y)}\, e^{-i\omega(k)t} \qquad (a)$$

amplitude

Transverse wave function

quantum numbers: $\{k_x, k_y\} = \underline{k}$

e.g. $\mathcal{E}_k(z) = \sin\left(\frac{\pi z}{L}\right) \bar{\mathcal{E}}$



Dispersion $\omega(k) = c\sqrt{\left(\frac{\pi}{L}\right)^2 + k^2} \simeq \frac{\pi c}{L} + \frac{cLk^2}{2\pi} + \dots$

setting $\omega_0 = \frac{\pi c}{L}$

$$\hbar\omega(k) = \underbrace{\hbar\omega_0}_{\text{rest energy}} + \frac{1}{2}\frac{c^2}{\hbar\omega_0}\hbar^2 k^2$$

kinetic **mass** $m = \frac{\hbar\omega_0}{c^2}$ as well!

Compton wavelength (of the cavity photon) $\lambda\!\!\!^{-} = \frac{\hbar}{mc} = \frac{\hbar}{\hbar\omega_0/c^2 \cdot c} = \frac{c}{\omega_0} = \frac{L}{\pi}$!

the general solution (a) can be written in terms of
field equation for the envelope

$$E(x,y,t) = \int \frac{d^2k}{(2\pi)^2}\, e_k\, e^{ik_x x}\, e^{ik_y y}\, e^{-i\omega(k)t}$$

$$i\partial_t E = \omega_0 E - \frac{\hbar}{2m} \nabla_\perp^2 E$$

$\downarrow$

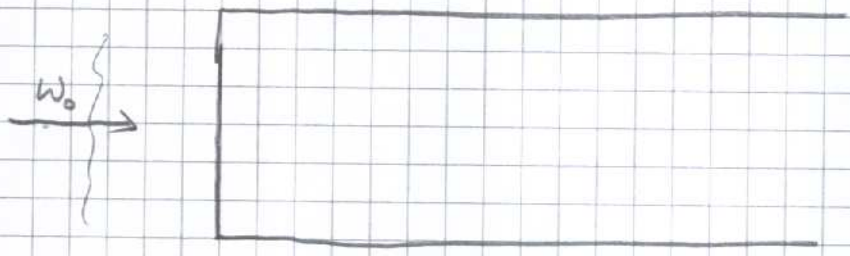
∇ in xy plane

which has the shape of a Schrödinger equation.

In the presence of nonlinearities $\longrightarrow$ Gross-Pitaevskii equation for photon BEC.

$\rightarrow$ I. Carusotto and C. Ciuti, Rev. Mod. Phys. 85, 299 (2013)

Paraxial light propagation in nonlinear crystal



* Monochromatic incident beam @ ω_0

* Transverse e.m. field profile @ $z=0$:

$E_0(x,y)$ slowly varying w/r. $\lambda = \frac{2\pi c}{\omega_0}$

$\hookrightarrow$ only small angles around $\hat{z}$

* Neglect polarization effects

* Neglect back-reflection

Maxwell eqs $\Longrightarrow$ $\frac{1}{c^2}\partial_t^2 E - \nabla^2 E = -\frac{4\pi}{c^2}\chi\,\partial_t^2 E$

for monochromatic $\quad \frac{\omega_0^2}{c^2}(1+4\pi\chi)E + \nabla^2 E = 0$

Choose $\quad \chi = \underbrace{\chi_0}_{\text{background}} + \underbrace{\delta\chi(x,y,z)}_{\text{(weak) modulation}} + \underbrace{\chi_{nl}|E(x,y,z)|^2}_{\text{nonlinear effects (Kerr)}}$

$$\frac{\omega_0^2}{c^2}(1+4\pi\chi_0)E + \frac{4\pi\omega_0^2}{c^2}\left[\delta\chi\, E + \chi_{nl}|E|^2 E\right] + \nabla_\perp^2 E + \partial_z^2 E = 0.$$

set $k_0 = \frac{\omega_0}{c}\sqrt{1+4\pi\chi_0}$

and $E(x,y,z) = \tilde{E}(x,y,z)\, e^{ik_0 z}$

$$\partial_z^2 E = e^{ik_0 z}\left(\partial_z^2 \tilde{E} + 2ik_0\partial_z\tilde{E} - k_0^2\tilde{E}\right)$$

$$\cancel{\frac{\omega^2}{c^2}(1+4\pi\chi_0)}\tilde{E}\,\cancel{e^{ik_0 z}} + \frac{4\pi\omega^2}{c^2}(\delta\chi + \chi_{nl}|\tilde{E}|^2)\tilde{E}\,\cancel{e^{ik_0 z}} +$$

$$+ \nabla_\perp^2\tilde{E}\,\cancel{e^{ik_0 z}} + \cancel{e^{ik_0 z}}\left(\partial_z^2\tilde{E} + 2ik_0\partial_z\tilde{E} - \cancel{k_0^2\tilde{E}}\right) = 0$$

$$2ik_0\partial_z\tilde{E} + \cancel{\partial_z^2\tilde{E}} = -\nabla_\perp^2\tilde{E} - \frac{4\pi\omega^2}{c^2}(\delta\chi + \chi_{nl}|\tilde{E}|^2)\tilde{E}$$

→ higher order ⇒ can be neglected

(only forward propagating light considered!)

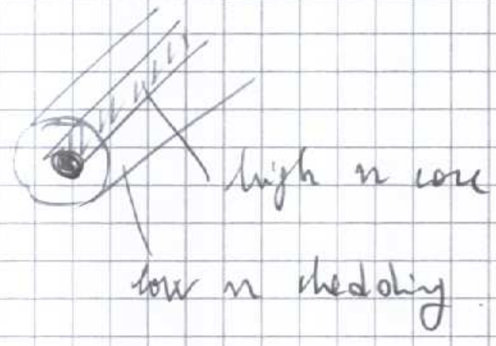
$$i\partial_z\tilde{E} = -\frac{1}{2k_0}\nabla_\perp^2\tilde{E} - \frac{2\pi k_0}{c}(\delta\chi + \chi_{nl}|\tilde{E}|^2)\tilde{E}$$

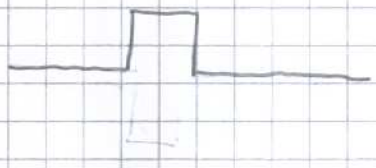
has the form of <u>(nonlinear) Schrödinger equation</u>:

* $z \longrightarrow$ time t
* $\frac{1}{k_0} \longrightarrow$ mass $\frac{\hbar}{m}$
* $-\frac{2\pi k_0}{c}\delta\chi \longrightarrow$ external potential
* $-\frac{2\pi k_0}{c}\chi_{nl}|\tilde{E}|^2 \longrightarrow$ interaction term

examples of application:

1) linear waveguide



$\chi(x,y) =$ 

$\Rightarrow$ potential well which transversally confines photons.

$\Rightarrow$ light attracted by higher n's!

2) self-focusing nonlinearity $\chi_{nl} > 0$

$\Longrightarrow$ plane wave collapses under attractive interactions

Many experiments

* solitons
* vortex lattices
* superfluidity
* BEC transition
* Synthetic gauge fields ...