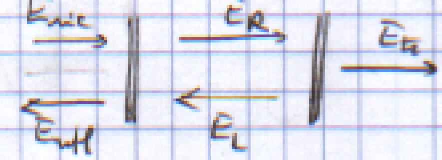
Reformulation in terms of field dynamics.



internal field :

$$E_R(\omega) = \frac{t}{1 - r^2 e^{2i\omega L/c}} \, E_{inc}(\omega)$$

$$E_L(\omega) = r\, e^{-2i\frac{\omega}{c}L} E_R(\omega) = \frac{tr}{1 - r^2 e^{2i\omega L/c}} \, e^{-2i\frac{\omega}{c}L} \, E_{inc}(\omega)$$

Around single resonance at ω_0, $\Delta\omega = \omega - \omega_0$:

$$E_R(\omega) \simeq \frac{t}{1 - r^2 e^{2i\Delta\omega L/c}} \, E_{inc}(\omega) \simeq \frac{t}{1 - r^2 - 2i\Delta\omega L/c \; r^2} \, E_{inc}(\omega) =$$

$$= \frac{i\, ct/2L}{\omega - \omega_0 + i\frac{c}{2L}(1 - r^2)} \, E_{inc}(\omega)$$

i.e.:

$$\left(\omega - \omega_0 + i\Gamma/2\right) E_R(\omega) = i\frac{c}{2L} \cdot t \cdot E_{inc}(\omega)$$

in real-time, this gives :

$$\left(i\frac{d}{dt} - \omega_0 - i\Gamma/2\right) E_R(t) = i\frac{c}{2L} \cdot t \; E_{inc}(t)$$

$$\left[\frac{d}{dt} E_R = -i\left(\omega_0 - i\Gamma/2\right) E_R + \frac{c}{2L} t \; E_{inc}(t) \right]$$

- $E_L(t) = E_R(t - 2L/c)$, $E_{tr}(t) = t \cdot E_R(t - L/c)$

* forced harmonic oscillator

  - natural frequency ω_0
  - damping Γ
  - forcing term proportional to incident field, coupling coefficient proportional to mirror t

Model hamiltonian (neglecting damping):

↳ non-hamiltonian process, does not conserve phase-space volume

$$\hat{H} = \hbar\omega_0 \, \hat{a}^\dagger \hat{a} + i\left(\alpha t \, E_{inc}(t) \, \hat{a}^\dagger - \alpha^* t^* E^*_{inc}(t) \, \hat{a}\right)$$

with internal field operator $\hat{E}_{L,R} = \mathcal{E}_{L,R} \, \hat{a}$.

More complex processes can be included at the level of field dynamical eq and/or model hamiltonian:

* $\chi^{(3)}$ optical nonlinearity:

$$\frac{d}{dt} E_R = \dots - i\,\omega_{nl} |E_R|^2 E_R$$

$$H^{(3)} = \hbar\,\omega_{nl} |\mathcal{E}_R|^2 \, \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}$$

* coupling with emitter: $H_{int} = \hbar\Omega\left(a^\dagger b + a b^\dagger\right)$