BEC
BOSE EINSTEIN CONDENSATION

Excitations and dynamics of fractional quantum Hall fluids of light (and of atoms)

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PROVINCIA
AUTONOMA DI TRENTO

Q@TN

European Commission

Horizon 2020
European Union funding
for Research & Innovation

PhoQuS
Photons for Quantum Simulation

MIR-BOSE

Why not hydrodynamics of light ?

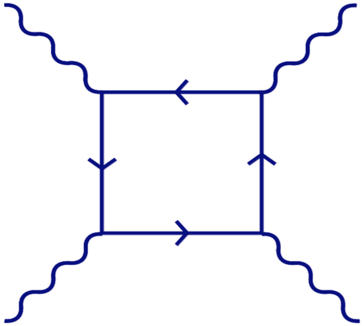
Light field/beam composed by a huge number of photons

- in vacuo photons travel along straight line at *c*
- (practically) do not interact with each other
- in standard cavity, thermalization via walls and absorption/emission

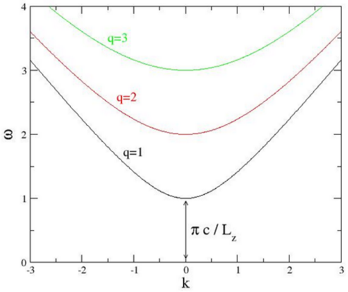
optics in vacuo typically dominated by single-particle physics

In suitable photonic structures:

- spatial confinement → effective photon mass
- $\chi^{(3)}$ nonlinearity → photon-photon interactions



Collective behaviour of *quantum fluid of light*



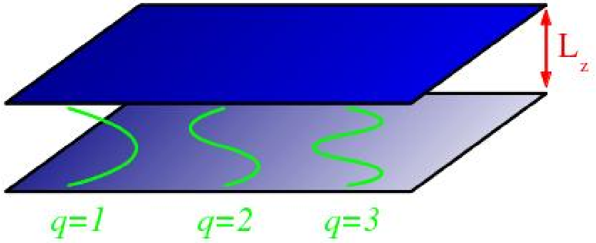
IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

What about mass?

In vacuo: photons massless, dispersion $\omega = c\,|k|$

In planar cavity → confinement along *z*, free propagation along *x,y*

Quantization along *z*: $k_z^{(q)} = q\,\pi / L_z$

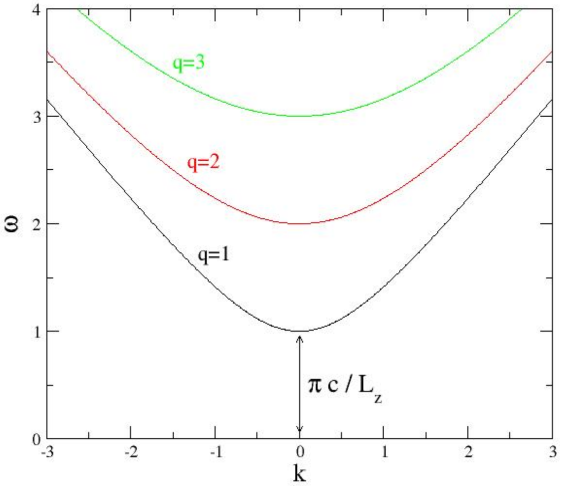


Massive dispersion along *x,y*:

$$\omega^{(q)}(\mathbf{k}_\parallel) = c\sqrt{[k_z^{(q)}]^2 + \mathbf{k}_\parallel^2} = c\sqrt{\left(\frac{q\pi}{L_z}\right)^2 + \mathbf{k}_\parallel^2} \simeq ck_z^{(q)} + \frac{c}{2k_z^{(q)}}\mathbf{k}_\parallel^2$$

Confinement gives effective photon mass $m_{ph}c^2 = \hbar c k_z^o$

- Rest mass → cut-off in the dispersion
- Inertial mass → curvature of dispersion



IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

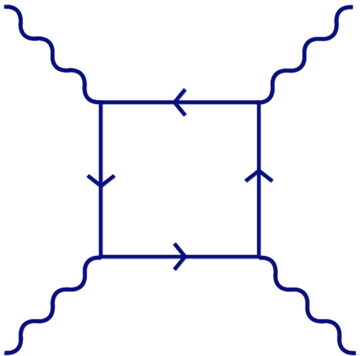
What about interactions?

Photon-photon interactions exist in QED:

Heisenberg-Euler processes via electron-positron exchange

… but cross section ridiculously small for visible light

(recent experiment in accelerator → Nat. Phys. 2017)



How to enhance it ?

$$\sigma \sim \alpha^4 \boxed{\frac{\hbar^2}{m^2 c^2}} \left(\frac{\hbar\omega}{mc^2}\right)^6$$

Compton λ → pm range

Replace electron-positron pair (E~1MeV) with

electron-hole pair (E~1eV) → gain factor $(10^6)^6=10^{36}$!!

In optical language:

- $\chi^{(3)}$ nonlinearity ↔ local photon-photon interactions
- typical material → spatially local (or quasi-local) $\chi^{(3)}$

Modern exceptional media:

- Rydberg atoms
  - Ultra-large, long-range nonlinearity in Rydberg-EIT config.
- Superconducting circuits
  - Strong coupling to macroscopic oscillation mode of superconductor device

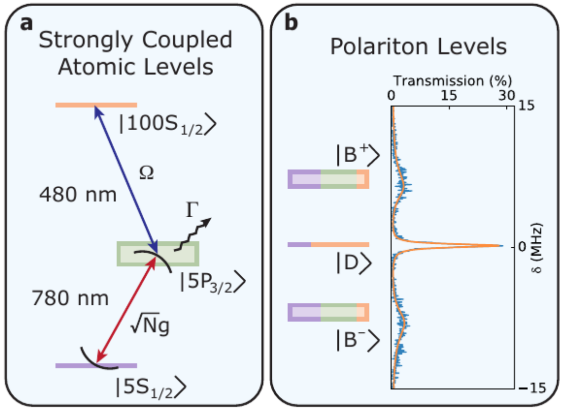
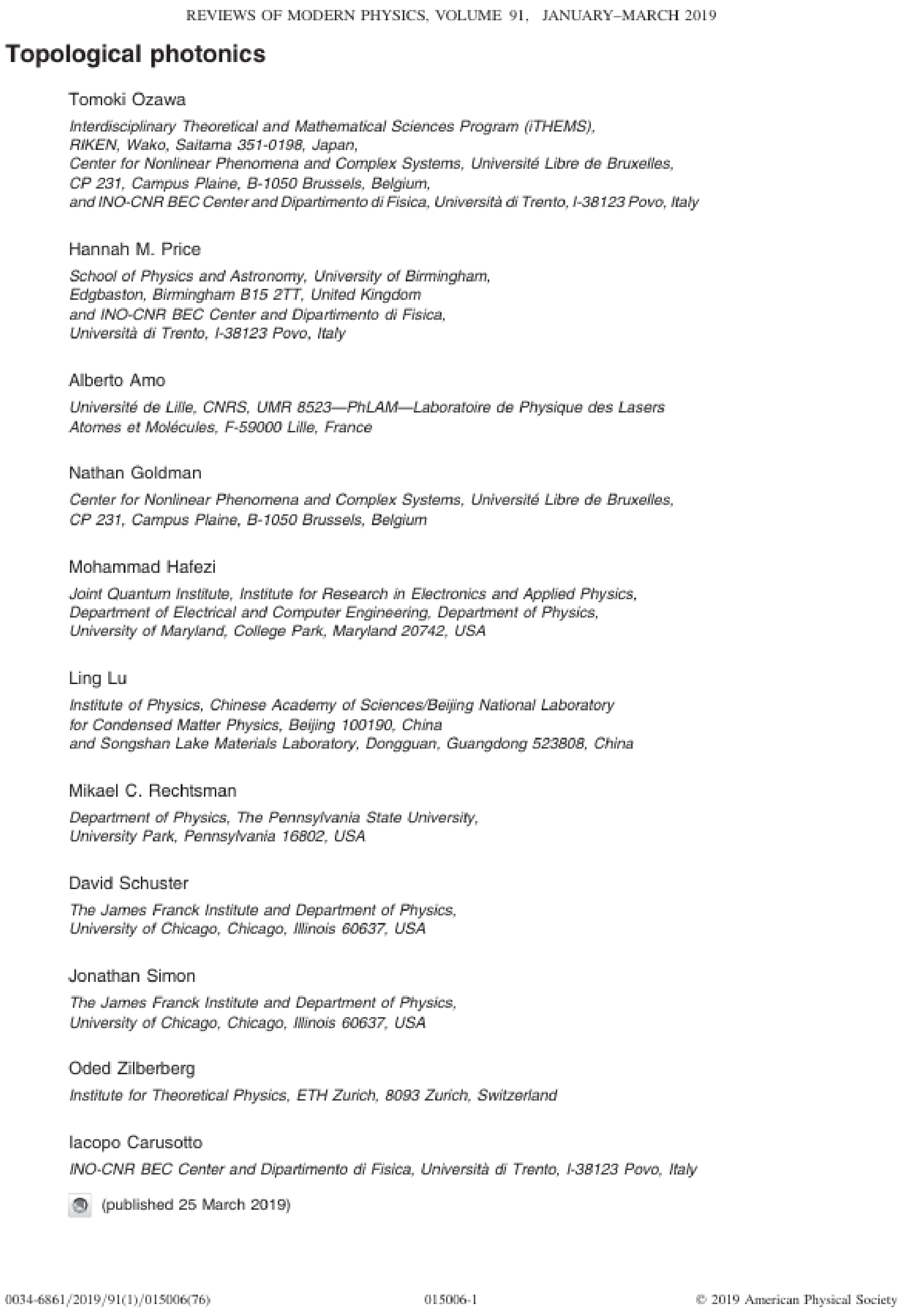


Figure: J. Simon's group @ U. Chicago

What about (orbital) magnetic effects ?



REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Topological photonics

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Interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS), RIKEN, Wako, Saitama 351-0198, Japan, Center for Nonlinear Phenomena and Complex Systems, Université Libre de Bruxelles, CP 231, Campus Plaine, B-1050 Brussels, Belgium, and INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy

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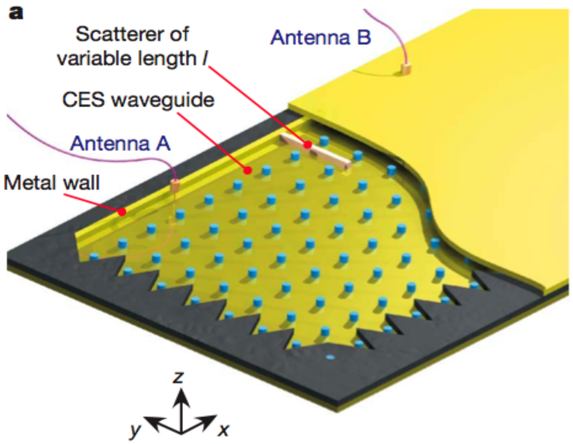
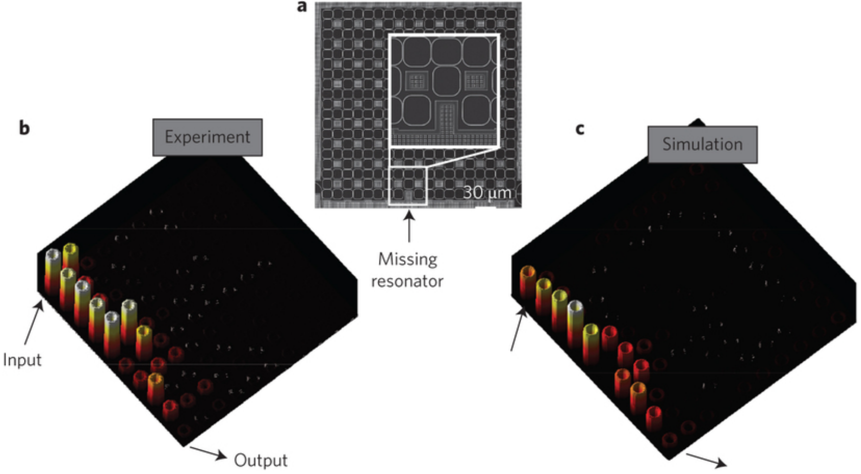
Magneto-optical photonic crystal



Z Wang et al., Nature 461, 772 (2009)

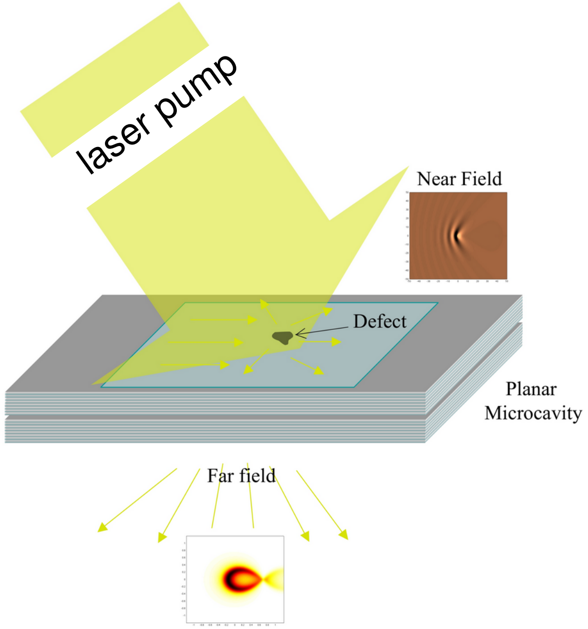
Array of ring resonators in Si-photonics



Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

And many other equally exciting experiments...

Non-equilibrium: a bug or a feature ?



Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

A variety of sophisticated pumping schemes available; no need for cooling

- Coherent laser pump: directly injects photon BEC in cavity, may lock BEC phase
- Incoherent (optical or electric) pump: BEC transition similar to laser threshold
  spontaneous breaking of U(1) symmetry

Quantum correlations of in-plane field directly transfer to emitted radiation in real time

IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

Milestone I: 2006 - Photon BEC

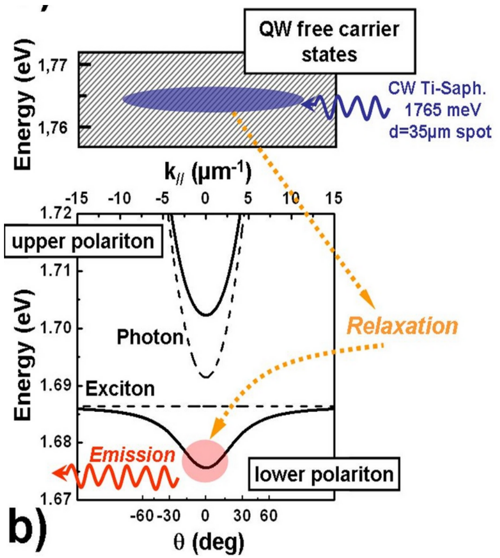
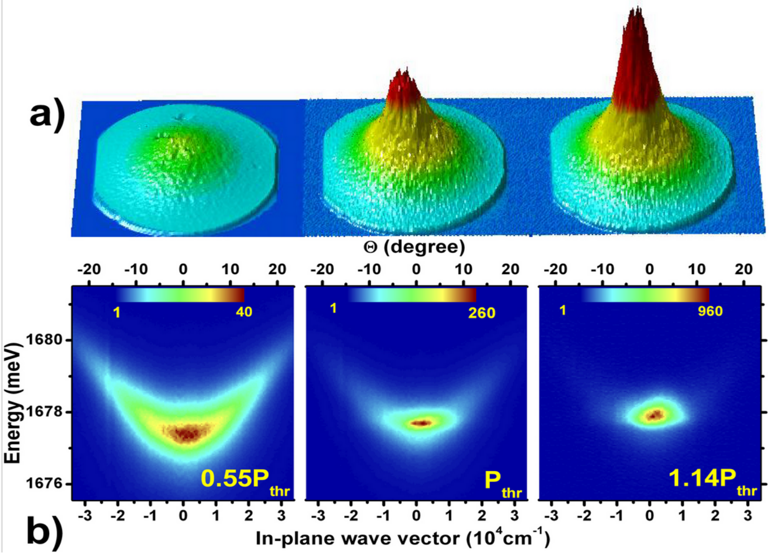
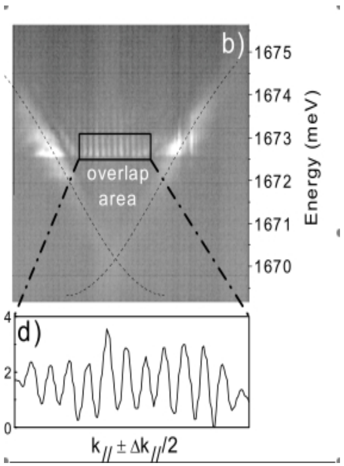


Figure from Kasprzak et al., Nature 2006



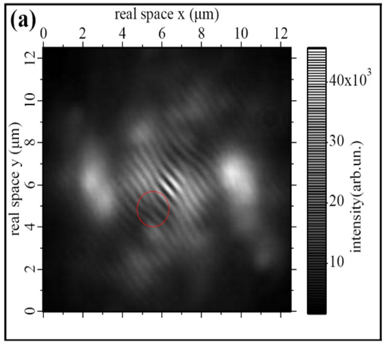
Momentum distribution

Kasprzak et al., Nature **443**, 409 (2006)



Interference

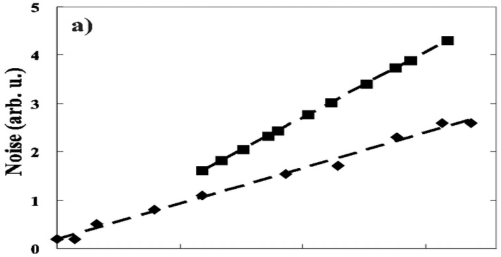
Richard et al., PRL **94**, 187401 (2005)



Quantized vortices

K. Lagoudakis *et al.*

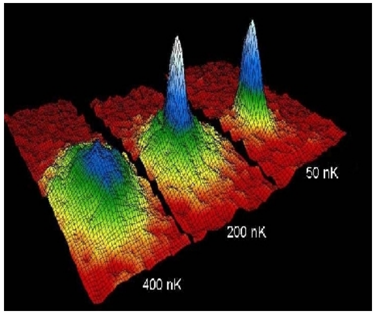
Nature Physics 4, 706 (2008).



Suppressed fluctuations

A. Baas et al., PRL **96**, 176401 (2006)

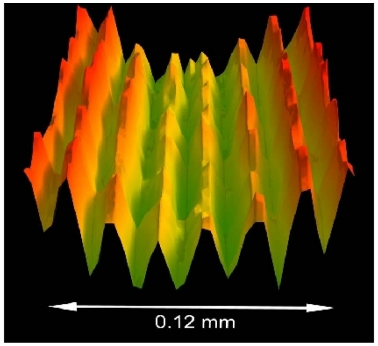
Many features very similar to atomic BEC



The first atomic BEC

M. H. Anderson et al.

Science **269**, 198 (1995)

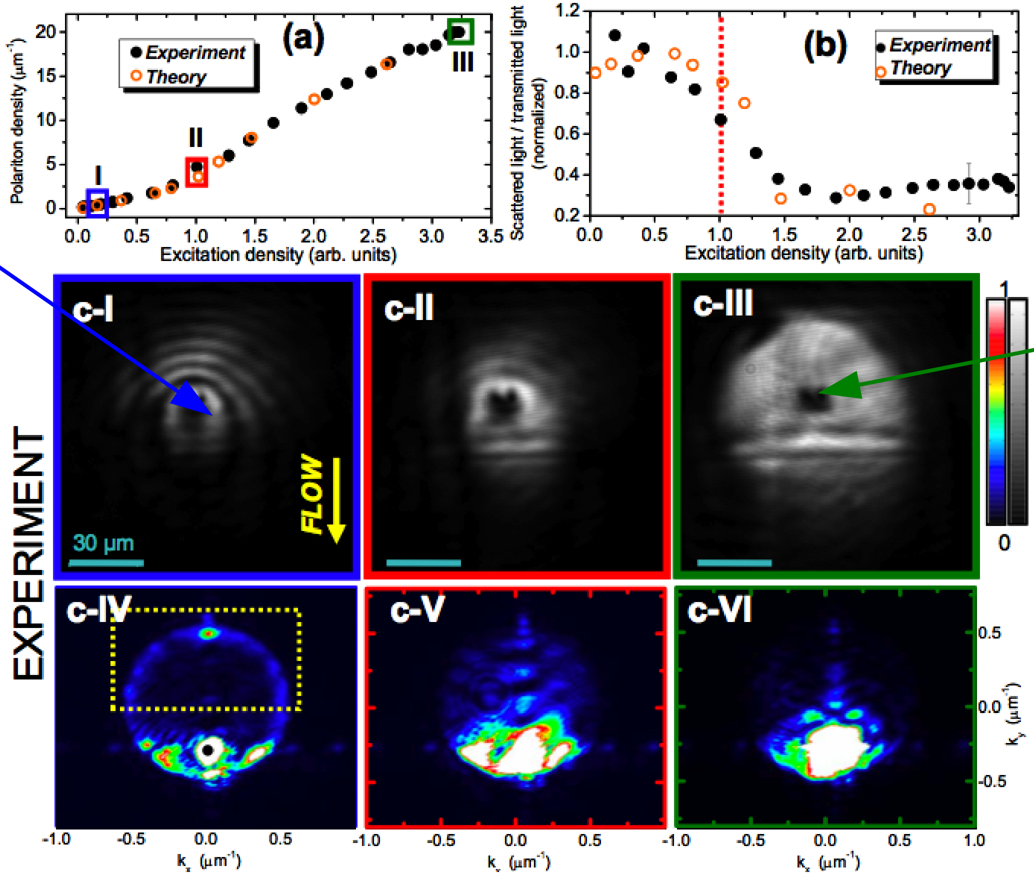


Interference pattern of two expanding atomic BECs

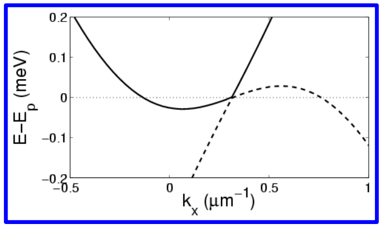
M. R. Andrews, Science **275**, 637 (1995)

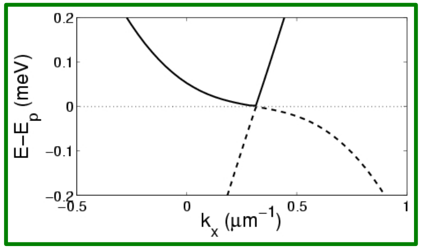
Milestone II: 2008 - Superfluid light

scattering
on weak defect



superfluid flow





increase polariton density

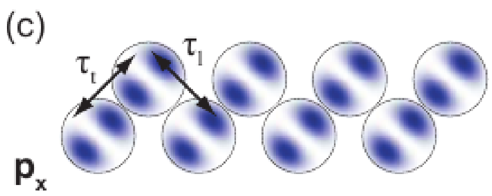
Figure from LKB-P6 group:
A.Amo, J. Lefrère, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

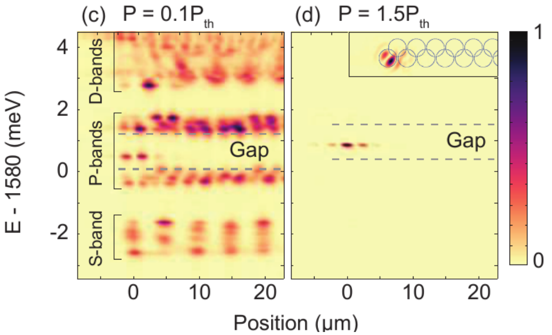
Theory: IC and C. Ciuti, PRL **93**, 166401 (2004).

Milestone III: 2017 - Topological lasing

a.k.a. non-equilibrium BEC in a topological edge state

In other words: what happens if gain added to topological photonics model ?



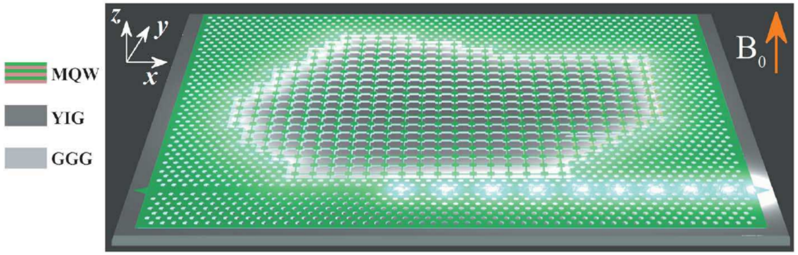


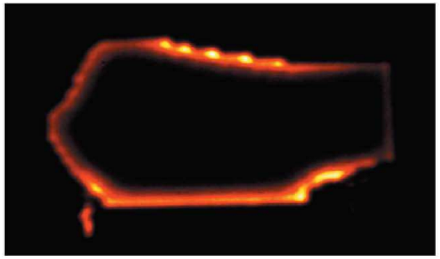
St. Jean, et al., Nat. Phot. ‘17

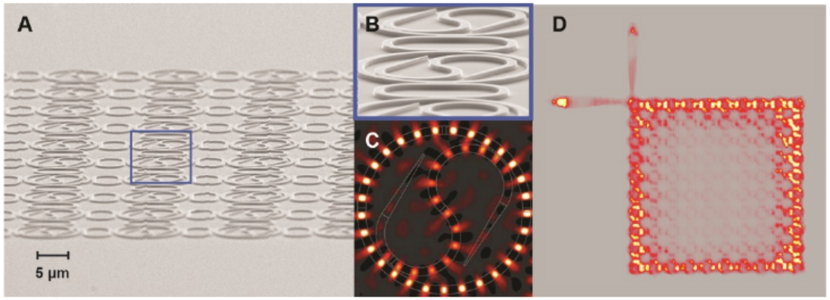
System: 1D SSH array of micropillar cavities for exciton-polaritons under incoherent pump

Bahari et al., Science 2017

System: 2D photonic crystal slab, amplification by QWs, magnetic field to break T







Bandres et al., Science 2018

System: array of Si-based ring resonators with optically pumped III-V amplifier layer.
Tai-Ji shape to break inversion symmetry

Exciting for opto-electronic applications:

- robust platform to ensure large-area coherence in high-power laser source
- interesting Kardar-Parisi-Zhang stat-mech – Amelio-IC, PRX 2021; Fontaine et al., Nature 2022

The future:

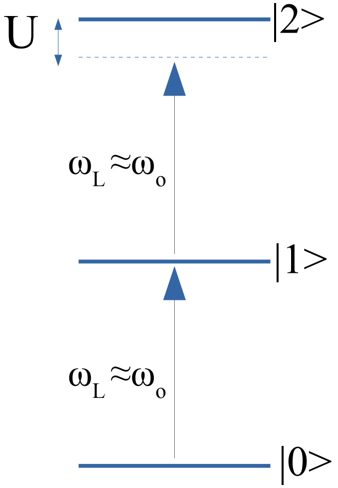
Strongly interacting fluids of light

Mott insulators &
Fractional Quantum Hall liquids

Photon blockade

Driven-dissipative Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i F_i(t)\, \hat{b}_i + h.c.$$

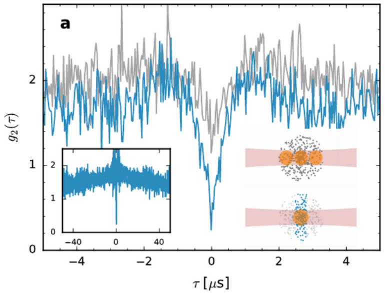


- Array of single-mode cavities at ω_o, tunnel coupling J, losses γ
- Polariton interactions: on-site interaction U due to optical nonlinearity
- If U >> γ & J, coherent pump resonant with 0→1, but not with 1→2.

Photon blockade → Effectively impenetrable photons

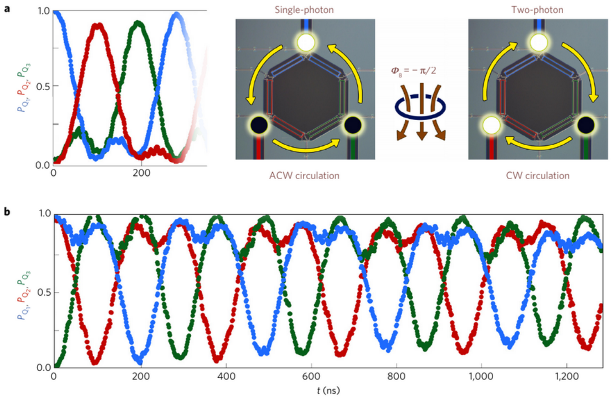
Opposite regime than non-interacting photons of Maxwell's eqs.

Single-cavity blockade observed in many platforms since the 2000s, present challenge → scale up to many-cavity geometry

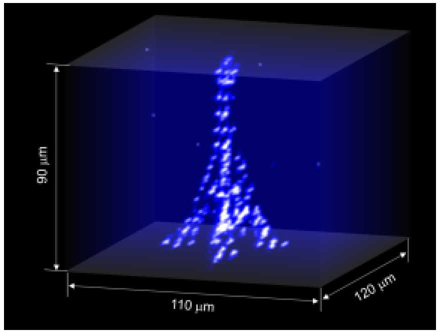


Polariton blockade via Rydberg-EIT (@ Chicago)

Circuit QED device (@GoogleLabs)



Fluid of spin excitations in lattice of Rydberg atoms. (Broways, Lukin,...)



IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

Photon blockade + synthetic gauge field = FQHE for light

Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

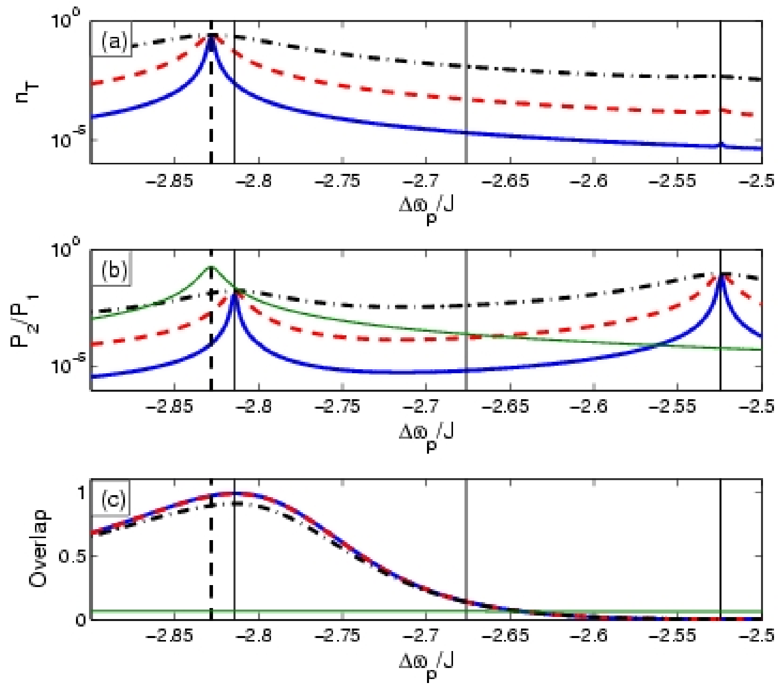
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1,\ldots,z_N) = \mathcal{N}_L F^{(l)}_{\mathrm{CM}}(Z) e^{-\pi\alpha\sum_i y_i^2} \times \prod_{i<j}^{N}\left(\vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\left(\frac{z_i - z_j}{L}\middle| i\right)\right)^2$$

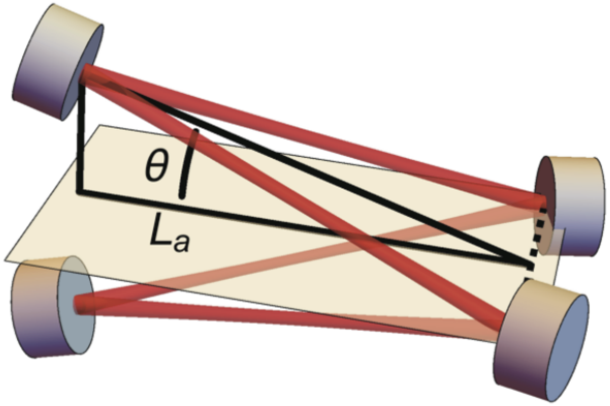
- no need for adiabatic following, etc....

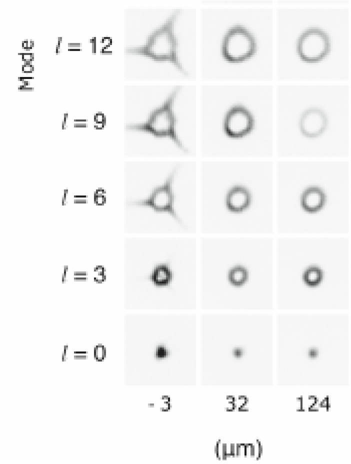


R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)
See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225

Experiment @ Chicago

A far smarter design



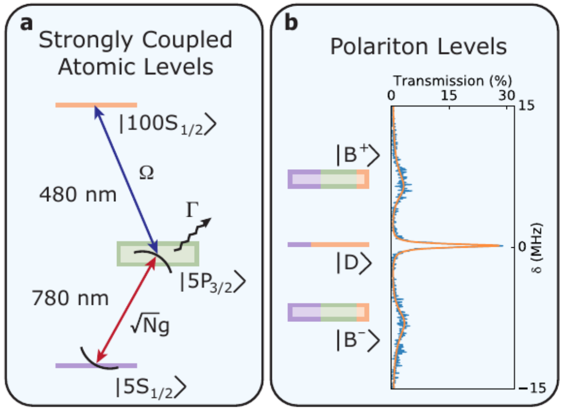


Non-planar ring cavity:

- Parallel transport→ synthetic B
- Landau levels for photons observed

Crucial advantages:

- Narrow frequency range relevant
- Integrated with Rydberg-EIT reinforced nonlinearities



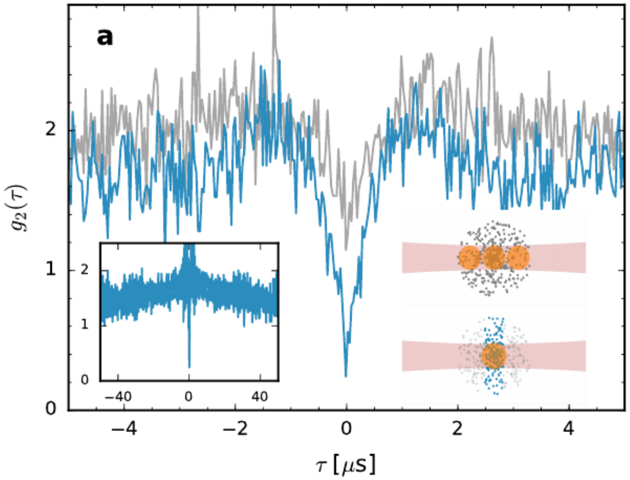
Polariton blockade on lowest (0,0) mode

- Equivalent to $\Delta_{\text{Laughlin}} > \gamma$

Easiest strategy for Laughlin

- Coherent pumping→ multi-photon peaks to few-body states
- Laughlin state → quantum correlations between orbital modes

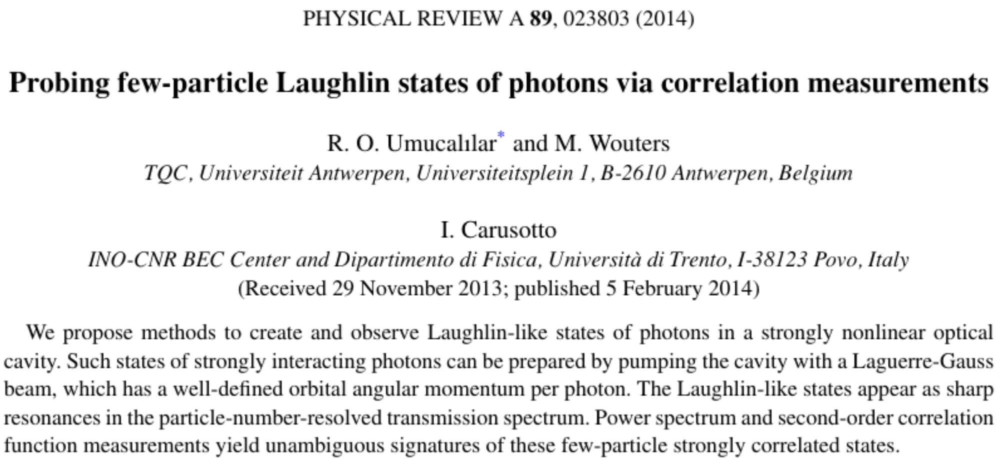
(Umucalilar-Wouters-IC, PRA 2014)



Breaking news: 2-photon Laughlin state realized
(Clark et al., Nature 2020)

Figures from J. Simon's group @ U. Chicago
Schine et al., Nature 2016; Jia et al. 1705.07475

Experiment @ Chicago (II)



PHYSICAL REVIEW A **89**, 023803 (2014)

Probing few-particle Laughlin states of photons via correlation measurements

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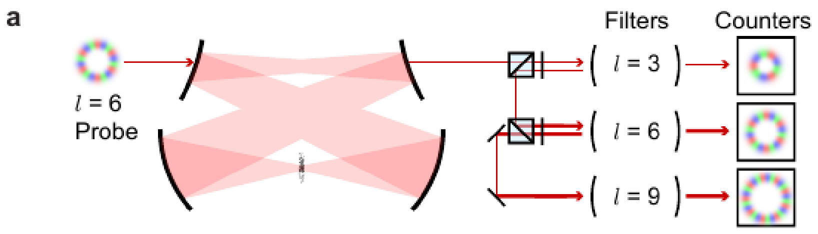
(Received 29 November 2013; published 5 February 2014)

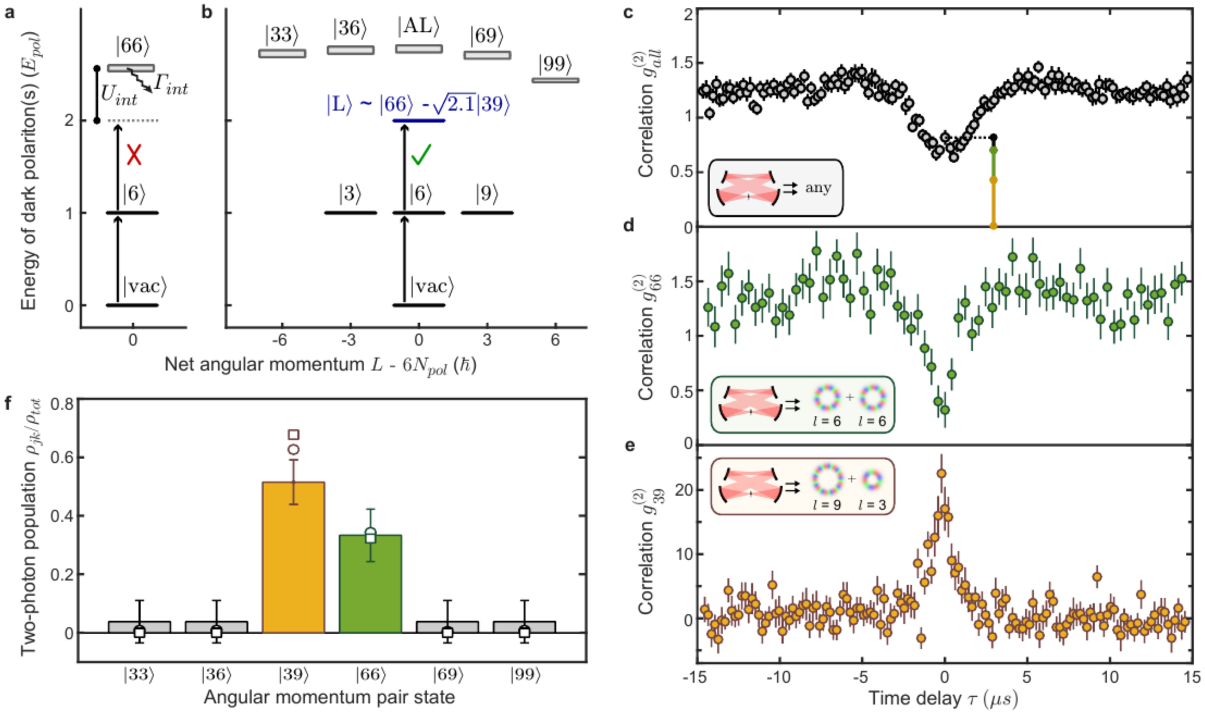
We propose methods to create and observe Laughlin-like states of photons in a strongly nonlinear optical cavity. Such states of strongly interacting photons can be prepared by pumping the cavity with a Laguerre-Gauss beam, which has a well-defined orbital angular momentum per photon. The Laughlin-like states appear as sharp resonances in the particle-number-resolved transmission spectrum. Power spectrum and second-order correlation function measurements yield unambiguous signatures of these few-particle strongly correlated states.



Quantum optical tricks
to perform state tomography:
highlight generation of two-photon
Laughlin state

Challenge: scale up to larger
number of particles

Coherent pump scheme scales
badly with N for topological states



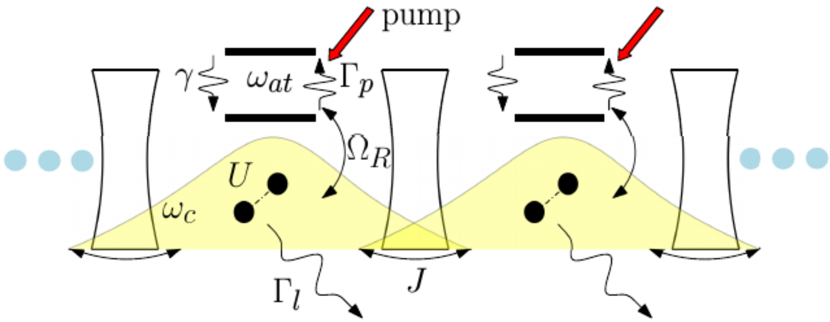
L. W. Clark, N. Schine, C. Baum, N. Jia, J. Simon, *Observation of Laughlin states made of light*, Nature 2020

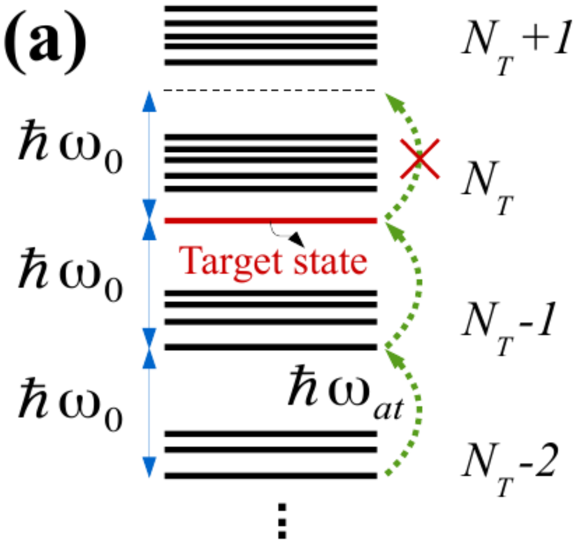
How to access larger particle numbers

Coherent pump only able to selectively excite few-photon states

→ Frequency-dependent incoherent pumping, e.g. collection of inverted emitters

- Lorentzian emission line around ω_{at}
  sophisticated schemes → other spectral shapes
- Emission only active if many-body transition is near resonance
- Injects photons until band is full (MI) or many-body gap develops (FQH)
- Many-body gap blocks excitation of higher states





Umucalilar-IC, PRA 2017
Lebreuilly, Biella et al., PRA 2017

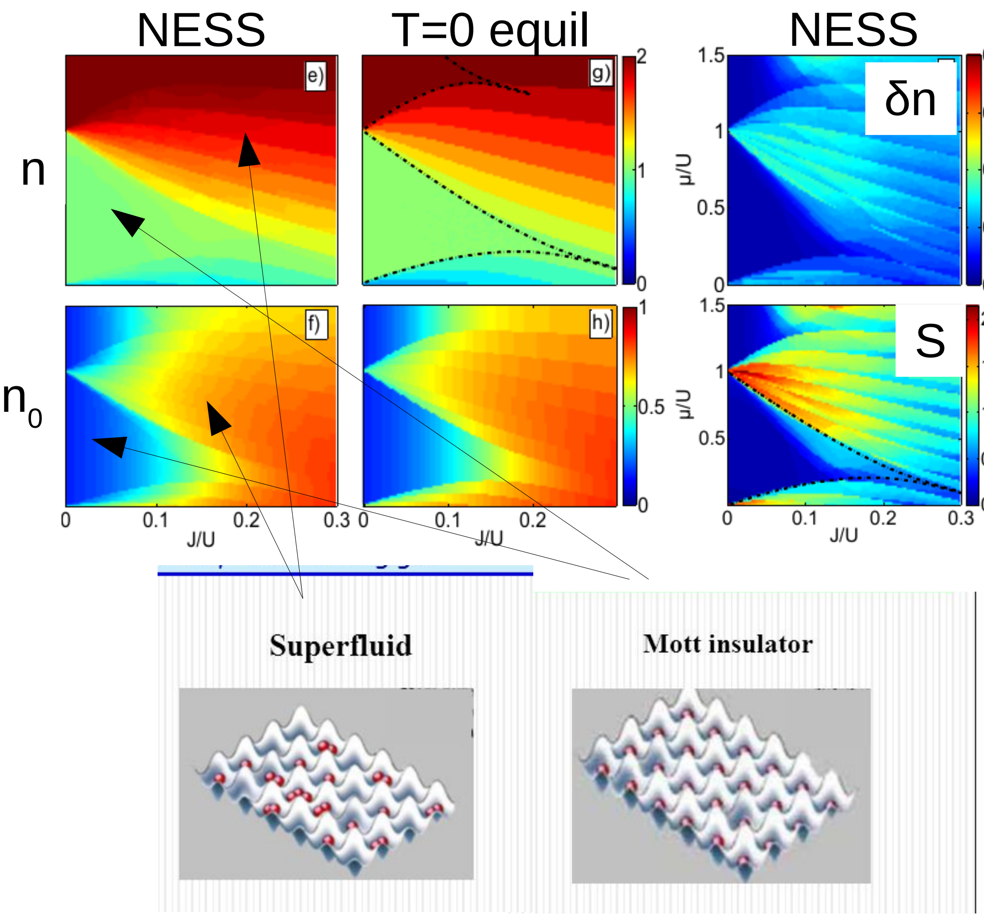
General idea:
Kapit, Hafezi, Simon, PRX 2014
Lebreuilly et al. CRAS (2016)

Mott insulators of light

$$\bar{\mathcal{L}}_{\rm em}(\rho_{\rm ph}) = \frac{\Gamma_{\rm em}}{2}\sum_{i=1}^{k}\left[2\bar{a}_i^\dagger \rho_{\rm ph}\bar{a}_i - \bar{a}_i \bar{a}_i^\dagger \rho_{\rm ph} - \rho_{\rm ph}\bar{a}_i \bar{a}_i^\dagger\right]$$

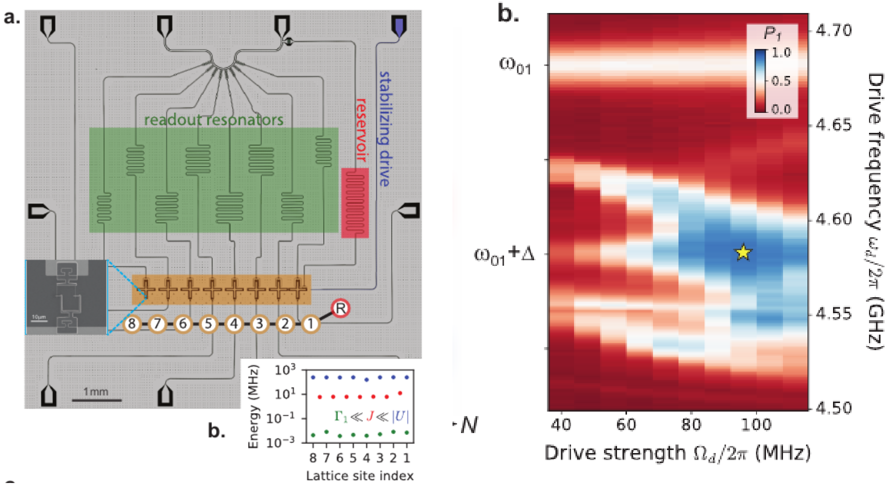
$$\langle f'|\,\bar{a}_i^\dagger\,|f\rangle = \frac{\Gamma_{\rm pump}/2}{\sqrt{(\omega_{\rm at}-\omega_{f',f})^2+(\Gamma_{\rm pump}/2)^2}}\langle f'|\,a_i^\dagger\,|f\rangle$$

- non-Markovian master equation:
  frequency-dependent emission
  → rescaled jump operators
- driven-dissipative steady state stabilizes
  strongly correlated many-body states
  e.g. Mott-insulator, FQH…
- resembles low-T equilibrium
  (but interesting deviations in some cases)
- (in principle) no restriction to small $N_{\rm ph}$
  only requirement → many-body energy gap



Lebreuilly, Biella et al., 1704.01106 & 1704.08978
(published on PRA, 2017)

Related work in Kapit, Hafezi, Simon, PRX 2014



First expt: Ma *et al.* Nature 2019

What about large FQH fluids?

Coherent pump:

- Able to selectively generate few-body states
- Limited by (exponentially) decreasing matrix element for larger systems

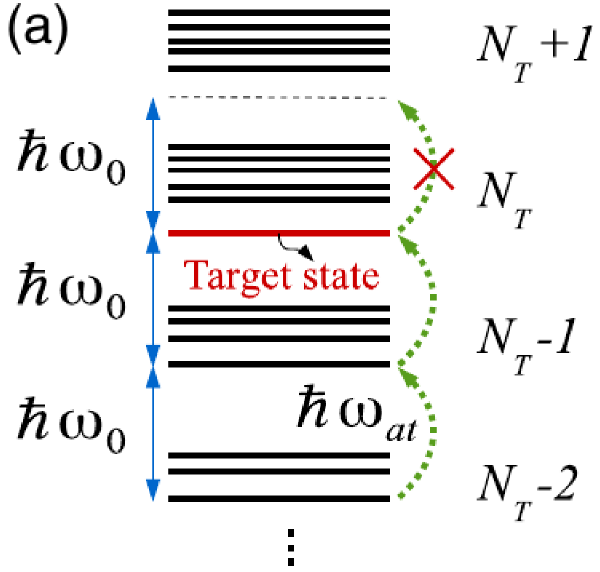
Frequency-dependent incoherent pump:

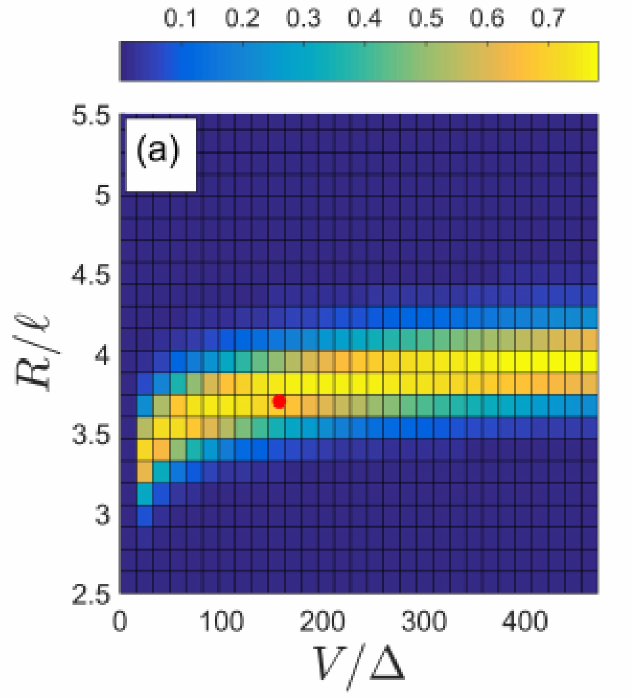
- Interactions → many-body gap Δ
- Edge excitations not gapped.
  Hard-wall confinement gives small δ
- Non-Markovianity blocks excitation to higher states

Calculations only possible for small systems:

- Large overlap with Laughlin states
- Excitations localized mostly on edge

Open question: what are ultimate limitations of this pumping method?





R. O. Umucalilar, IC, *Generation and spectroscopic signatures of a fractional quantum Hall liquid of photons in an incoherently pumped optical cavity*, PRA 2017.
R. O Umucalilar, J. Simon, IC, *Autonomous stabilization of photonic Laughlin states through angular momentum potentials*, PRA 2022
Similar works by Lebreuilly & Girvin (2022)

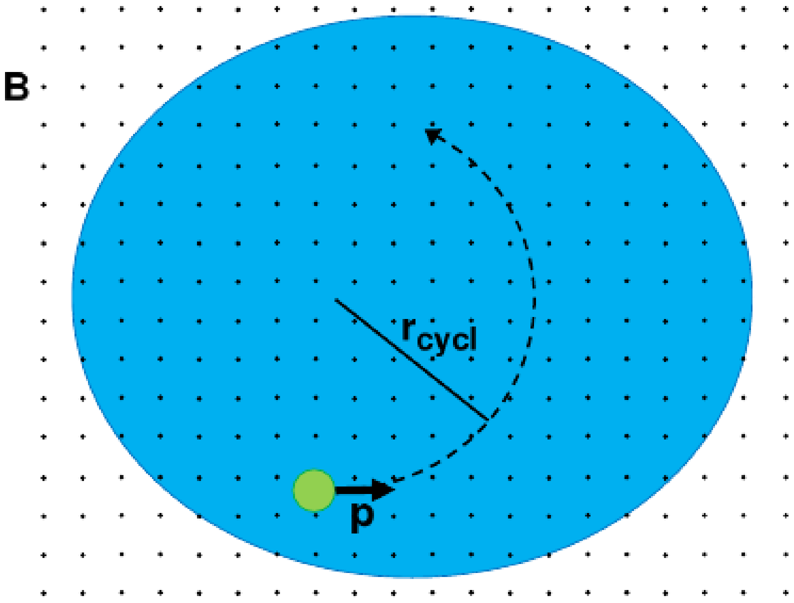
A long-term objective:

Probing anyonic statistics of quasi-holes

Quantum mechanics of anyons (I) – single particle

Laughlin wavefunction of Fractional Quantum Hall:

- quasi-holes → no E_{kin}, no independent life
- dressed by heavy impurity → anyonic molecule
- full-fledged mechanical degree of freedom



Born-Oppenheimer approx:

- Heavy impurity→ slow Degree of Freedom
- Light FQH particles → fast DoF

$$H_{\text{eff}} = \frac{\left[-i\nabla_{\mathbf{R}} - (Q - \nu q)\,\mathbf{A}(\mathbf{R})\right]^2}{2\mathcal{M}}$$

- Mass $\mathcal{M}$ → M (impurity) + QH dragging effect
- Impurity & FQH particles feel (Synth-)B,
  so synth-Charge → Q (impurity) – ν q (QH)

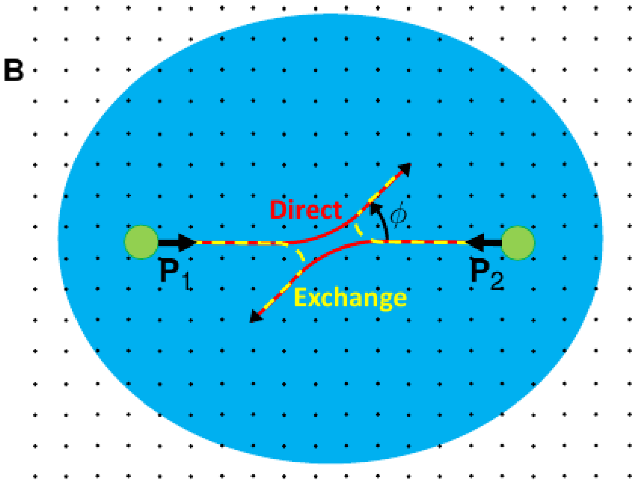
Cyclotron orbit → fractional charge and BO mass correction

A. Muñoz de las Heras, E. Macaluso, IC, PRX 2020

Quantum mechanics of anyons (II) – two particles

Each particle → attached flux

$$\mathcal{A}_j(\mathbf{R}) = \mathcal{A}_{\mathrm{q}}(\mathbf{R}_j) + \mathcal{A}_{\mathrm{stat},j}(\mathbf{R})$$
$$= \frac{\mathcal{B}_{\mathrm{q}}}{2}\mathbf{u}_{\mathrm{z}} \times \mathbf{R}_j + (-1)^j \frac{\nu}{R_{\mathrm{rel}}^2}\mathbf{u}_{\mathrm{z}} \times \mathbf{R}_{\mathrm{rel}}$$



Relative motion:

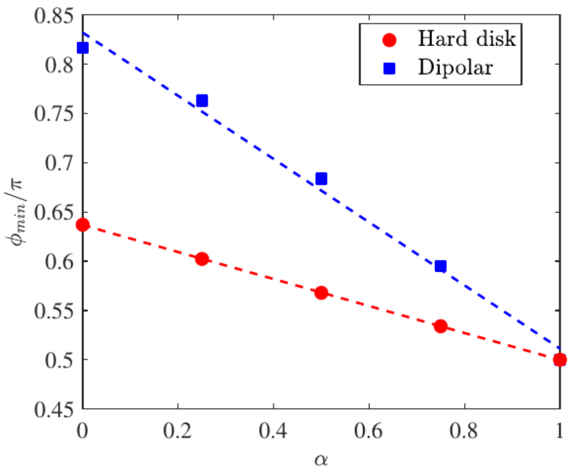
- inter-particle potential
- statistical A$_{\mathrm{rel}}$ due to attached flux

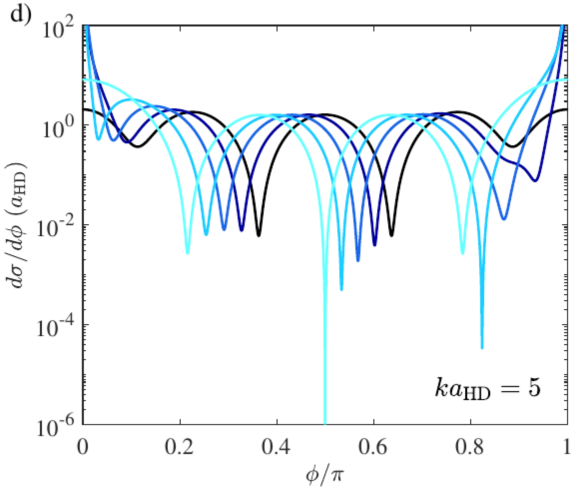
$$H_{\mathrm{rel}} = \frac{\left[\mathbf{P}_{\mathrm{rel}} + \mathbf{A}_{\mathrm{rel}}(\mathbf{R}_{\mathrm{rel}})\right]^2}{2\mathcal{M}_{\mathrm{rel}}} + V_{\mathrm{ii}}(R_{\mathrm{rel}})$$

2-body scattering: interference of direct & exchange

- fringes in differential cross section
- fringe position depends on attached flux, i.e. measure fractional statistics

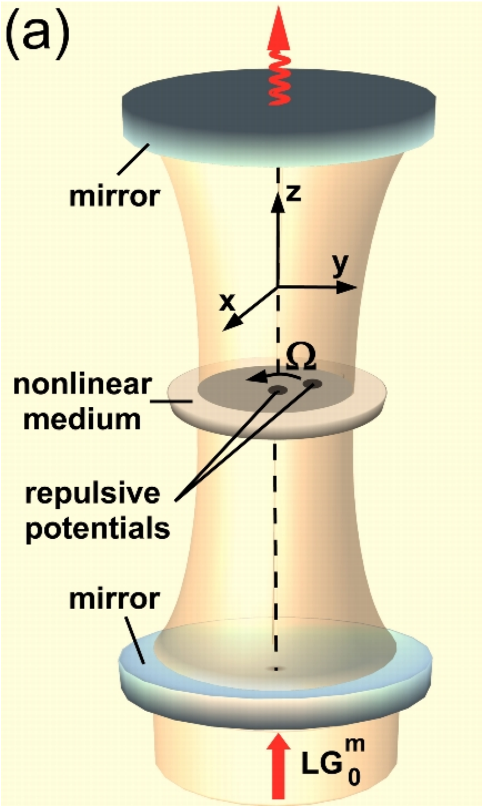
- Scheme works with polar molecules (heavy + long-range interactions) in atoms (light FQH gas)
- What about Rydberg polaritons?





A. Muñoz de las Heras, E. Macaluso, IC, PRX 2020

Optical signatures of the anyonic braiding phase



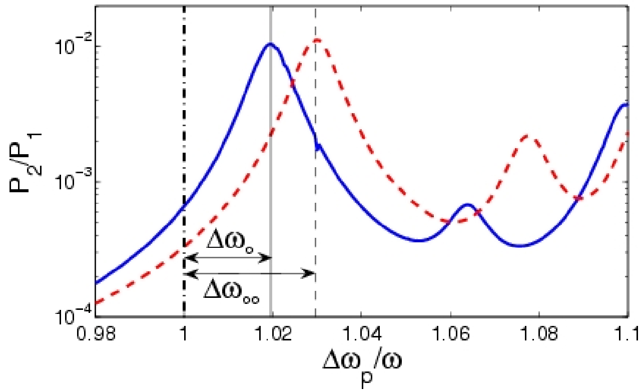
Anyonic statistics of quasi-holes: many-body Berry phase ϕ_{Br} when positions swapped during braiding

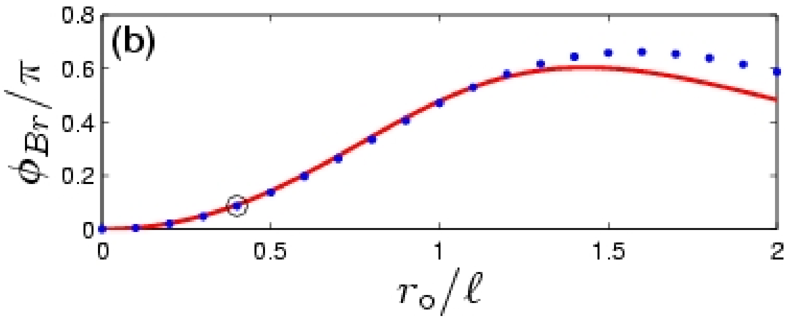
In a photonic system:

- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_{o})\, T_{rot} \quad [2\pi]$$

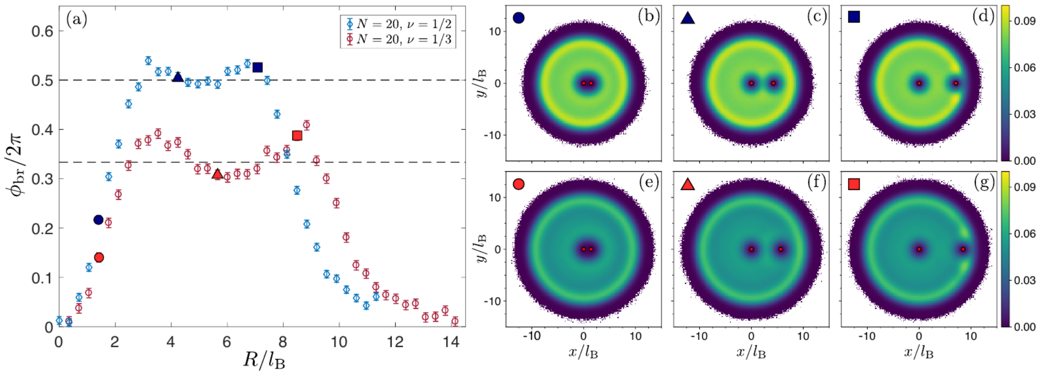
How to measure ϕ_{Br} without an actual braiding?





R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Observing anyonic statistics via time-of-flight measurements



Braiding phase → Berry phase when two quasi-holes are moved around each other

$$\varphi_{\rm B}(R) = i \oint_R \langle \Psi(\theta)|\partial_\theta|\Psi(\theta)\rangle d\theta.$$

Braiding operation can be generated by rotations, so braiding phase related to L_z

$$\varphi_{\rm B}(R) = \frac{1}{\hbar}\oint_R \langle \Psi(\theta)|L_z|\Psi(\theta)\rangle d\theta = \frac{2\pi}{\hbar}\langle L_z\rangle$$

Self-similar expansion of lowest-Landau-levels → L_z can be measured in time-of-flight via size of the expanding cloud

$$\langle r^2\rangle_{\rm tof} = \frac{1}{N}\left(\frac{\hbar t}{\sqrt{2}Ml_B}\right)^2\left(\frac{\langle L_z\rangle}{\hbar} + N\right) = \left(\frac{\hbar t}{2Ml_B^2}\right)^2\langle r^2\rangle$$

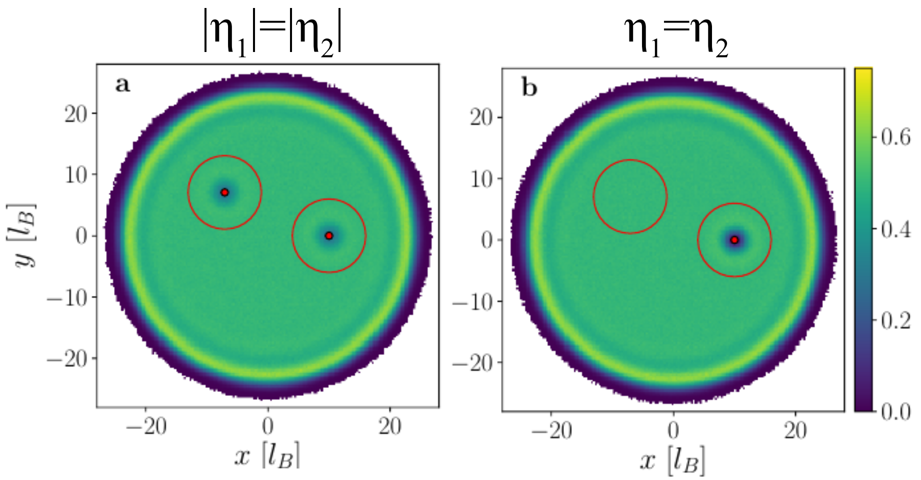
Can be applied to both cold atoms or to fluids of light looking at far-field emission pattern
Difficulty → small angular momentum difference of QH compared to total L_z

Umucalilar, Macaluso et al., *Observing anyonic statistics via time-of-flight measurements*, PRL (2018)

Quasi-Hole structure vs. anyon statistics (I)

- Compare (two) single quasi-holes and overlapping pair of quasi-holes:

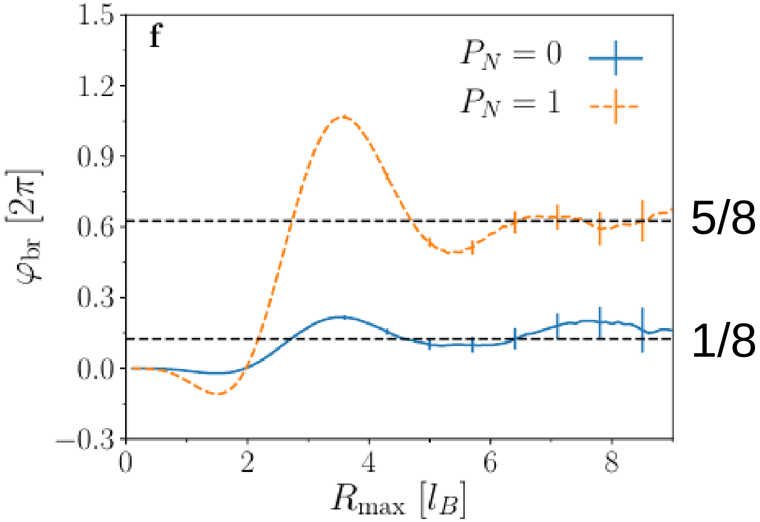
$$\frac{\varphi_{\rm br}}{2\pi} = \frac{1}{\hbar}\left[\langle \hat{L}_z\rangle_{|\eta_1|=|\eta_2|} - \langle \hat{L}_z\rangle_{\eta_1=\eta_2}\right].$$



- Relates to difference of density profiles:

$$\frac{\varphi_{\rm br}}{2\pi} = \frac{N}{2l_B^2}\left[\langle r^2\rangle_{|\eta_1|=|\eta_2|} - \langle r^2\rangle_{\eta_1=\eta_2}\right],$$

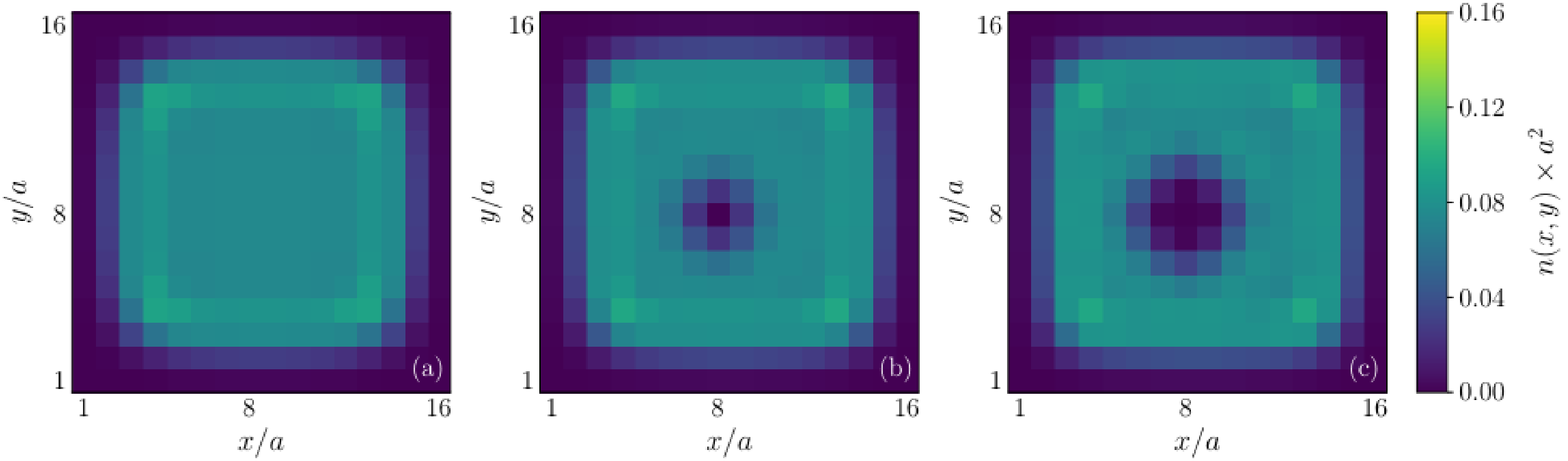
- Incompressibility → external region unaffected
- Statistics inferred from local density difference around QH core, i.e. variance of density depletion



- Insensitive to spurious excitation of (ungapped) edge states
- Proposal realizable in Chicago's twisted cavity set-up
- Numerical calculation using Moore-Read wavefunction allows to distinguish fusion channels of even/odd total particle number

E. Macaluso, T. Comparin, L. Mazza, IC, *Fusion channels of non-Abelian anyons from angular-momentum and density-profile measurements*, PRL 2019

Quasi-Hole structure vs. anyon statistics (II)



N	L	α	Ω/t
12	16	0.15	1×10^{-3}

Discrete lattice model → Harper-Hofstadter-Bose-Hubbard

Ground state using Tree-Tensor-Network ansatz

- experimentally realistic “large” system
- open boundary conditions with harmonic trap
- repulsive potentials to pin quasi-holes

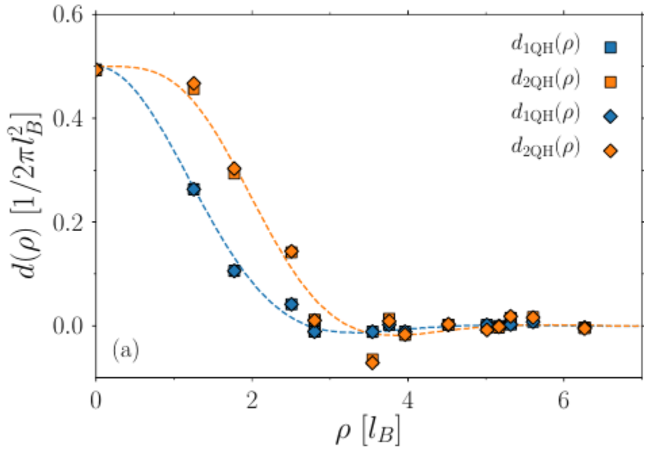
Apply discretized version of braiding phase formula

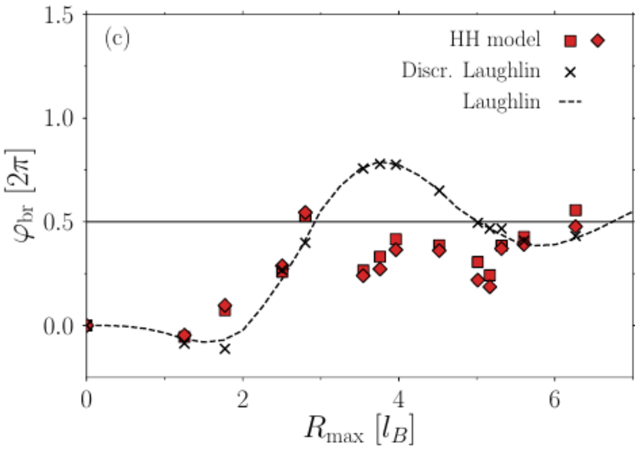
$$\frac{\varphi_{\mathrm{br}}}{2\pi} = \frac{N}{2l_B^2}\left[\langle r^2\rangle_{|\eta_1|=|\eta_2|} - \langle r^2\rangle_{\eta_1=\eta_2}\right],$$

to physical ground state wavefunction

→ Accurate reconstruction of anyonic statistics

→ Experiment accessible in state-of-the-art circuit-QED systems





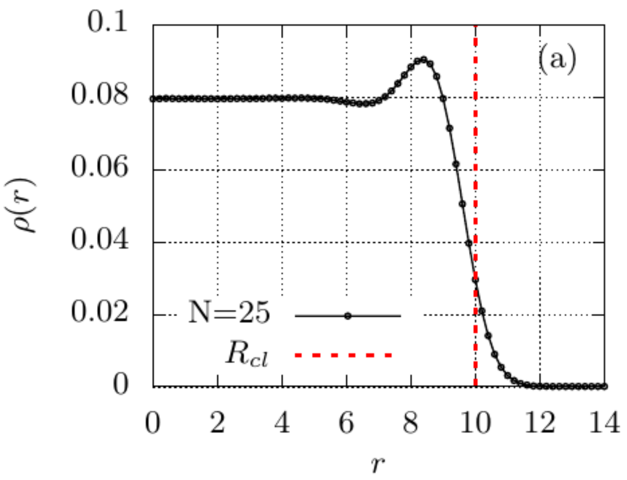
E. Macaluso *et al.*, *Charge and statistics of lattice quasiholes from density measurements: a Tree Tensor Network study,* Phys. Rev. Research (2020)

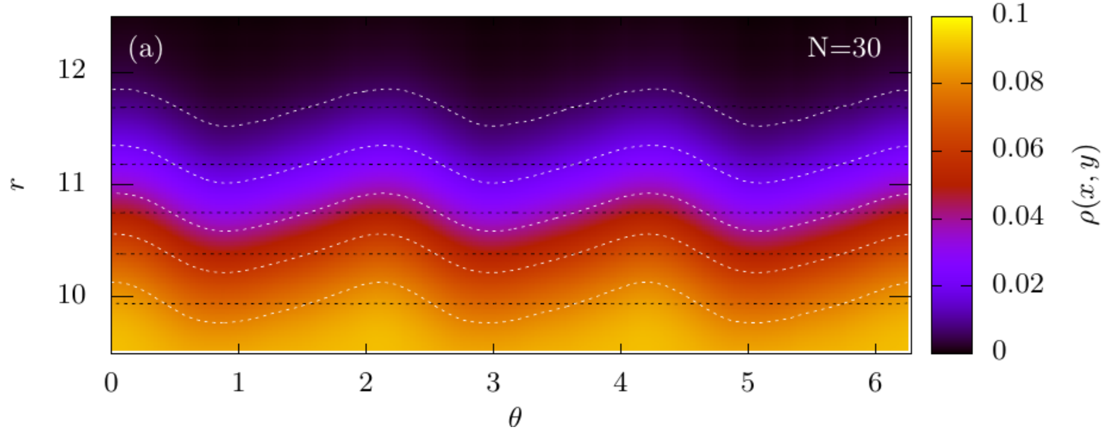
On-going work:

Linear and nonlinear edge dynamics of FQH clouds

- A. Nardin, IC, Non-linear edge dynamics of an integer quantum Hall fluid, Europhys. Lett. 132, 10002 (2020).
- A. Nardin, IC, *Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm*, arXiv:2203.02539

Response of trapped FQH cloud to external potential (I)





Trapping potential $V_{conf}(r) = \lambda\, r^{\delta}$

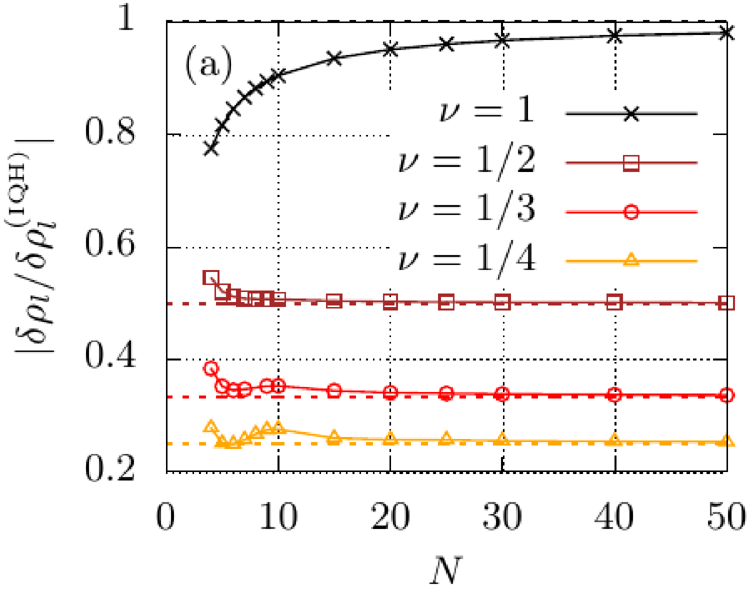
ED calculations by MC evaluation of matrix elements via Metropolis (works well upto ~50 particles)

Time-dependent perturbation U(r,θ;t):

- generates oscillatory perturbation on edge

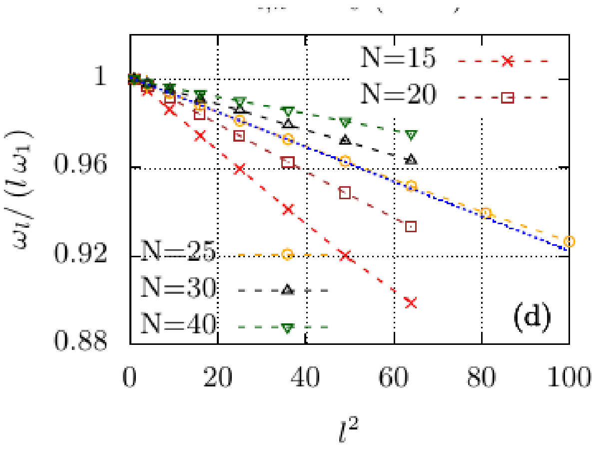
Weak perturbation limit:

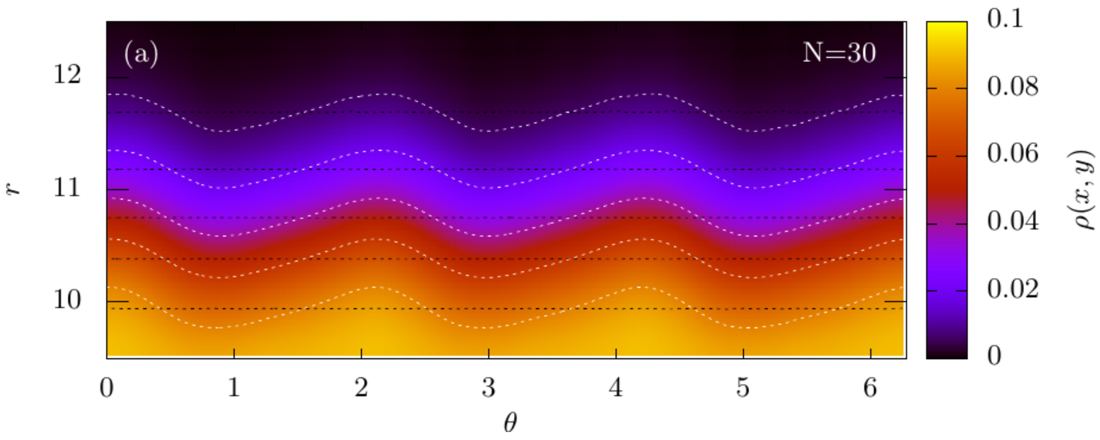
- linear response proportional to filling ν
- related to quantized transverse conductivity of FQH
- matches chiral Luttinger liquid picture...



A. Nardin, IC, *Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm*, arXiv:2203.02539

Response of trapped FQH cloud to external potential (II)





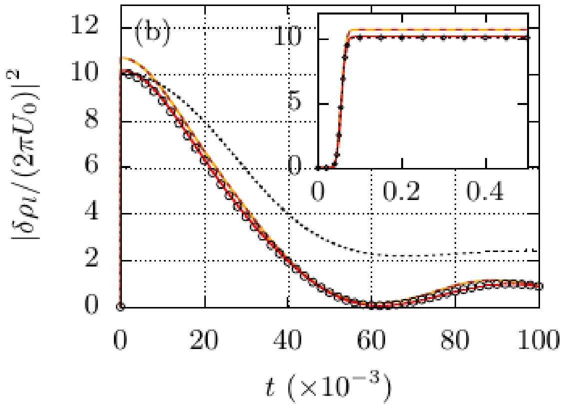
Linear response matches χLL...but much more physics hidden in edge perturbation σ(z,t) :

- free oscillation frequency shift ~ k^3 → group velocity dispersion
- nonlinear effects → frequency shift proportional to amplitude σ
  (due to radially increasing trapping force)

Well captured by
classical evolution eq.

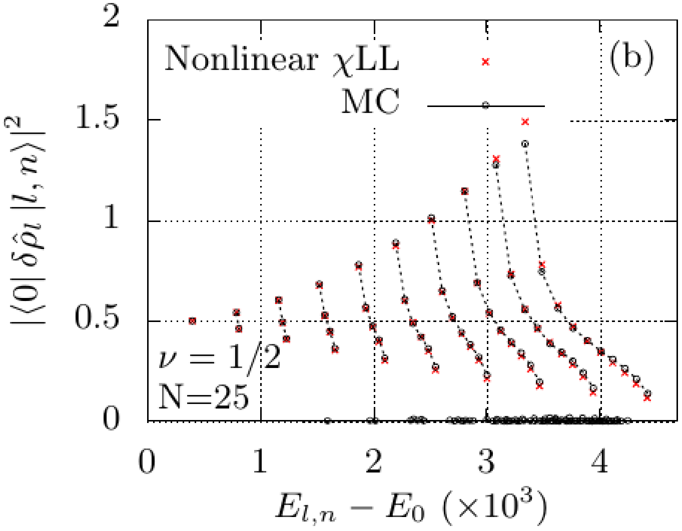
$$\frac{\partial\sigma}{\partial t} = -\left[v_0 \boxed{- \frac{2\pi\tilde{c}_0}{\nu}\sigma}\right]\frac{\partial\sigma}{\partial\zeta} - \boxed{\beta_m\tilde{c}_0\frac{\partial^3\sigma}{\partial\zeta^3}} - \frac{\nu}{2\pi}\frac{\partial U}{\partial\zeta}$$

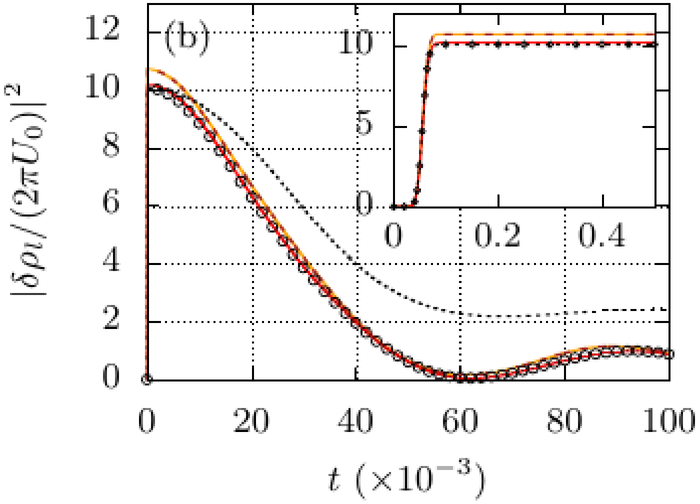
- but also temporal decay of oscillation…
  ...which requires further refinements...

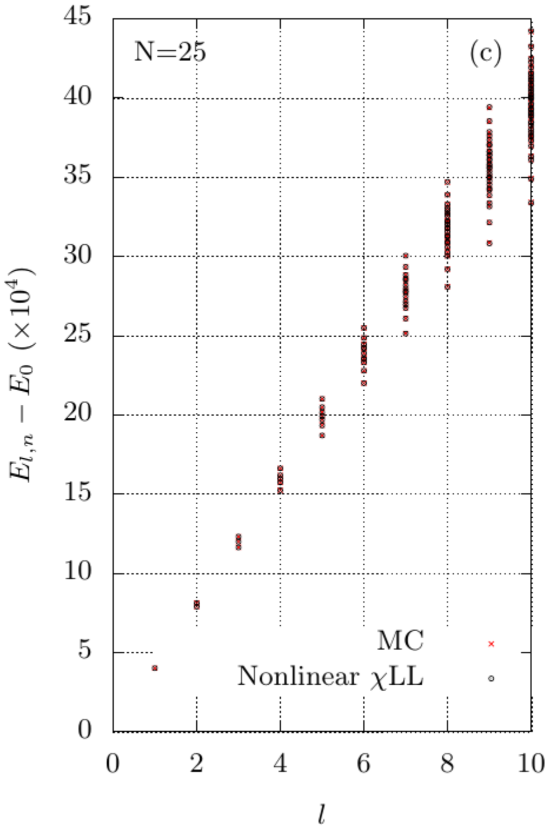


A. Nardin, IC, *Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm*, arXiv:2203.02539

Response of trapped FQH cloud to external potential (III)







Broadening associated to damping captured by quantum-χLL

$$\hat{H}^{NL}_{\chi\mathrm{LL}} = \int d\zeta \left[\frac{\pi\, v_0}{\nu}\,\hat{\sigma}^2 \boxed{-\,\frac{\pi\,\beta_m \tilde{c}_0}{\nu}\left(\frac{\partial\hat{\sigma}}{\partial\zeta}\right)^2} + \boxed{\frac{2\pi^2\tilde{c}_0}{3\nu^2}\hat{\sigma}^3} + U(\zeta,t)\,\hat{\sigma}\right]$$

with $[\hat{\sigma}(\zeta),\hat{\sigma}(\zeta')] = -i\,\frac{\nu}{2\pi}\,\partial_\zeta\,\delta(\zeta-\zeta')$.

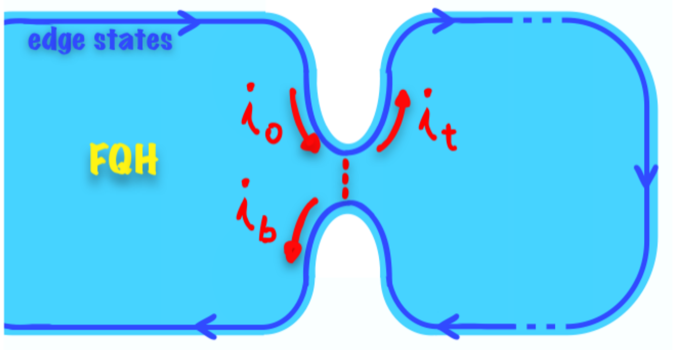
Quantum-χLL eigenstates well match ED results, as well as temporal evolution of observables

A. Nardin, IC, *Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm*, arXiv:2203.02539

(Preliminary) Quantum dynamics at quantum point contact

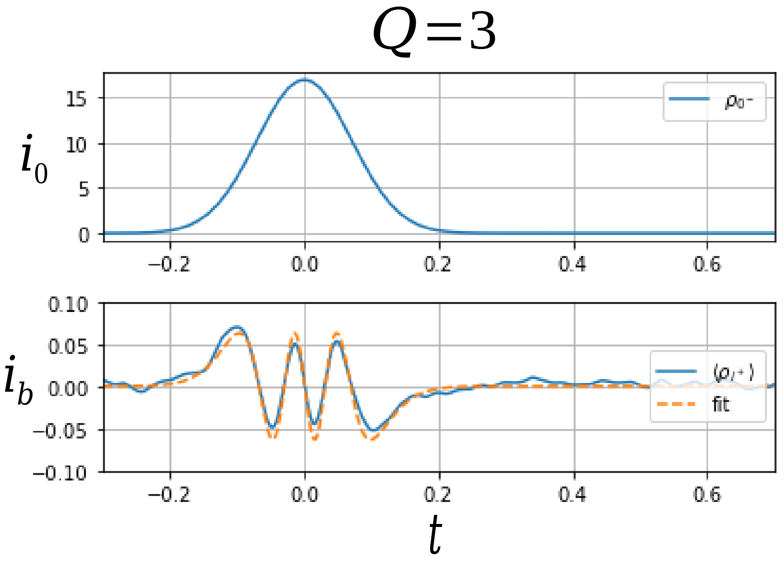
χLL dynamics of edge, intrinsic nonlinearity at junction

Truncated-Wigner description of bosonic χLL DoF



$$\partial_t \rho_s(x,t) + v\,\partial_x \rho_s(x,t) = -\Gamma \sin(2\pi q_s(t))\,(\delta(x) - \delta(x-l))$$

$$\rho_s(x,t=0) = \sum_{k>0} r_k \alpha_k e^{ikx} + h.c. \qquad \langle \rho_s(0^-,0)\,\rho_s(0^-,t)\rangle \propto -\frac{1}{m t^2}$$



- Reproduces many features of FQH edge dynamics:
- Crystallization of high-charge wavepackets
- Gives hints of fractional shot-noise in current
- Better performance for large 1/ν: fractional charge ν of FQH makes semi-classical picture more accurate

Z. Bacciconi, *Fractional quantum Hall edge dynamics from a quantum optics perspective*, MSc thesis at UniTrento (2021); arXiv:2111.05858
Z. Bacciconi, A. Nardin, IC, in preparation (2022)

Conclusions

1-body magnetic/topological effects for photons in synthetic gauge field:

- Unidirectional and topologically protected edge states (2009-)
- Geometrical properties of bulk & anomalous current (2016-)
- Landau levels in cylindrical trap; smart tricks to generate conical geometry (2016-)

Interplay of (weak) nonlinearities and topology:

- Topological solitons (2020-)
- Topolasing, a.k.a. BEC in topological edge state (2017-) → optoelectronics applications

First steps in strongly correlated many-body physics:

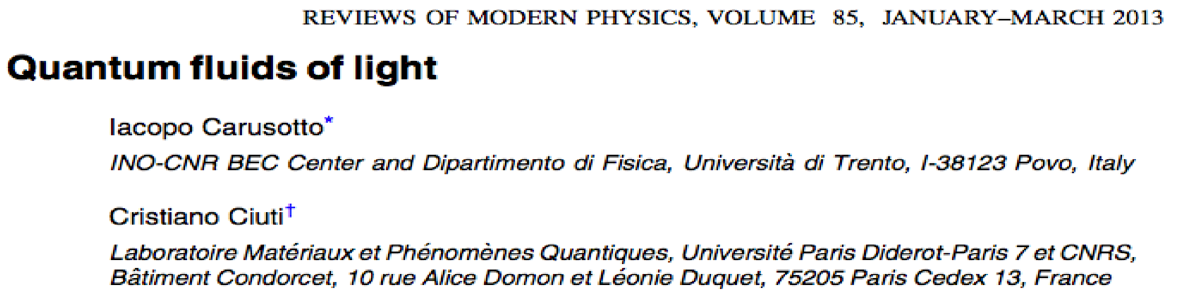
- Mott-insulator → recent experimental observation @ Chicago – Schuster/Simon
- Chain of strongly interacting bosons in synthetic gauge field → expt @ GoogleLabs
- Few-body Laughlin states → recently 2-body baby-Laughlin-state @ Chicago – Simon

Dynamical properties of FQH clouds:

- Apply equally well to photon as well as to atomic fluids
- Observable signatures of fractional anyonic statistics of quasi-hole excitations
- Rich linear and nonlinear dynamics of IQH and FQH edges. Even richer in presence of quantum point contact

If you wish to know more...

BEC



REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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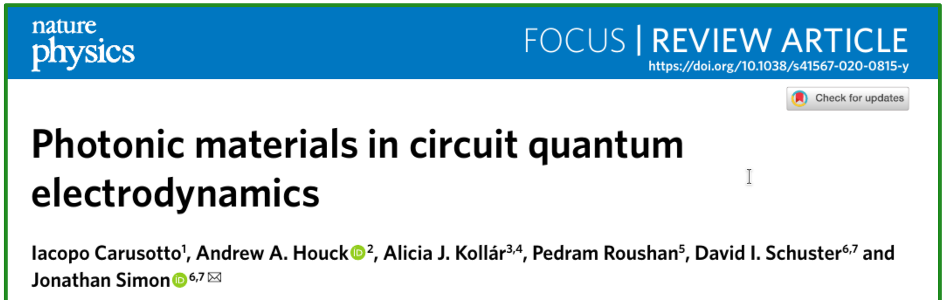
I. Carusotto, C. Ciuti, Rev. Mod. Phys. **85**, 299 (2013)

Come and visit us in Trento!



nature physics

FOCUS | REVIEW ARTICLE
https://doi.org/10.1038/s41567-020-0815-y

Check for updates

Photonic materials in circuit quantum electrodynamics

Iacopo Carusotto[1], Andrew A. Houck[2], Alicia J. Kollár[3,4], Pedram Roushan[5], David I. Schuster[6,7] and Jonathan Simon[6,7]

Review article on Nature Physics (2020)

REVIEWS OF MODERN PHYSICS, VOLUME 91

Topological photonics

Review article arXiv:1802.04173 by Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC,
RMP **91**, 015006 (2019)

We acknowledge generous financial support from:

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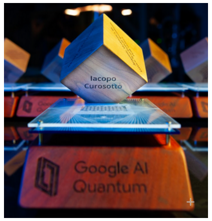


Spontaneous coherence in spatially extended photonic systems: Non-Equilibrium BEC,
J. Bloch, IC, M. Wouters
arXiv:2106.11137

European Commission

Horizon 2020
European Union funding
for Research & Innovation

PhoQuS
Photons for Quantum Simulation

MIR-BOSE

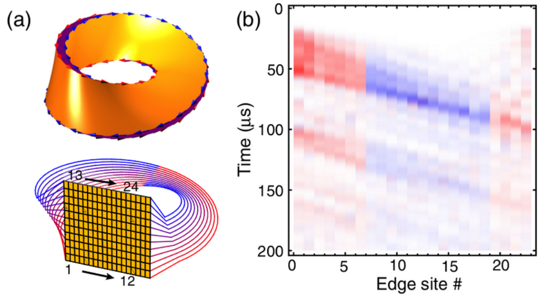
Topological lasers

What new physics in there?

Perspectives for quantum technologies

Fractional quantum Hall fluids in topologically non-trivial geometries

→ topologically degenerate ground state



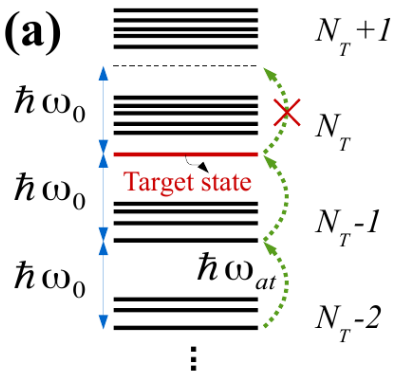
Topological photonics on Moebius band
Ningyual et al., PRX 2015

No mixing by 1-body observables: $\langle\psi_i \,|\, \hat{O} \,|\psi_j\rangle \sim O\,\delta_{ij}$

- Information can be stored in topological ground states
- Quantum operations by braiding non-abelian anyonic excitations
  → in large systems, quantum operations protected by topology

In optical systems:

- standard quantum superpositions $|\mathrm{coh}{:}\alpha\rangle + |\mathrm{coh}{:}{-\alpha}\rangle$ fragile against losses
- which-way information leaks out into dissipation, e.g. radiative emission

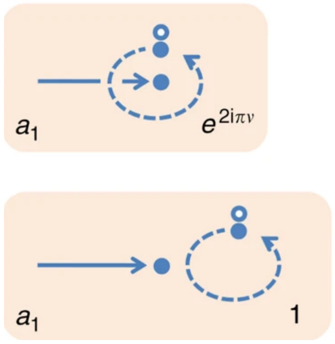


Topological ground states not distinguishable from their emission

→ no which-way info leaks into losses, quantum superposition robust

In other terms, loss of 1 particle creates several quasi-holes:

- mixing of topological states requires quasi-holes to be randomly braided
- this not possible if pump refilling is quick enough



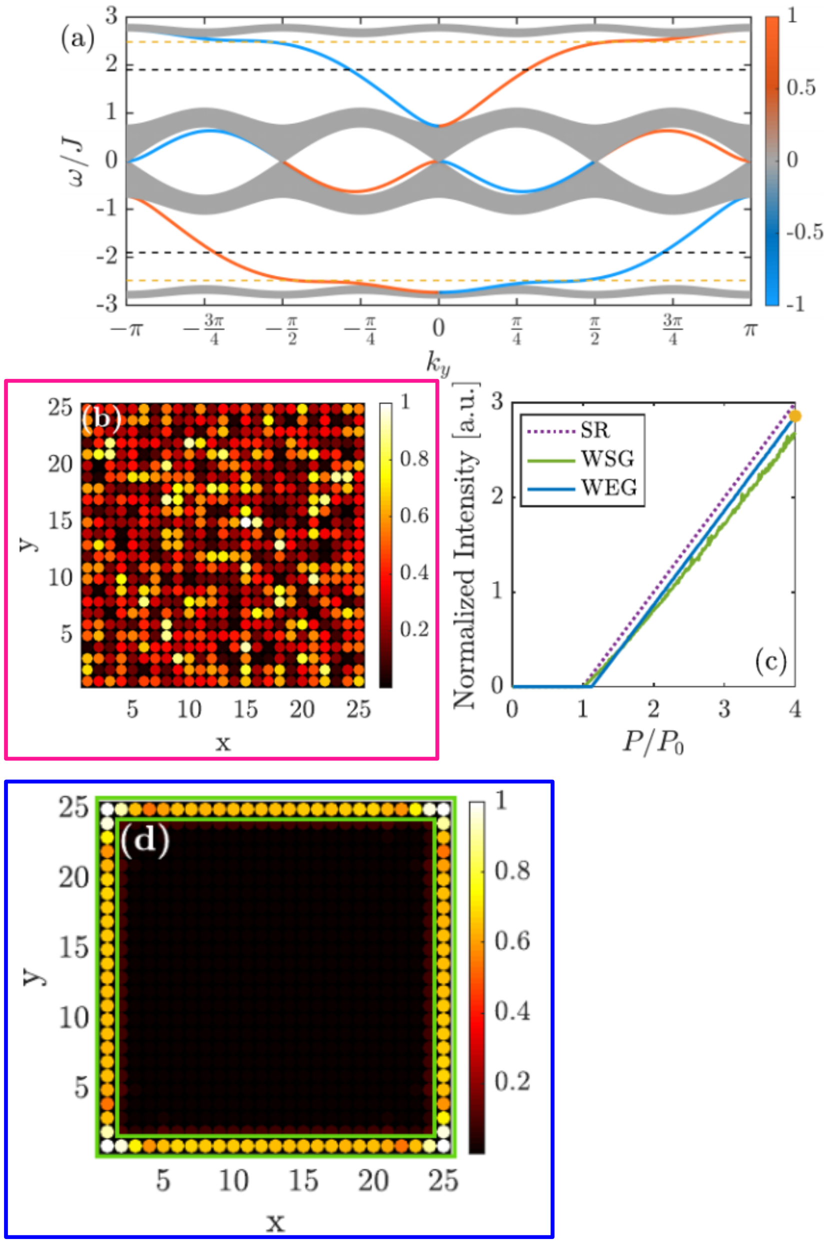
Sketch from Han et al.,
Nat. Comm. 2016

Non-Abelian fractional quantum Hall fluids of light

→ a promising avenue for topological quantum computing

Original idea: Kapit-Hafezi-Simon PRX 2014. General review: Nayak et al., RMP 2008

Topological lasing in 2D models: why interesting?



Topologically trivial system:

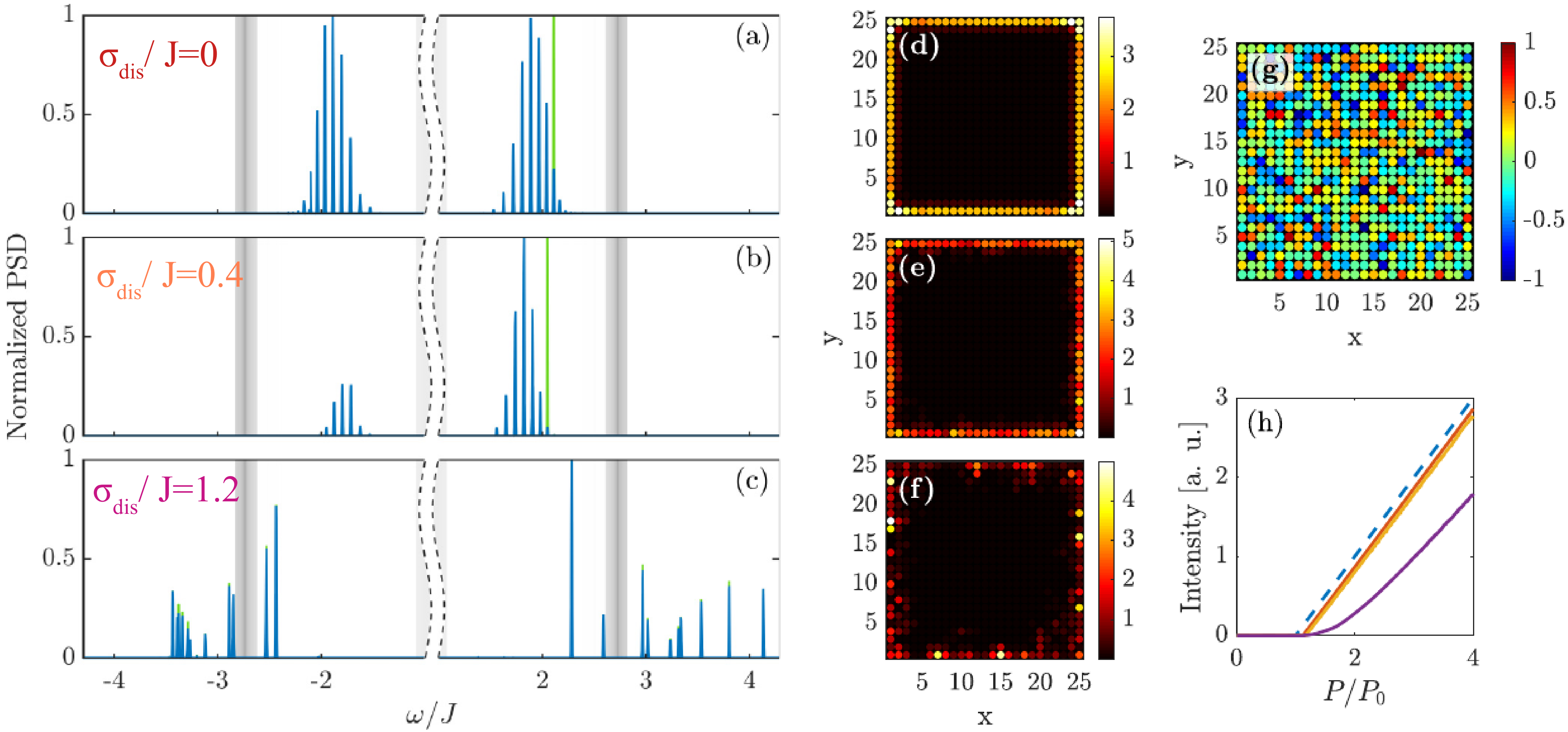
- pumping many cavities gives complicate many-mode emission
- hard to preserve coherence and fully exploit gain when gain distributed on many sites to increase emission power
- serious technological problem for high-power semiconductor laser applications

Topological system:

- 2D Topolaser operation into edge mode when edge only is pumped (WEG)
- Chiral motion → phase lock many individual sites

Figures from M. Seclì's Msc thesis @ UniTN, 2017 and Seclì *et al.*, Phys. Rev. Research 2019
See also: Harari et al., Science 2018.

Topological lasing in 2D models: immune to disorder

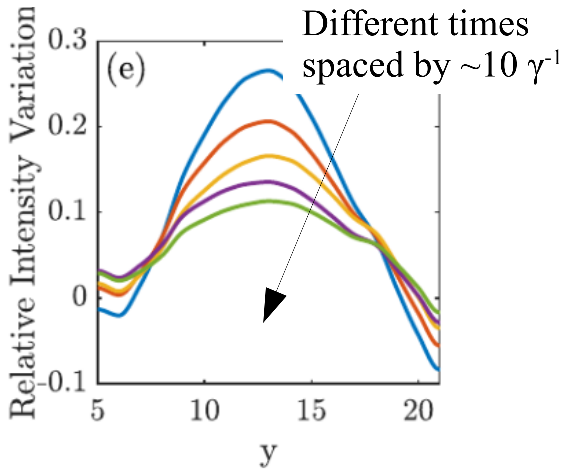


- Chiral propagation immune to disorder
- Efficient single mode lasing with high slope efficiency
- Lasing mode randomly selected via periodic boundary conditions
  – like in any ring laser

Mean-field
(semiclassical)
calculation

M. Seclì’s Msc thesis @ UniTN, 2017 and Seclì *et al.*, Phys. Rev. Research 2019
See also: Harari et al., Science 2018.

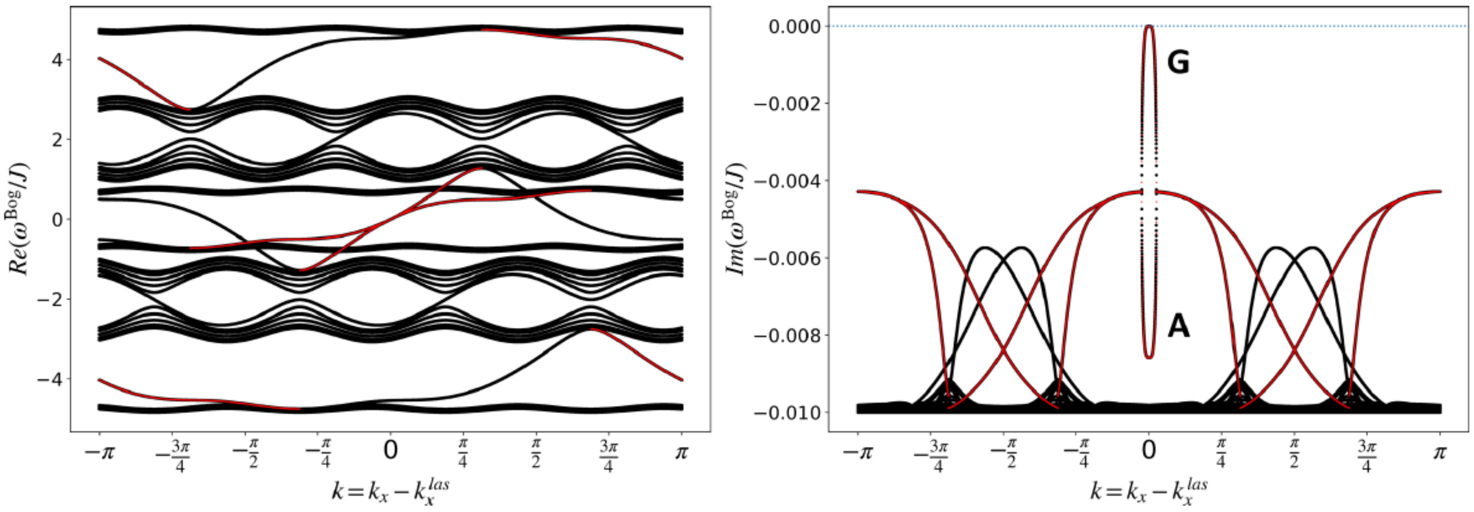
Goldstone mode & slow relaxation



Different times spaced by ~10 γ^{-1}

Ultra-slow relaxation of fluctuations

Simulation from Seclì et al., Phys. Rev. Research 2019



Collective excitations:

- Goldstone mode → (almost) flat real part, small imaginary part
- Higgs-like fast amplitude mode

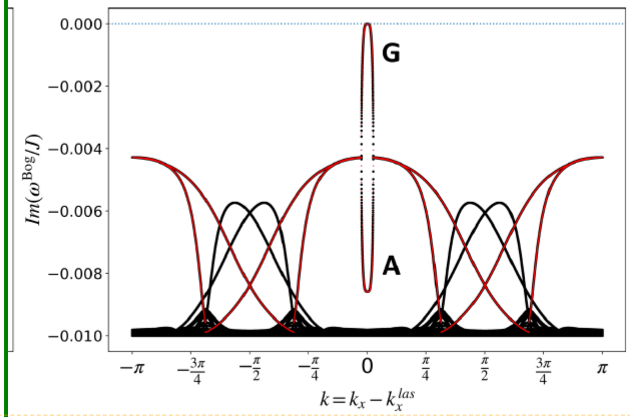
A. Loirette-Pelous' stage report @ ENS-Cachan, 2019
A. Loirette-Pelous et al., *Linearized theory of the fluctuation dynamics in 2D topological lasers*, PRA 2021
See also interesting related work by Zapletal et al., Optica 2020

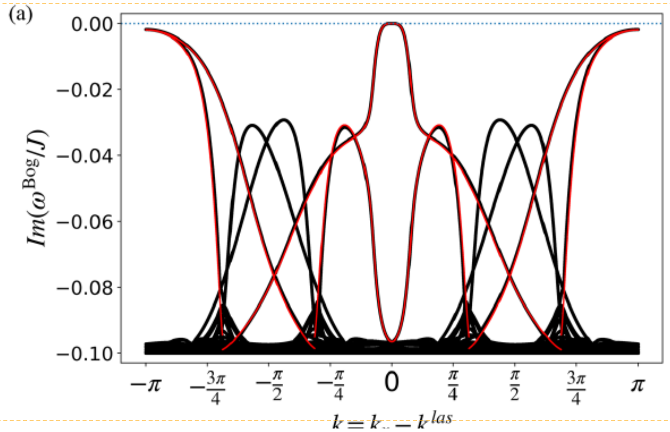
Fluctuation dynamics & topolaser stability

Class A topolaser $\gamma_R \gg \gamma$

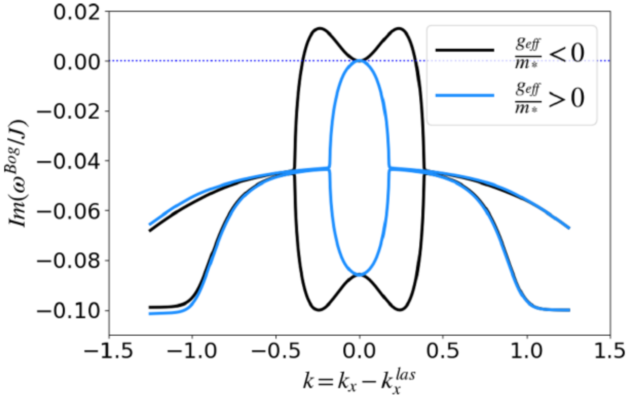
Class B topolaser $\gamma_R \leq \gamma$

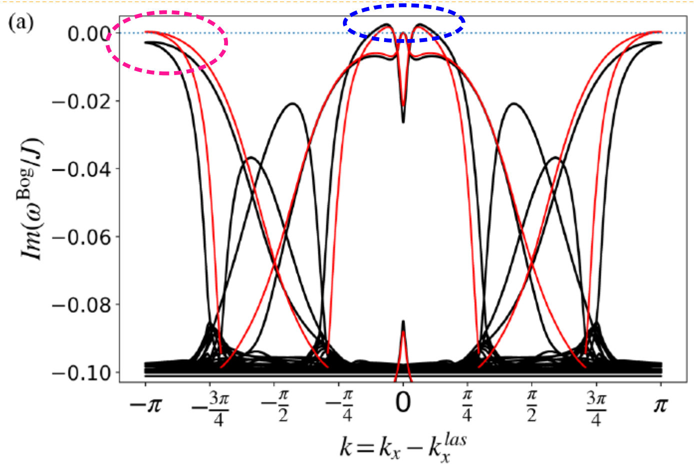
only gain saturation;
no intensity-dependent
refractive index
$g=g_R=0$





$\chi^{(3)}$ oonlinearity g
+
interactions with carriers g_R





Modulational instability
stable topolasing in one gap only

Interactions with reservoir
cf. polariton BECs: Baboux et al., Optica 2018
can be stabilized via edge localization

Counter-propagating edge mode
frequency-dependeny gain
single-gap topological model

A. Loirette-Pelous et al., *Linearized theory of the fluctuation dynamics in 2D topological lasers*, PRA 2021

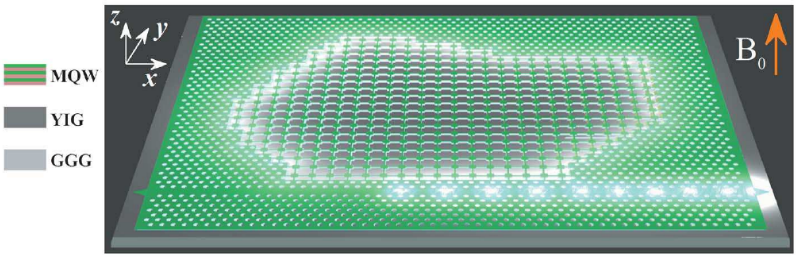
Coherence properties
of
topological lasers

What are the ultimate limits on coherence?

Coherence of topolaser emission (I)

Important fundamental & applied questions:

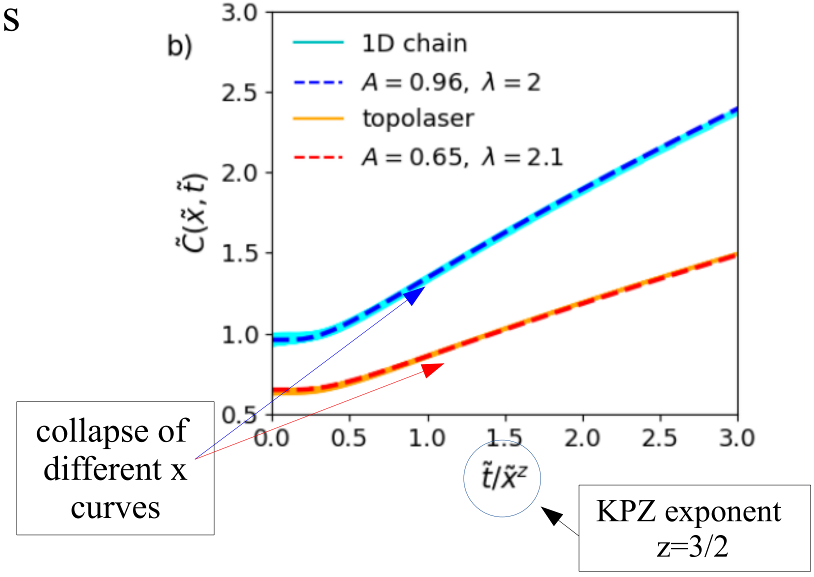
- What are ultimate limitations of coherence?
- How robust is coherence to disorder?
- What advantage over standard lasers?



Laser operation in spatially extended system:

- Include quantum fluctuations, e.g. as noise terms in dynamics
- Linearized theory not enough, crucial role of nonlinearities
- Kardar-Parisi-Zhang model of non-equilibrium stat mech (Altman/Diehl, Gladilin/Wouters, Canet/Minguzzi)
- spatio-temporal scaling properties of phase-coherence

$$\tilde{C}(\tilde{t},\tilde{x}^z) \equiv -2(\phi^*)^{-2}\tilde{x}^{-2\chi}\log g^{(1)}_{\rm CM}(\tilde{x},\tilde{t})$$



collapse of different x curves

KPZ exponent z=3/2

Topological laser:

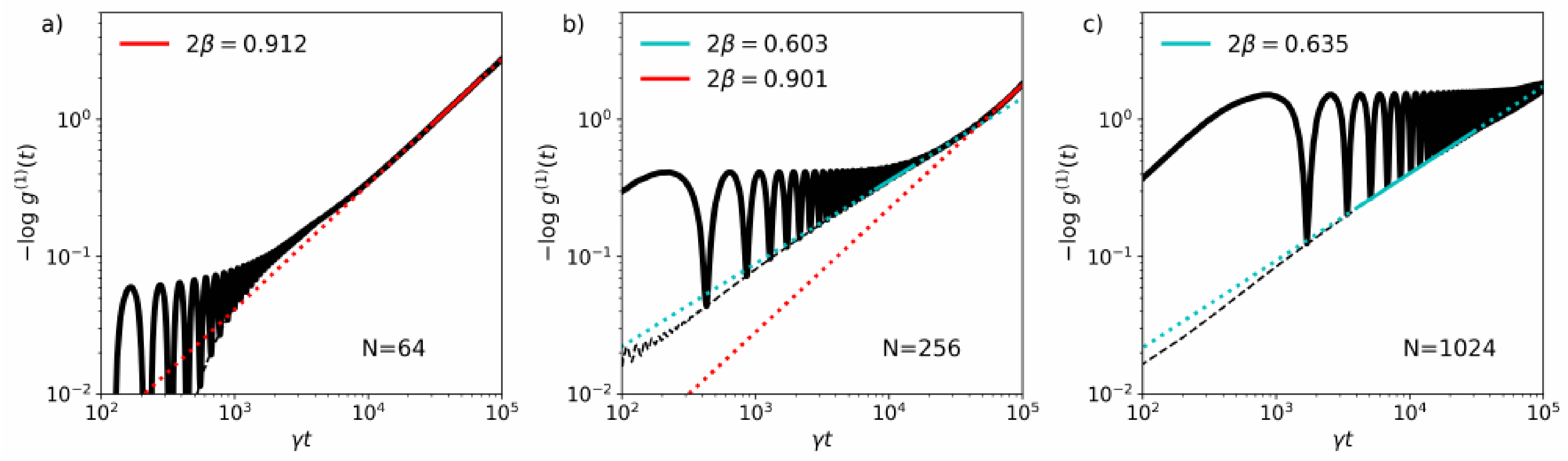
- One-dimensional edge state gives effective 1D dynamics
- KPZ spatio-temporal scaling of $g^{(1)}(x,t)$
- Periodic boundary conditions around device

Teaser: experimental evidence of KPZ using polariton quasi-condensates @ C2N

don't miss Quentin Fontaine's poster

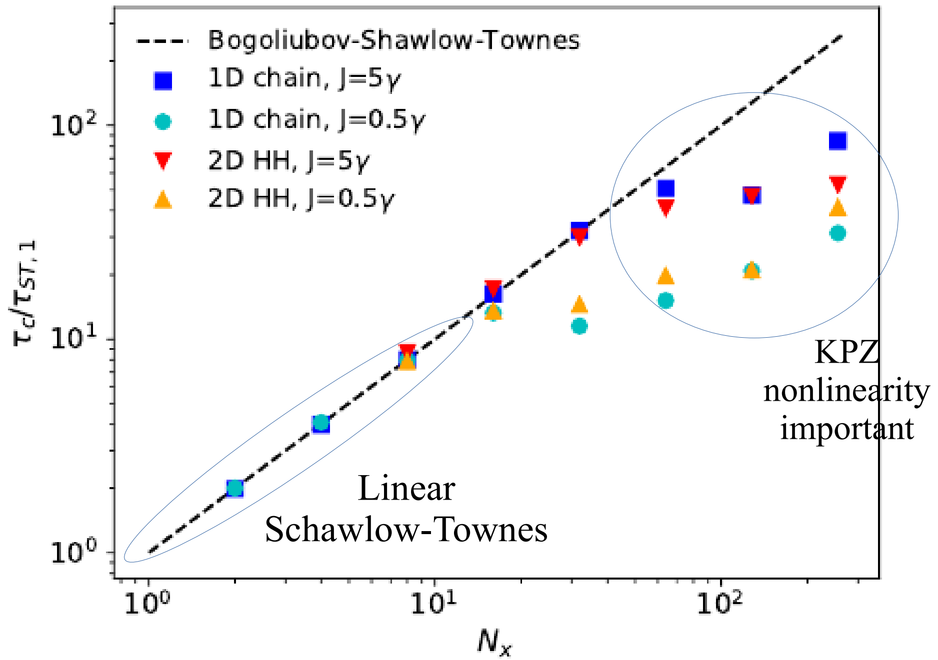
I. Amelio and IC, PRX 10, 041060 (2020)

Coherence of topolaser emission (II)



Coherence of laser emission:

- Physical system necessarily finite
  → crossover from stretched-exp to exp decay
  at long times for given size N_x
- crossover from Schawlow-Townes τ_c to KPZ
  at long times for increasing size N_x
- imposes fundamental limitation to τ_c
- physics similar to 1D chains, but...

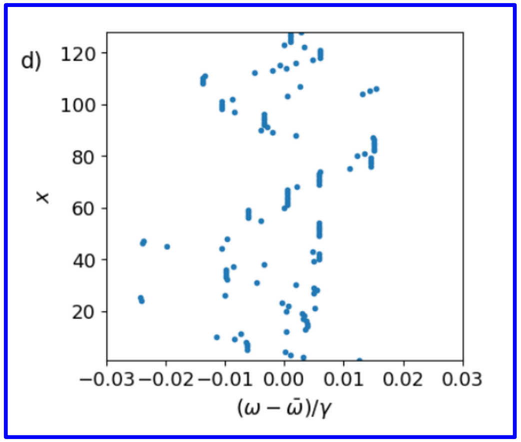


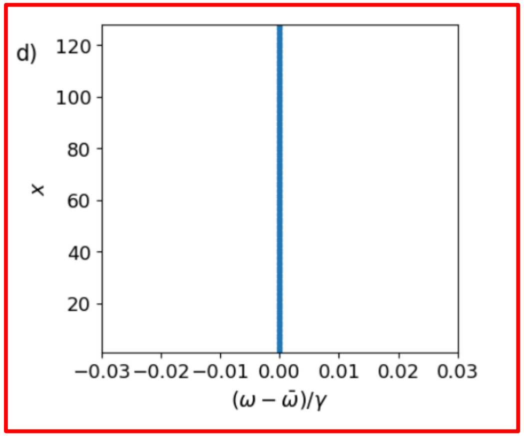
KPZ nonlinearity important

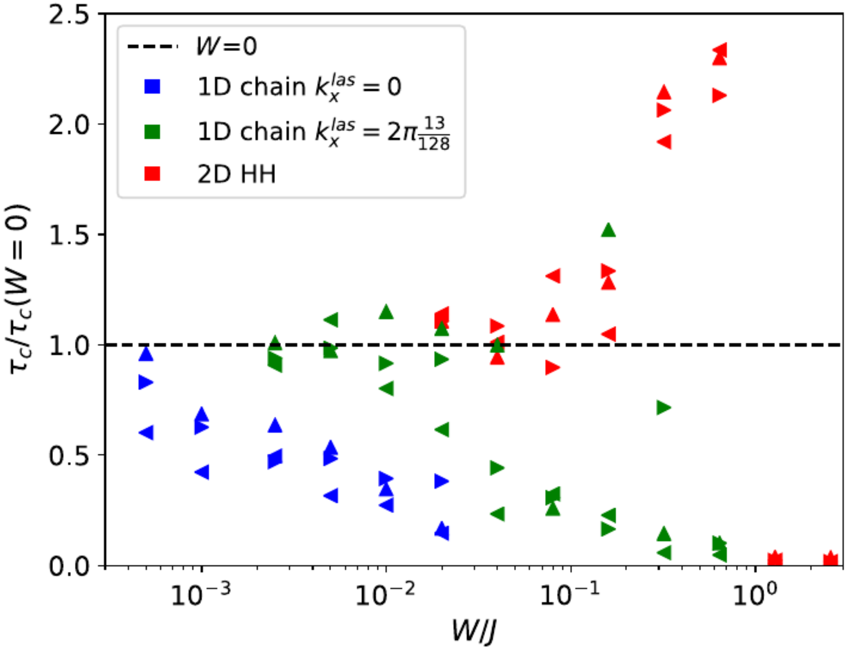
Linear Schawlow-Townes

I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 10, 041060 (2020)

Coherence of topolaser emission (III)







<u>In the presence of static disorder:</u>

- Non-Topological: weak disorder suppresses temporal coherence (mode fragmentation, multimode emission, localization, etc.)
- Topological: robust spatio-temporal coherence, chiral propagation travels through/around defects without backscattering.

<u>Technologically important in (semiconductor) laser technology:</u>

Allows to phase lock many individual lasers → strong intensity and high coherence

<u>Next steps:</u> extend theory to Class-B lasers. Control instabilities and maintain single-mode emission

<u>Fundamental question</u> → effect of convective/absolute instability on coherence properties

I. Amelio and IC, *Theory of the coherence of topological lasers*, PRX 10, 041060 (2020)

Conservative dynamics in circuit-QED experiment:
interplay of strong interactions & synthetic magnetic field

Ring-shaped array of qubits in a superconductor-based circuit-QED platform

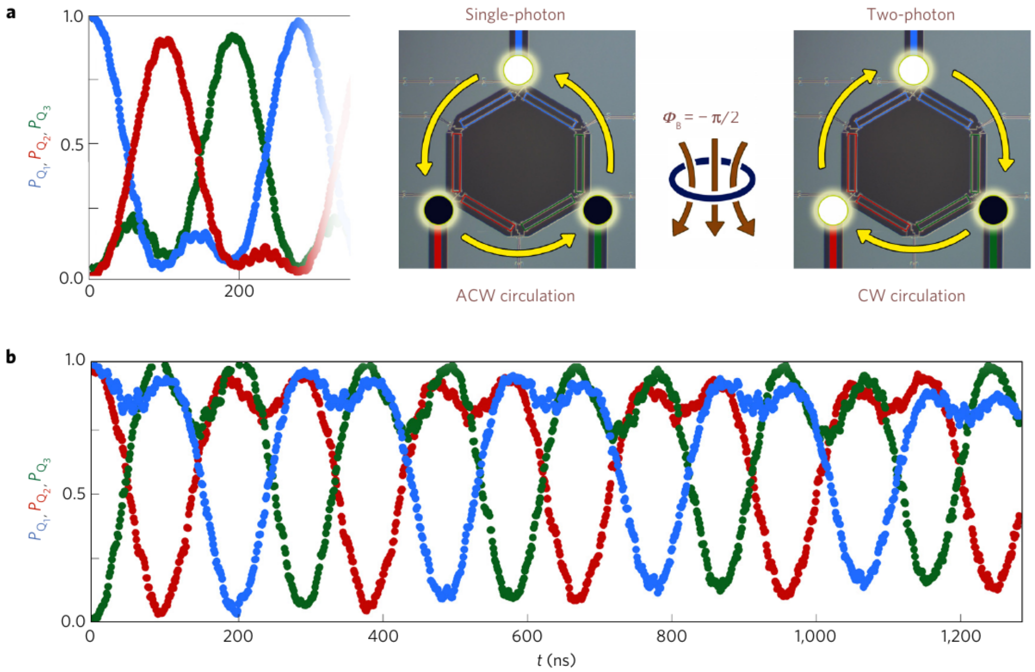
- Transmon qubit: two-level system
  → Impenetrable microwave photons

- Time-modulation of couplings
  → synthetic gauge field

- Independently initialize sites

- Follow unitary evolution until bosons lost
  (microwave photons → long lifetime)

- Monitor site occupation in time



Roushan et al., Nat. Phys. 2016

“Many”-body effect:

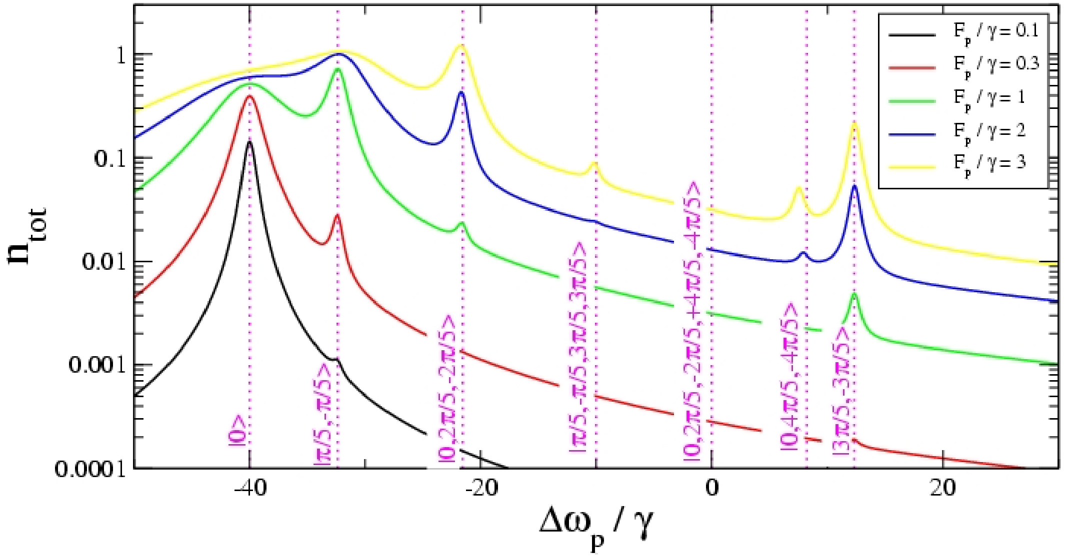
two-photon state → opposite rotation compared to one-photon state

(similar to cold-atom experiment in Greiner’s lab: Tai et al., Nature 2017)

Impenetrable "fermionized" photons in 1D necklaces

Many-body eigenstates of
Tonks-Girardeau gas
of impenetrable photons

Coherent pump
selectively addresses
specific many-body states



Transmission spectrum as a function pump frequency for fixed pump intensity:

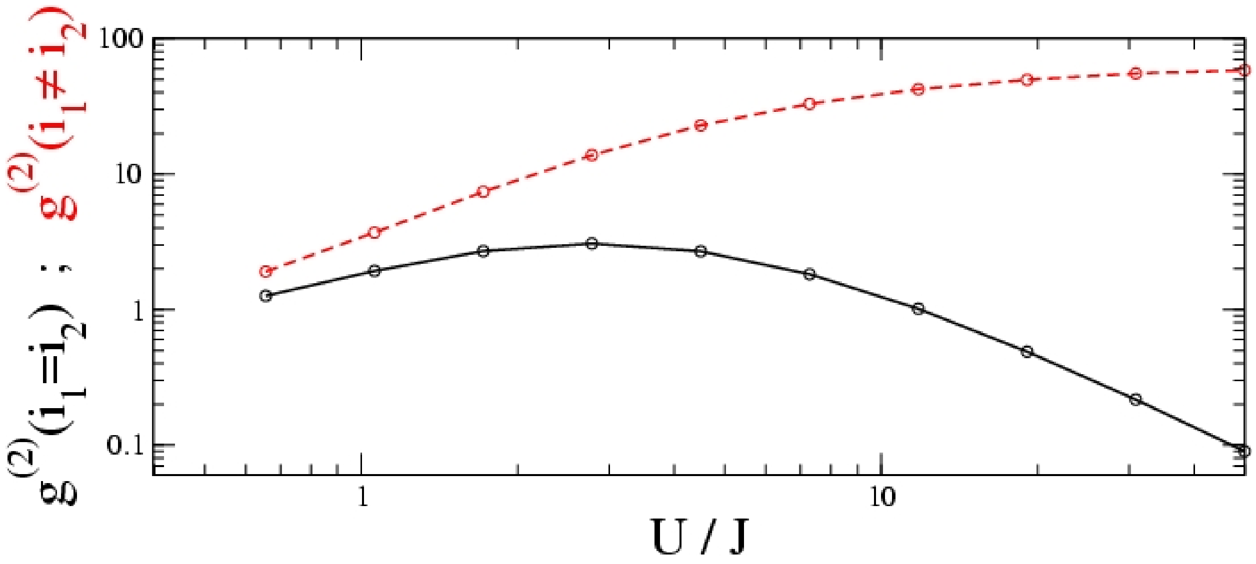
- each peak corresponds to a Tonks-Girardeau many-body state $|q_1, q_2, q_3 \ldots>$
- q_i quantized according to PBC/anti-PBC depending on N=odd/even
- U/J >> 1: efficient photon blockade, impenetrable photons.

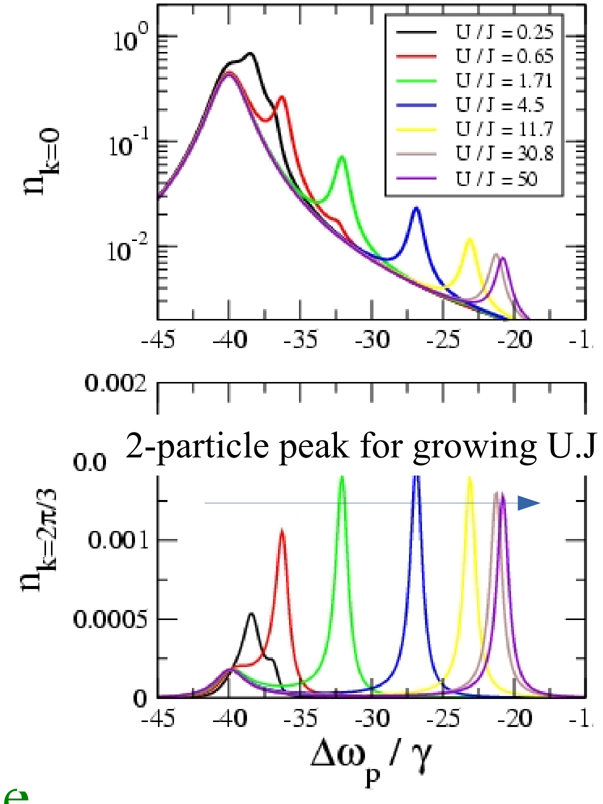
N-particle state excited by N photon transition:

- Plane wave pump with $k_p=0$: selects states of total momentum P=0
- Monochromatic pump at ω_p: resonantly excites states of many-body energy E such that $\omega_p = E / N$

IC, D. Gerace, H. E. Türeci, S. De Liberato, C. Ciuti, A. Imamoglu, PRL **103**, 033601 (2009)
See also related work D. E. Chang et al, Nature Physics (2008)

State tomography from emission statistics



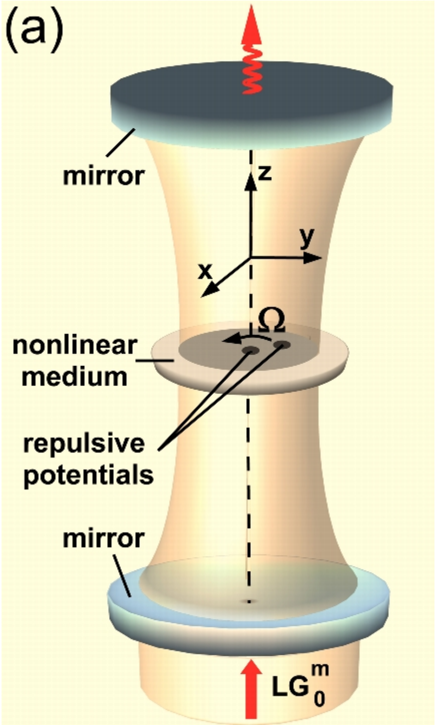


Finite U/J, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ, larger probability of having N=0 or 2 photons than N=1
  - low U<<J: bunched emission for all pairs of i_1, i_2
  - large U>>J: antibunched emission from a single site
    positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

Continuous space FQH physics

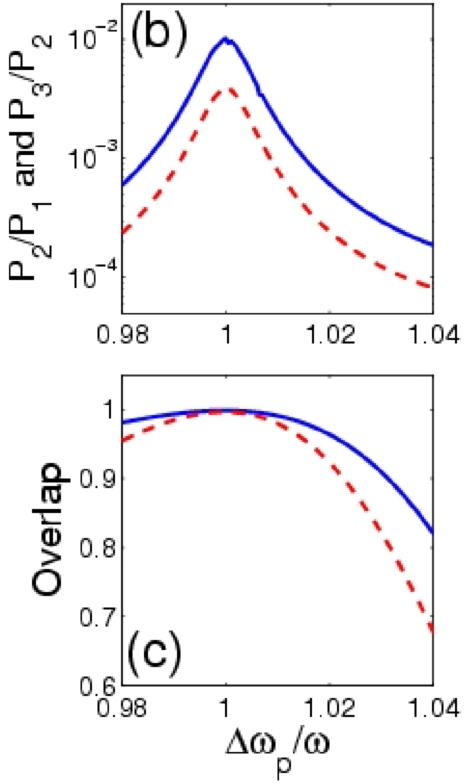
Single cylindrical cavity. No need for cavity array



same form

Coriolis $F_c = -2m\Omega \times v$

Lorentz $F_L = e\, v \times B$



Photon gas injected by Laguerre-Gauss pump

with finite orbital angular momentum

Strong repuls. interact., e.g. layer of Rydberg atoms

Resonant peak in transmission due to Laughlin state:

$$\psi(z_{1,}\dots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070