Strongly interacting photons in strongly nonlinear cavities

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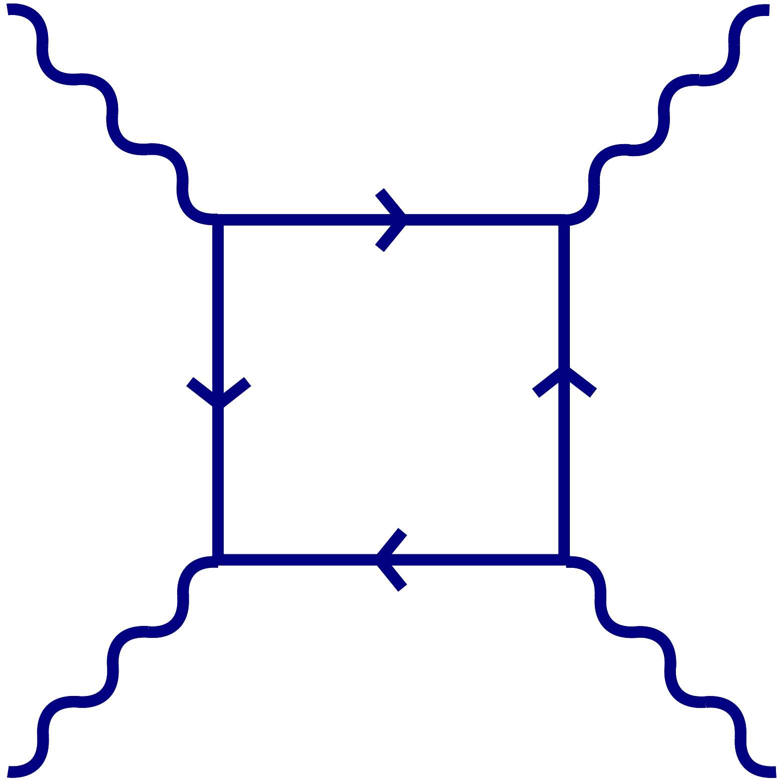
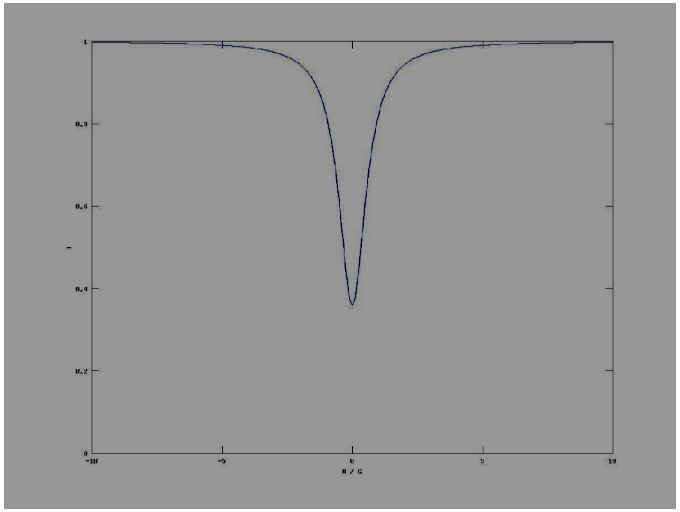
Photon-photon scattering



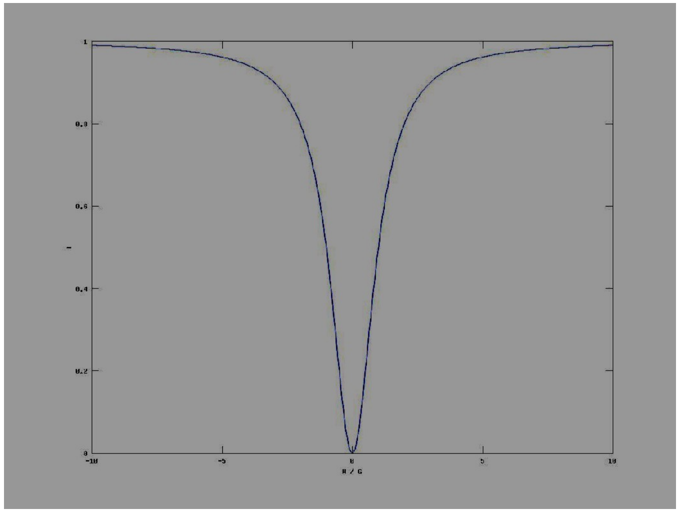
$$\sigma \sim \alpha^4 \frac{\hbar^2}{m^2 c^2} \left(\frac{\hbar \omega}{mc^2} \right)^6$$

Transmission spectra

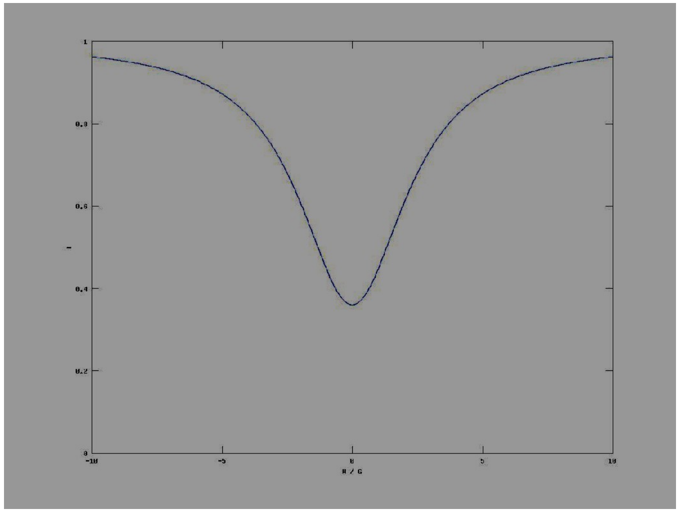


← Undercoupling regime

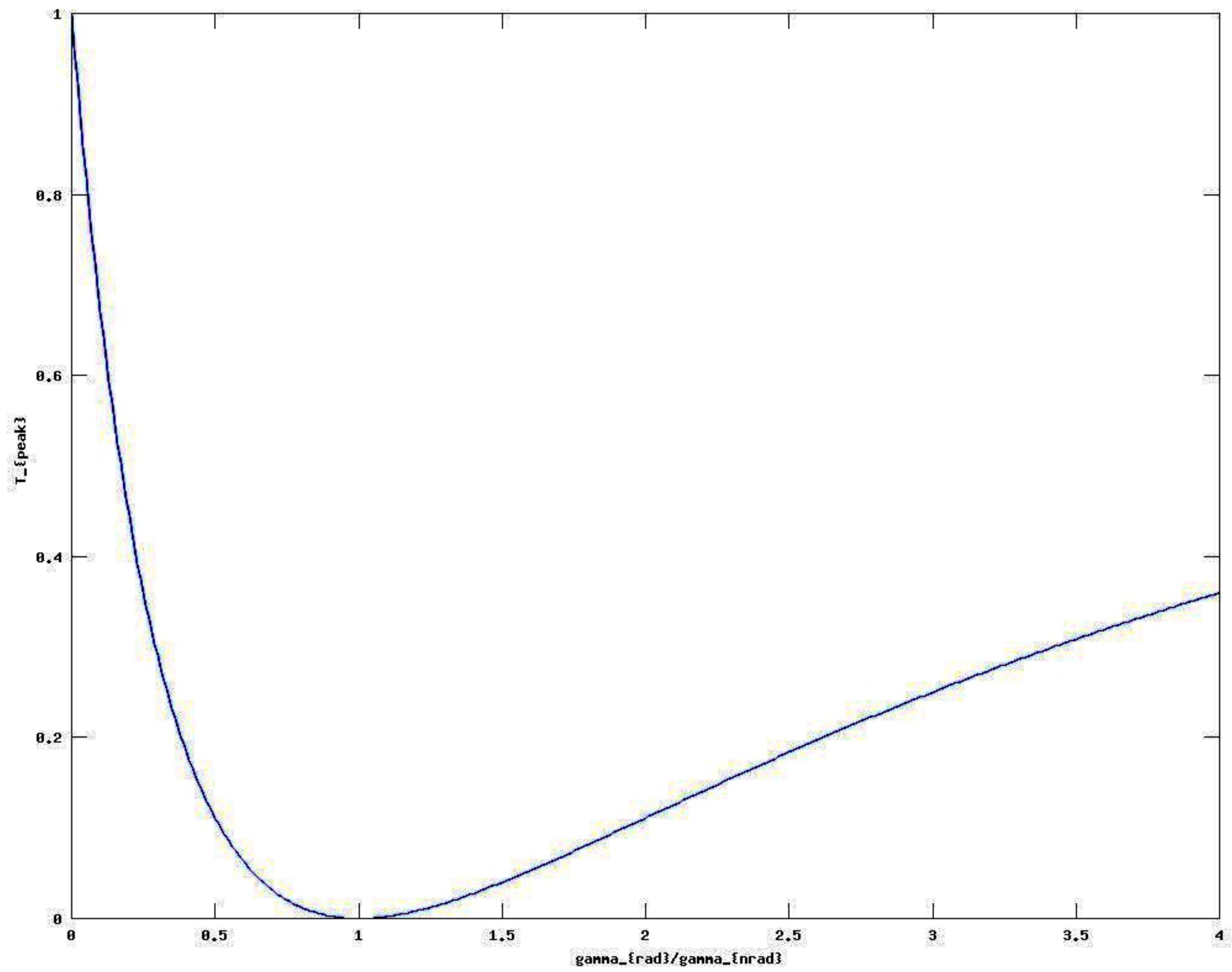
Critical coupling →



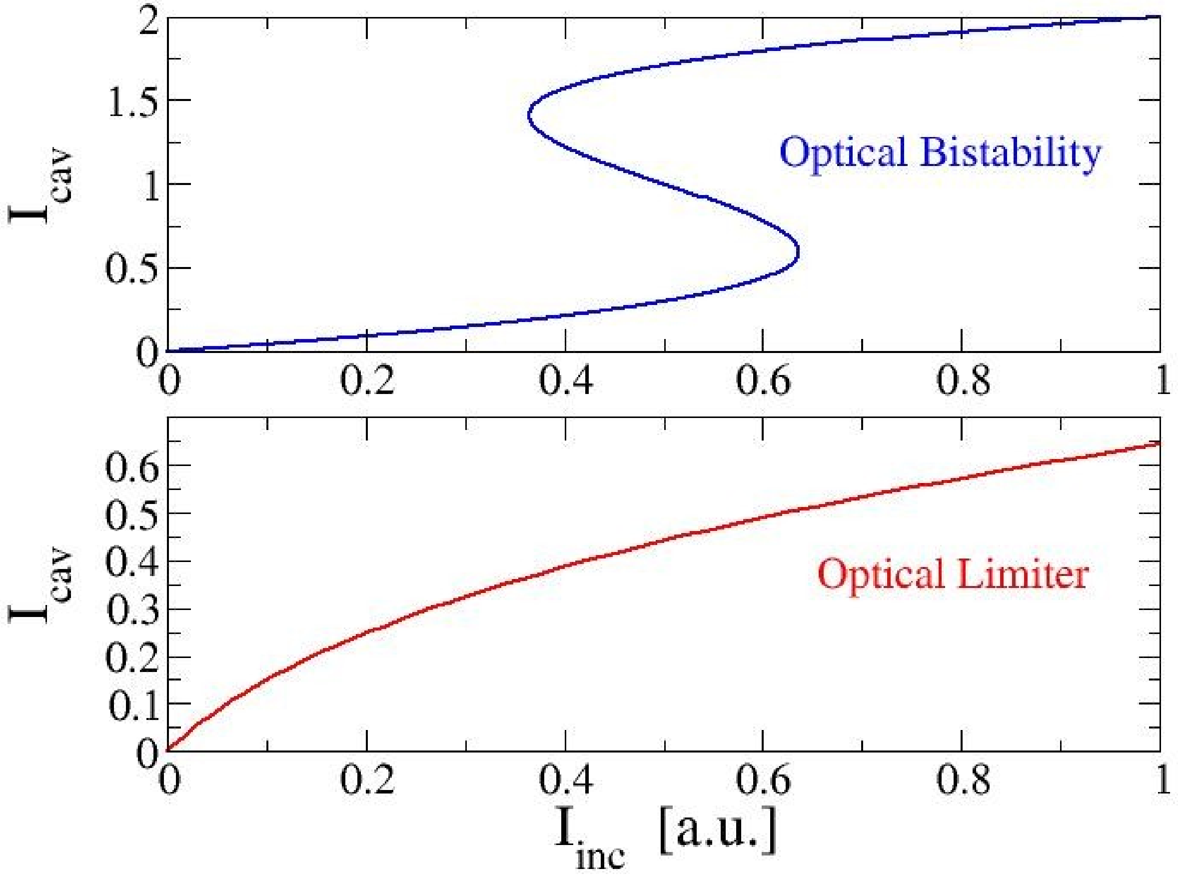
Overcoupling regime →



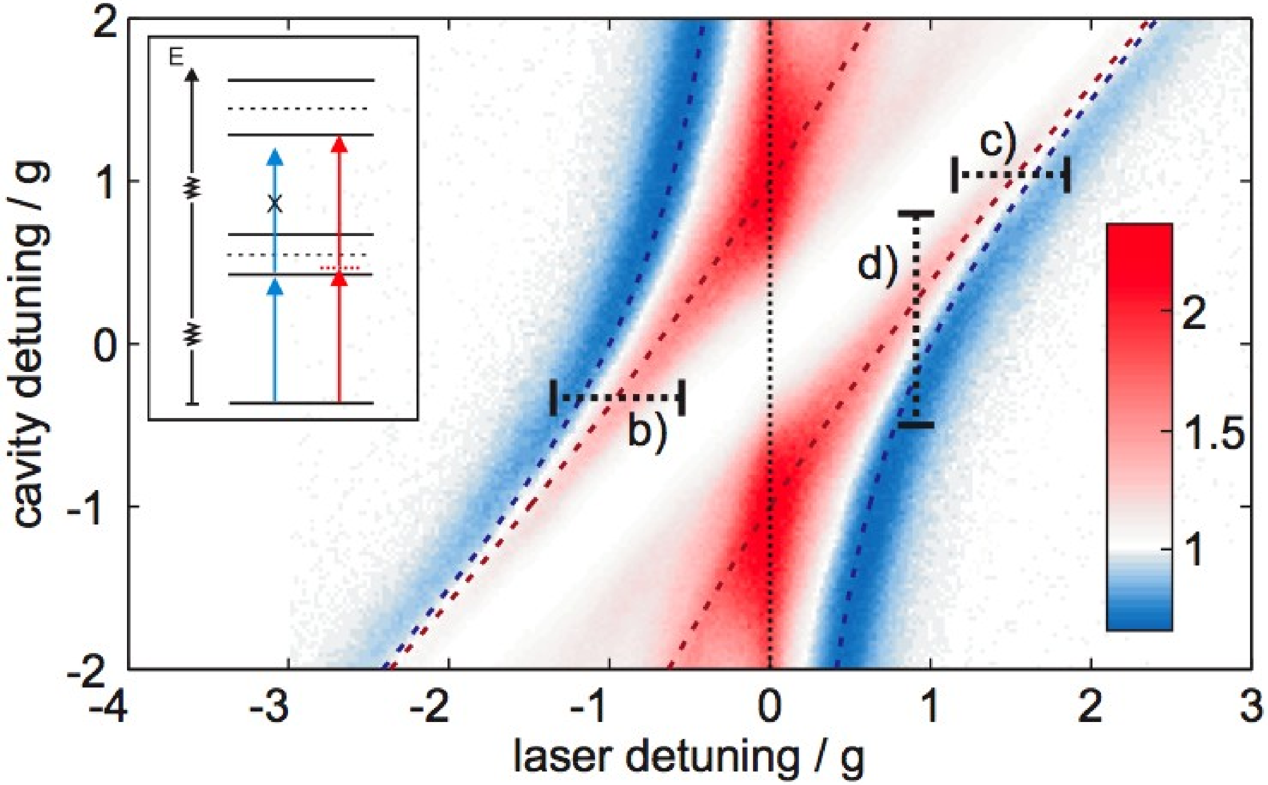
Under vs. over coupling



Optical bistability and limiting

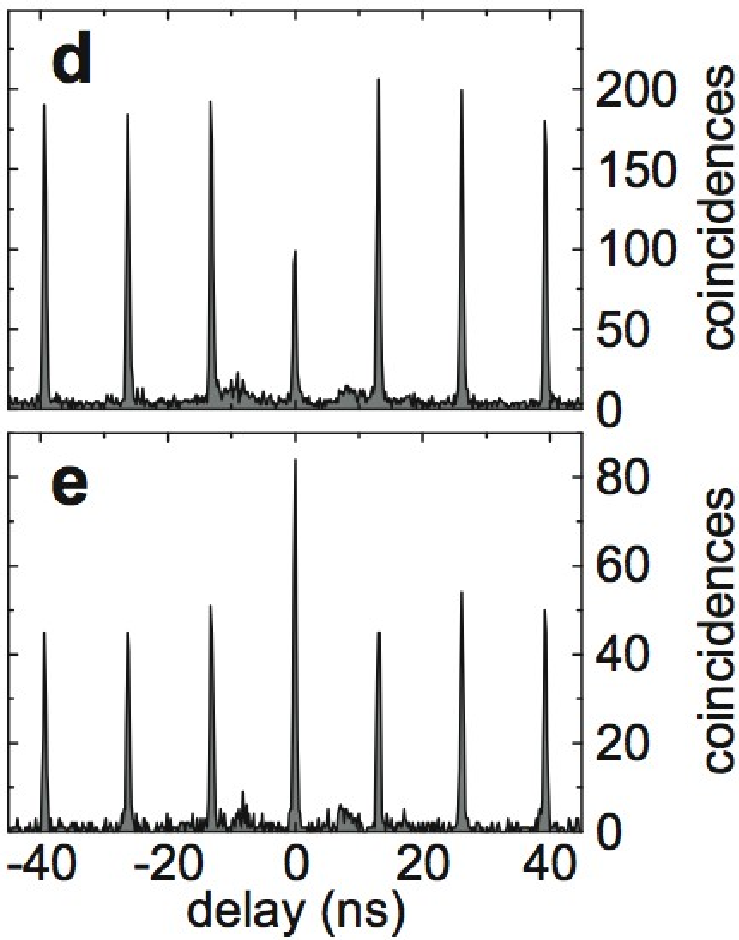


Bunching vs. Antibunching



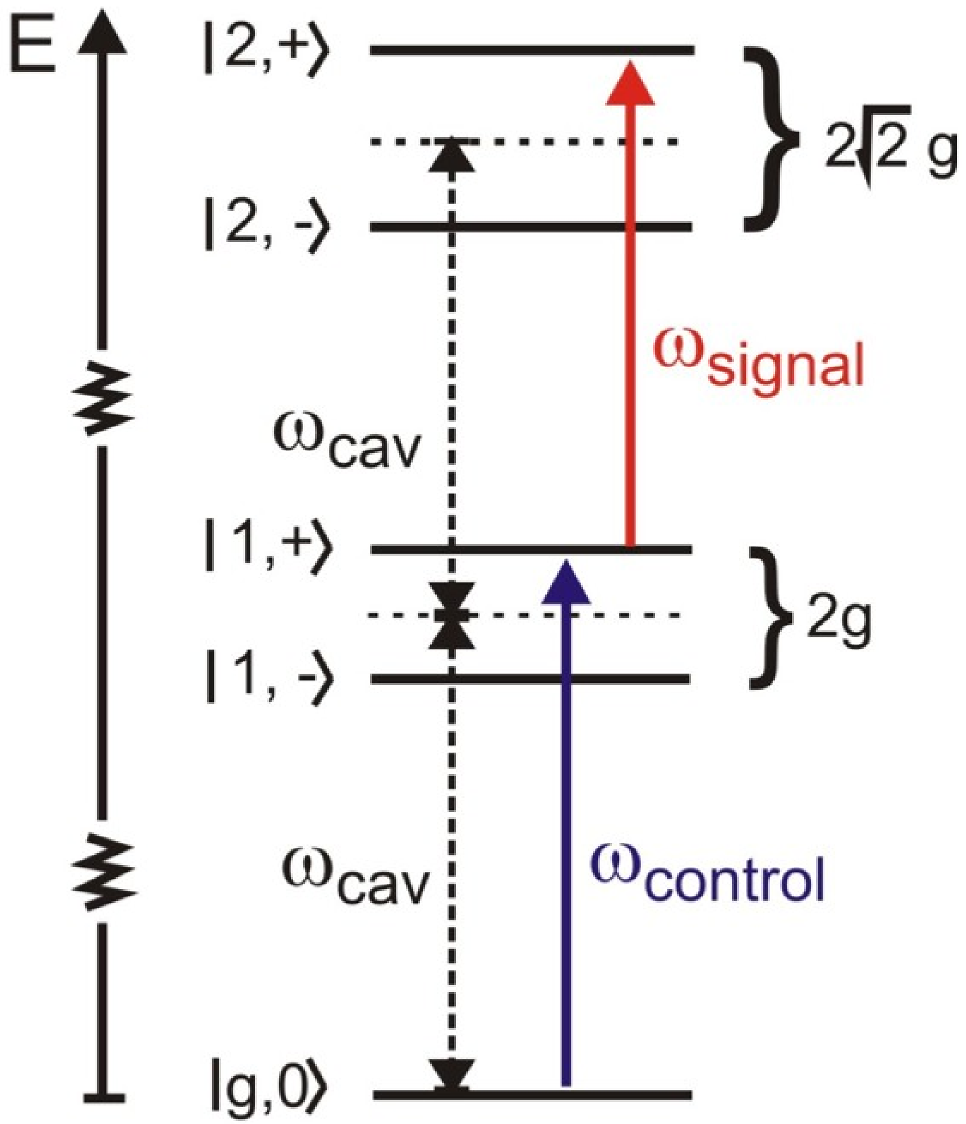
From: T. Volz et al., *Ultrafast all-optical switching by single photons,* Nature Photonics 6, 605–609 (2012)

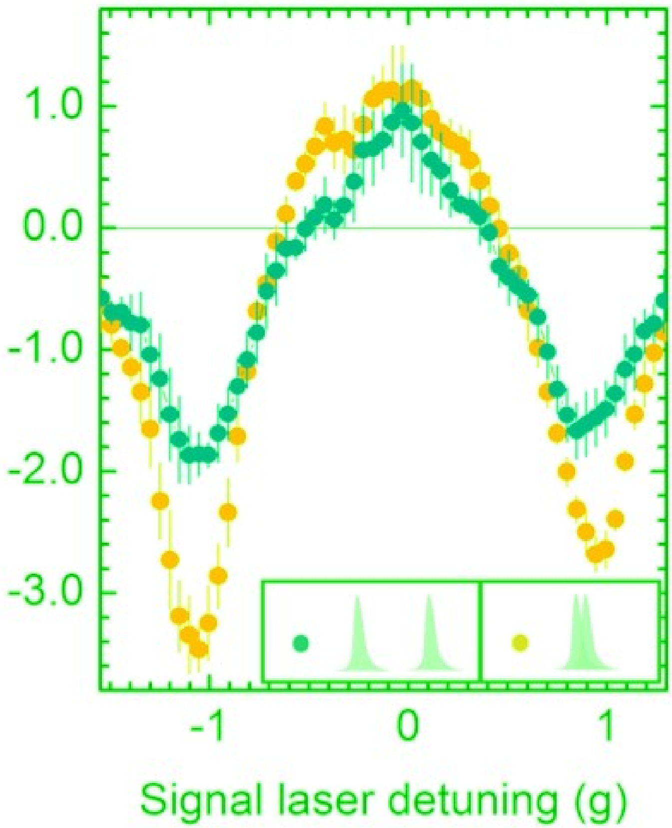
Correlation measurement



From: T. Volz et al., *Ultrafast all-optical switching by single photons,* Nature Photonics 6, 605–609 (2012)

Ultrafast pump & probe





From: A. Reinhard et al., *Strongly correlated photons on a chip,* Nature Photonics 6, 93-96 (2012)

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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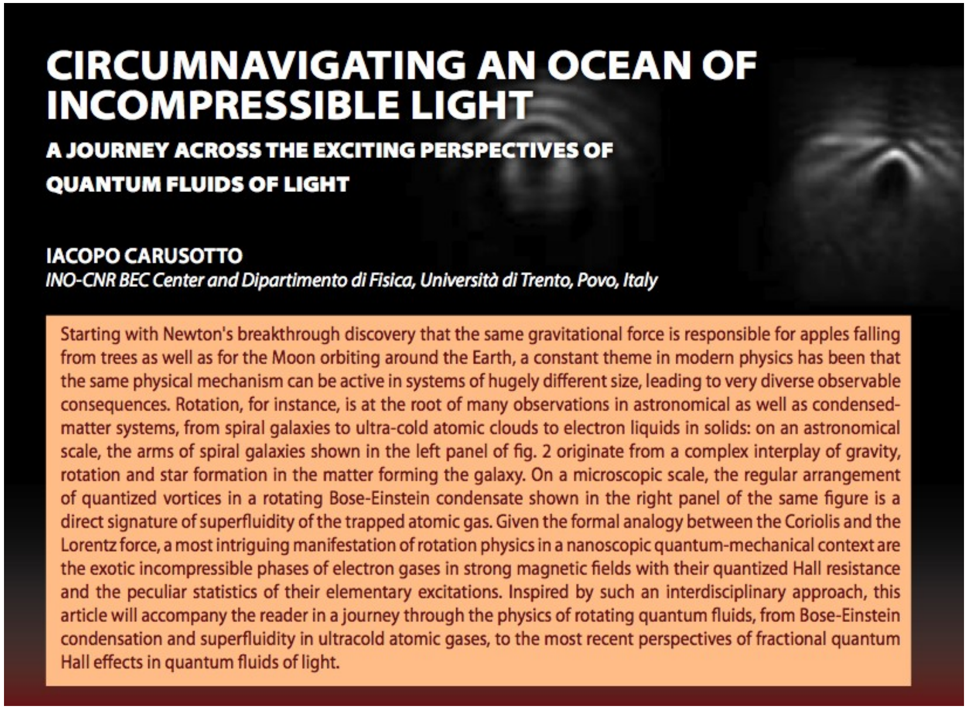

CIRCUMNAVIGATING AN OCEAN OF INCOMPRESSIBLE LIGHT

A JOURNEY ACROSS THE EXCITING PERSPECTIVES OF QUANTUM FLUIDS OF LIGHT

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Starting with Newton's breakthrough discovery that the same gravitational force is responsible for apples falling from trees as well as for the Moon orbiting around the Earth, a constant theme in modern physics has been that the same physical mechanism can be active in systems of hugely different size, leading to very diverse observable consequences. Rotation, for instance, is at the root of many observations in astronomical as well as condensed-matter systems, from spiral galaxies to ultra-cold atomic clouds to electron liquids in solids: on an astronomical scale, the arms of spiral galaxies shown in the left panel of fig. 2 originate from a complex interplay of gravity, rotation and star formation in the matter forming the galaxy. On a microscopic scale, the regular arrangement of quantized vortices in a rotating Bose-Einstein condensate shown in the right panel of the same figure is a direct signature of superfluidity of the trapped atomic gas. Given the formal analogy between the Coriolis and the Lorentz force, a most intriguing manifestation of rotation physics in a nanoscopic quantum-mechanical context are the exotic incompressible phases of electron gases in strong magnetic fields with their quantized Hall resistance and the peculiar statistics of their elementary excitations. Inspired by such an interdisciplinary approach, this article will accompany the reader in a journey through the physics of rotating quantum fluids, from Bose-Einstein condensation and superfluidity in ultracold atomic gases, to the most recent perspectives of fractional quantum Hall effects in quantum fluids of light.

Strongly interacting photons in strongly nonlinear resonators

In vacuo : do photons interact ?

* classical Maxwell's eqs are linear in the $\vec{E}, \vec{B}$ fields

⇒ response does not vary with intensity

$$\vec{I}_{out} = \mathbb{M}\, \vec{I}_{in} \quad , \quad \vec{E}_{out} = \mathbb{Q}\, \vec{E}_{in}$$

no transistor effect, no SHG, ...

* upon quantization → virtual e^-/e^+ pairs excited

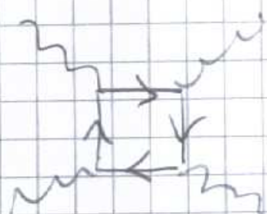


diagram contributes to effective $\chi^{(3)}$ of vacuum

practically irrelevant

$$\sigma \simeq \alpha^4 \left(\frac{\hbar}{mc}\right)^2 \left(\frac{\hbar\omega}{mc^2}\right)^6$$

$\alpha \to 1/137$

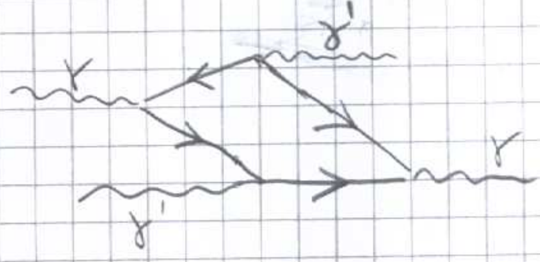
$\frac{\hbar}{mc} \to 0{,}4\,\mathrm{pm}$

$mc^2 \simeq 0.5\,\mathrm{MeV}$

How to make σ larger and observable?

What if mc^2 (energy of e^+/e^- pair) is replaced by $\hbar\omega_{gap}$ of semiconductor, i.e. $e^+/e^- \to e/h$ pair ?

Physical interpretation



→ photon γ creates e/h pair that modifies propagation of γ' photon

$\Longrightarrow$ $\chi^{(3)}$ non linear effect

Typical theory of non linear optics:

$$P = \underbrace{\chi^{(1)} E}_{\text{standard refractive index}} + \chi^{(2)} E^2 + \underbrace{\chi^{(3)} E^3}_{\text{e.g. intensity dependent refractive index}} + \dots$$

$$n = n_0 + n_{nl} |E|^2$$

$\chi^{(2)}$ effects → different family

* second harmonic generation
* parametric down conversion

etc.

Ring-shaped cavity (e.g. whispering gallery, microtoroid ...)

modes characterized by

$$\underbrace{k(\omega_N)}_{\text{dispersion of light along ring}} \cdot 2\pi R = 2\pi N$$

example: $k(\omega) = n \frac{\omega}{c}$ (refractive index n)

$\Rightarrow \omega_N = \frac{c}{Rn} N$ uniformly spaced by $\frac{c}{Rn}$

if $R = 10\,\mu m \rightarrow \Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{c}{2\pi R n} \simeq 15\,THz$.

$\Delta\lambda \simeq 30\,nm$

Variation of $n \rightarrow n + \delta n$: all modes shifted

$$\delta\omega \simeq -\omega \frac{\delta n}{n}, \quad \delta\lambda \simeq \lambda \frac{\delta n}{n}$$

Cavity Hamiltonian $\hat{H} = \sum_N \hbar\omega_N \hat{a}_N^\dagger \hat{a}_N$

Classically $E(r,t) = \sum_N \mathcal{E}_N(r)\, \alpha_N(t)$ with

free evolution $\dot{\alpha}_N = -i\omega_N \alpha_N$

$\alpha_N(t) = \alpha_N(0)\, e^{-i\omega_N t}$

Effect of nonlinear refractive index:

$$\omega_N \longrightarrow \omega_N \left(1 - \frac{\delta n}{n}\right) \simeq \omega_N - \omega_N \times \chi^{(3)} E^2$$

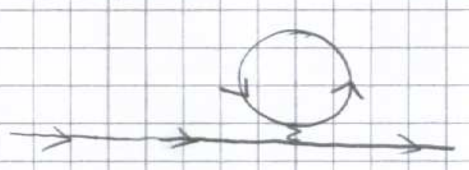
$$= \omega_N + \omega_{nl}|\alpha_N|^2 \quad \text{in single mode approximation.}$$

$$\dot{\alpha}_N = -i\left(\omega_N + \omega_{nl}|\alpha_N|^2\right)\alpha_N$$

i.e. $H = \hbar\omega_N a_N^\dagger a_N + \frac{\hbar}{2}\omega_{nl}\hat{a}_N^\dagger \hat{a}_N^\dagger \hat{a}_N \hat{a}_N$

↳ verify through commutators ...

Cavity coupled to waveguide:



* assume point-like coupling
* waveguide modes labelled by q

$$H = \underbrace{H_0}_{\text{isolated cavity}} + \underbrace{\int_q}_{\text{integral over } q \text{ modes}} \hbar g_q \left[a_N^\dagger b_q + b_q^\dagger a_N\right] + \int_q \hbar\omega_q \underbrace{b_q^\dagger b_q}_{\text{free wave propagation}}$$

q-mode in waveguide

q → indicates e.g. longitudinal momentum.

$$i\dot{a}_N = \omega_N a_N + g_q b_q$$

$$i\dot{b}_q = \omega_q b_q + g_q a_N$$

at $t \to -\infty$ $\hat{b}_q \simeq \beta_q$ coherent field, $\hat{a}_N \simeq 0$ vacuum

$\Rightarrow$ at all times $\hat{a}_N$ coherent $\rightarrow \alpha_N$

$$i\dot{\alpha}_N = \omega_N a_N + g_q \beta_q + g_q \delta\beta_q$$

$$i\delta\dot{\beta}_q = \omega_q \delta\beta_q + g_q \alpha_N$$

→ incident field

↳ field generated in wg by cavity

Solving system of eqs and summing over all q modes

$$i\dot{\alpha}_N = \omega_N a_N + g_q \beta_q - i\frac{\gamma}{2} a_N$$

example: monochromatic $\beta_q = \beta_q^0 e^{-i\omega_L t}$

$$\alpha_N = \frac{g_q}{\omega_L - \omega_N + i\gamma/2} \beta_q$$

* linear response in incident field
* strongly resonant @ ω_N
* linewidth $\gamma \propto g_q^2$ depends on cavity - waveguide coupling

Adding extra loss channels (i.e. absorption, etc.)

$$\alpha_N = \frac{g_q}{\omega_L - \omega_N + \frac{i}{2}(\gamma + \gamma_{nrad})} \beta_q$$

Transmitted field in waveguide: incident + reemitted field by cavity

$$\beta_q^{tr} = \beta_q - \frac{i\gamma}{\omega_L - \omega_N + \frac{i}{2}(\gamma+\gamma_{rad})}\,\beta_q$$

calculation of this factor requires a lot of care

$$= \beta_q \underbrace{\left[1 - \frac{i\gamma}{\omega_L-\omega_N+\frac{i}{2}(\gamma+\gamma_{rad})}\right]}_{t(\omega)}$$

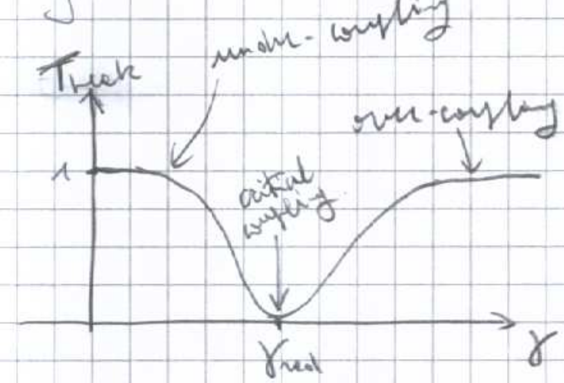
$t(\omega) \longrightarrow$ transmission amplitude

$T(\omega) = |t(\omega)|^2$ transmittivity

At resonance: $\omega_L = \omega_N$

$$\beta_q^{tr} = \beta_q\left[1 - \frac{i\gamma}{i\,\frac{\gamma+\gamma_{rad}}{2}}\right]$$

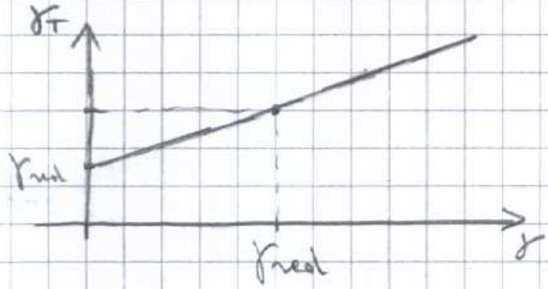
$$= \beta_q\left[\frac{\gamma_{rad}-\gamma}{\gamma_{rad}+\gamma}\right]$$

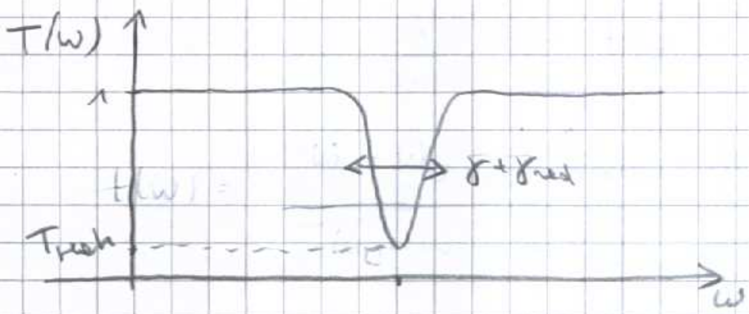


* T_{peak} minimum @ $\gamma=\gamma_{rad}$

* Linewidth $\gamma_T = \gamma+\gamma_{rad}$

  monotonically growing





What happens in the presence of nonlinearity?

$$\omega_N \longrightarrow \omega_N + \omega_{nl} |\alpha_N|^2$$

$$i \dot{\alpha}_N = \omega_N \alpha_N + \omega_{nl} |\alpha_N|^2 \alpha_N + g_q \beta_q - i \frac{\gamma}{2} \alpha_N$$

for monochromatic pump $\beta_q(t) = \beta_q^0 e^{-i\omega_L t}$

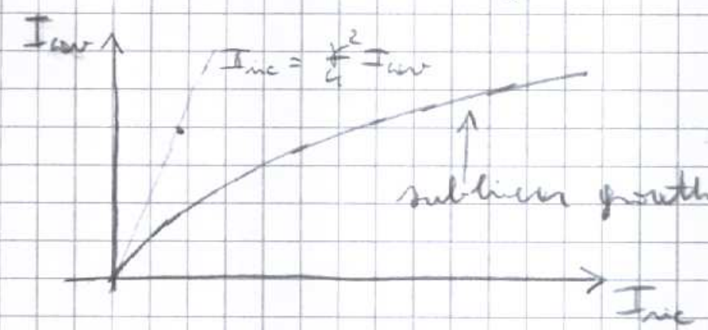
$$\Big[\omega_L - \underbrace{\omega_N - \omega_{nl} |\alpha_N|^2}_{\text{shifted resonance}} + i \frac{\gamma}{2}\Big] \alpha_N = g_q \beta_q$$

$\hookrightarrow$ non linear algebraic equation

$$\underbrace{g_q^2 |\beta_q|^2}_{I_{inc}} = \underbrace{|\alpha_N|^2}_{I_{cav}} \Big[(\omega_L - \omega_N - \omega_{nl} |\alpha_N|^2)^2 + \frac{\gamma^2}{4} \Big]$$

Better solve $I_{inc}(I_{cav})$ [case $\omega_{nl} > 0$]

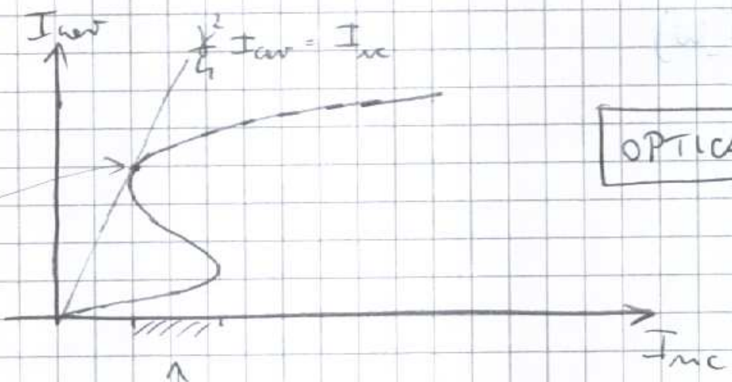
1) $\underline{\omega_L < \omega_N}$



sublinear growth of I_{cav} with I_{inc}

$\boxed{\text{OPTICAL LIMITER}}$

2. $\omega_L > \omega_N$



shifted resonance condition
$\omega_L = \omega_N + \omega_{nl}|\alpha_N|^2$

$\boxed{\text{OPTICAL BISTABILITY}}$

↑
3x solutions for a given I_{inc}

Actual I_{cav} depends on history → hysteresis

All these results rely on classical approx $\hat{a}_N \to \alpha_N \in \mathbb{C}$

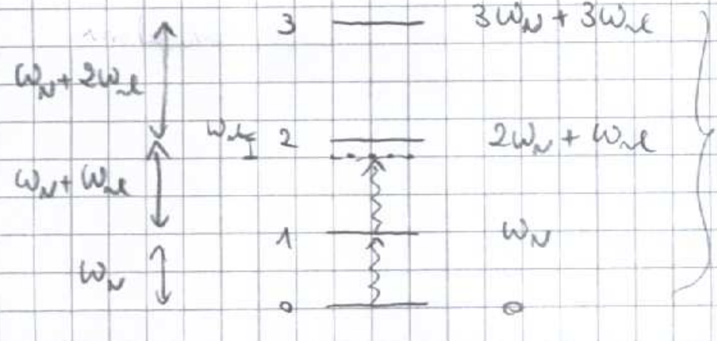
↳ accurate as long as $\gamma \gg \omega_{nl}$.

[IC, Phys. Rev A 63, 023610 (2000)]

General case requires full quantum mechanical treatment of the so-called "quantum fluctuations".

Description easy again if $\gamma \ll \omega_{nl}$:

isolated system $H = \hbar\omega_N a_N^\dagger a_N + \frac{\hbar\omega_{nl}}{2} a_N^\dagger a_N^\dagger a_N a_N$



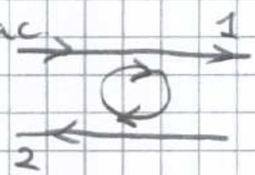
Anharmonic level scheme

If $\omega_L \approx \omega_r$ and $\omega_r \gg \gamma$, only $|0\rangle$ and $|1\rangle$ are involved in dynamics as $m \geq 2$-photon transitions are non-resonant because of anharmonicity

↳ Effective two-level system

Physics is simplest for transmission in another waveguide

$\hat{b}_{q'} = g_{q'} \, \hat{a}_N$



As cavity contains at most 1 photon, photons scattered into port 2 are separated in time

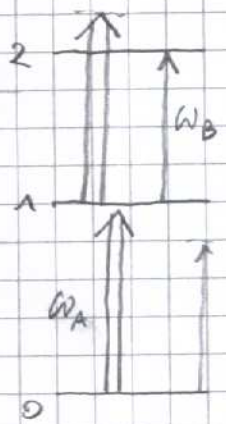
↳ same description as resonant fluorescence by two-level atom.

* Transition saturated by single photon ⟶ nonlinear optics at low-light intensity

* Mollow triplet of transmitted light

* Strong antibunching ⟶ single photon source

Two-color spectroscopy (Volz, Reinhard et al (Imamoglu group)



laser @ ω_A $\longrightarrow$ drives $0 \rightarrow 1$

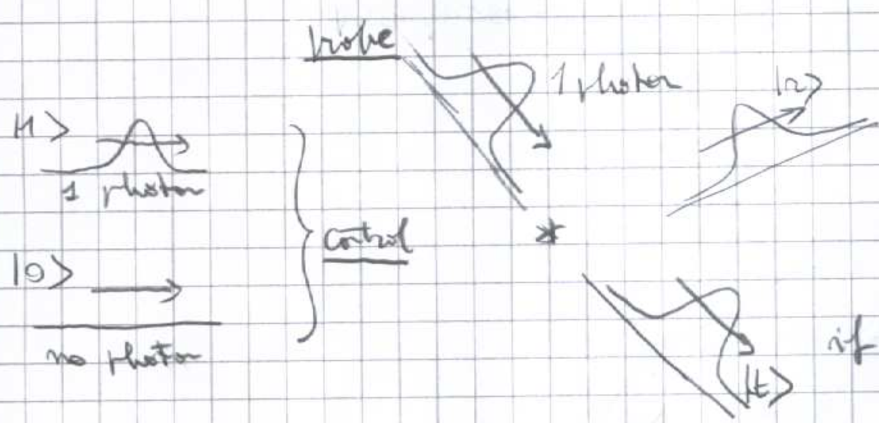
laser @ ω_B $\longrightarrow$ drives $1 \rightarrow 2$

all other combinations far off resonant.

Absorption of ω_B light conditioned upon driving of $0 \rightarrow 1$ transition by ω_A laser

↳ strong 2-photon absorption, no 1-photon absorption

First step towards "single photon quantum transistor":



if control $= |1\rangle$ reflected

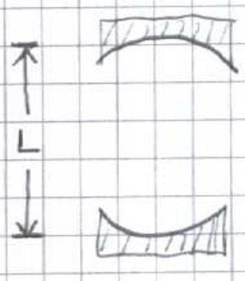
if control $= |0\rangle$ transmitted

Example of quantum gate:

$$|i\rangle \otimes \left[\alpha_1|1\rangle + \alpha_2|0\rangle\right] \longrightarrow \alpha_1|1\rangle \otimes |r\rangle + \alpha_2|0\rangle \otimes |t\rangle$$

entangles control photon with probe

Multi-mode cavities



spherical mirror cavity:

- distance L
- radius R

Separable: longitudinal q : $\omega_q \simeq \frac{c\pi}{L} q$

transverse n_x, n_y : $\Delta\omega_{n_x, n_y} = \omega_\perp (n_x + n_y + 1)$

$$\omega_{q, n_x n_y} = \frac{c\pi}{L} q + \omega_\perp \underbrace{(n_x + n_y + 1)}_{\text{transverse dynamics as in H.O.}}$$

$\omega_\perp = ..$

ex: Klaers et al (Weitz group), Nature (2010)

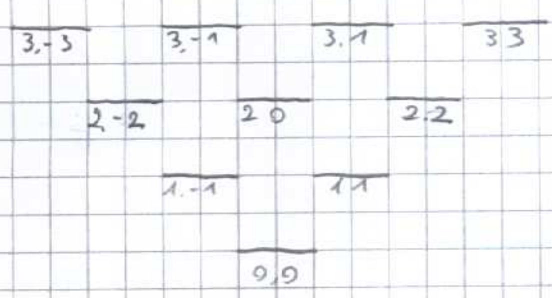
→ "BEC" of photons

(n_x, n_y) quantum numbers → Hermite-Gauss modes

(ℓ, m) quantum numbers → Laguerre-Gauss modes

↳ orbital angular momentum

Fix q:



Local photon-photon interactions (local $\chi^{(3)}$):

$$H_{int} = \frac{1}{2} \sum_{\substack{l_1 m_1 \\ l_2 m_2 \\ l_3 m_3 \\ l_4 m_4}} \hat{a}^{+}_{l_1, m_1} \hat{a}^{+}_{l_2 m_2} \hat{a}_{l_3, m_3} \hat{a}_{l_4 m_4} I^{l_3 m_3, l_4 m_4}_{l_1 m_1, l_2 m_2}$$

with $$I^{l_3 m_3 l_4 m_4}_{l_1 m_1 l_2 m_2} = \int d^2 r \; \phi^*_{l_4 m_4}(r) \, \phi^*_{l_3 m_3}(r) \cdot \phi_{l_2 m_2}(r) \, \phi_{l_1, m_1}(r)$$

Angular momentum conservation $m_1 + m_2 = m_3 + m_4$

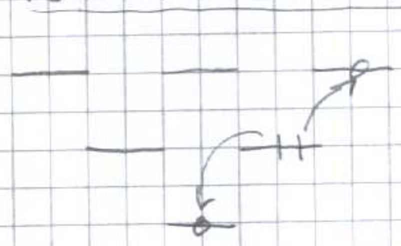
otherwise $I = 0$.

Two-photon spectroscopy with LG_{11} pump:

* 2x photons injected into $l=1$, $m=1$ mode
* interaction mix $(1,1)\,(1,1) \longrightarrow (2,2)\,(0,0)$

(no other process resonant)

$\Longrightarrow$ Parametric luminescence emitted @ ω_{00} and ω_{22} (if $g \leq \gamma$)



If $g \gg \gamma$: eigenstate of cavity H is
$\alpha |2:11\rangle + \beta |1:00\rangle |1:22\rangle$

Two-photon spectroscopy of cavity sees two peaks:

- 1 × ω_{11} → non-interacting eigenstate
- 1 × shifted by g → interacting state

With some calculation (homework!):

$$|\Psi_{non-int}(z_1, z_2)\rangle \simeq (z_1 - z_2)^2 \, e^{-|z_1|^2/2} \, e^{-|z_2|^2/2}$$

$$z_i = x_i + i y_i$$

Analogously for N-photon state:

$$|\Psi_{non-int}(z_1, z_2 \dots z_N)\rangle = \left[\prod_{i<j} (z_i - z_j)^2\right] e^{-|z_1|^2/2} \dots e^{-|z_N|^2/2}$$

<u>Laughlin wavefunction of fractional quantum Hall effect</u>

Umucalilar and IC, arXiv:1210...

<u>How far does analogy extend?</u>

- macroscopic number of particles ×
- many-body braiding phases ✓
- Anyonic character independent of path ×
- topological robustness, non-Abelian Anyons ?