Superfluids of atoms and of light as analog models of gravity: a fruitful synergy of gravity and quantum optics

Iacopo Carusotto

INO-CNR BEC Center and Università di Trento, Italy

BEC
BOSE EINSTEIN CONDENSATION
European Commission
Horizon 2020
European Union funding
for Research & Innovation
PhoQuS
Photons for Quantum Simulation

Part 1

Basics of analog models

analog Hawking radiation in atomic superfluids

Fishic black hole horizon

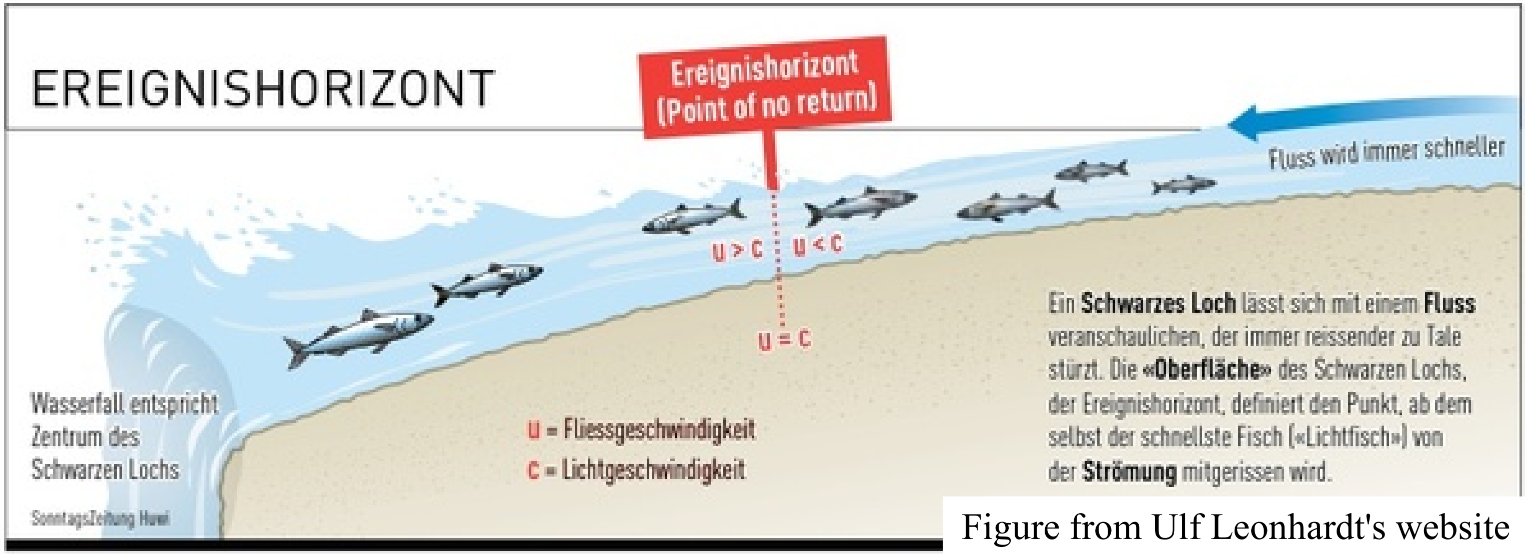


Figure from Ulf Leonhardt's website

Excitations (i.e. fish) propagate (i.e swim) at $v=c_s \pm v_{flow}$

- Horizon region separating sub-fishic flow (upstream) from super-fishic flow (downstream)
- Fish in super-fishic region can not swim back through fishic horizon

} Behavior analogous to astrophysical black hole horizon

- So, what happens with quantum fish ? Hawking radiation of fish ?

Acoustic Black Hole horizon

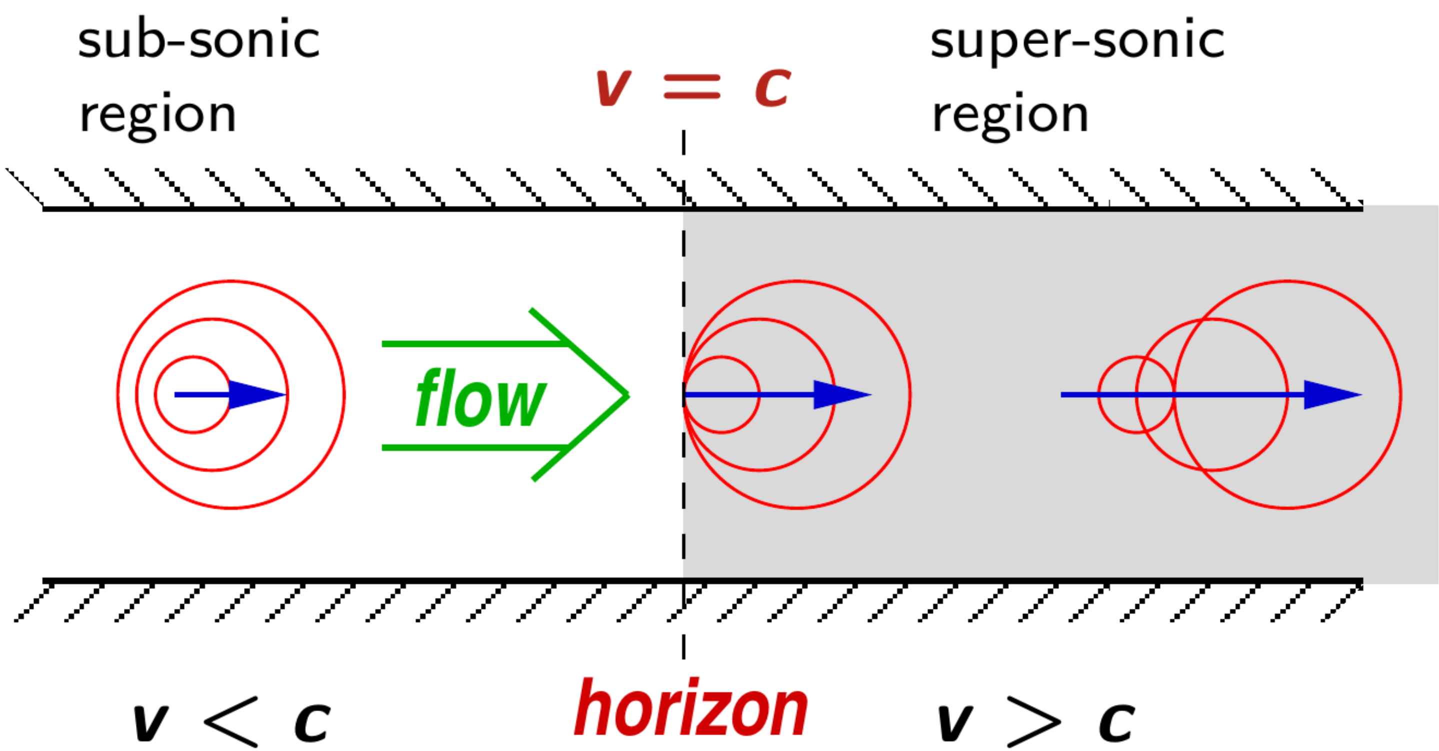


Figure from N. Pavloff's website

- Sound emitted in super-sonic region is dragged by the flow in the downstream direction
- Excitations in super-sonic region can not travel back through horizon

- What happens with quantized radiation field ? Hawking radiation of sound ?

Unruh, PRL 1981; Barceló, Liberati, Visser, Liv. Rev. Relativity **14**, 3 (2011)

Mathematical framework

Superfluid hydrodynamics of dilute BEC, e.g. ultracold atomic gas

Gross-Pitaevskii equation for BEC order parameter Ψ(x,t):

$$i\hbar\frac{\partial\Psi}{\partial t}=\frac{-\hbar^2}{2m}\nabla^2\Psi(x,t)+V(x)\Psi+g\,|\Psi(x,t)|^2\Psi(x,t)¿¿$$

Modulus-phase picture $\Psi(x,t)= n(x)^{1/2}\, e^{\,i\,\Phi(x,t)}$ gives hydrodynamic equations

Sonic dispersion of low-k phonons ω=c |k|.

Doppler shifted ω=c |k| - v · k in moving fluid at v

Relativistic eq for phase in inhomogeneous, moving BEC: $$\frac{1}{\sqrt{-G}}\,\partial_\mu\left[\sqrt{-G}\,G^{\mu\nu}\partial_\nu\right]\phi(x,t)=0$$

Equivalent to light propagation in curved space-time metric

$$ds^2=G_{\mu\nu}dx^\mu dx^\nu=\frac{n(x)}{c_s(x)}\left[-c_s(x)^2dt^2+(d\vec{x}-\vec{v}(x)dt)(d\vec{x}-\vec{v}(x)dt)\right]$$

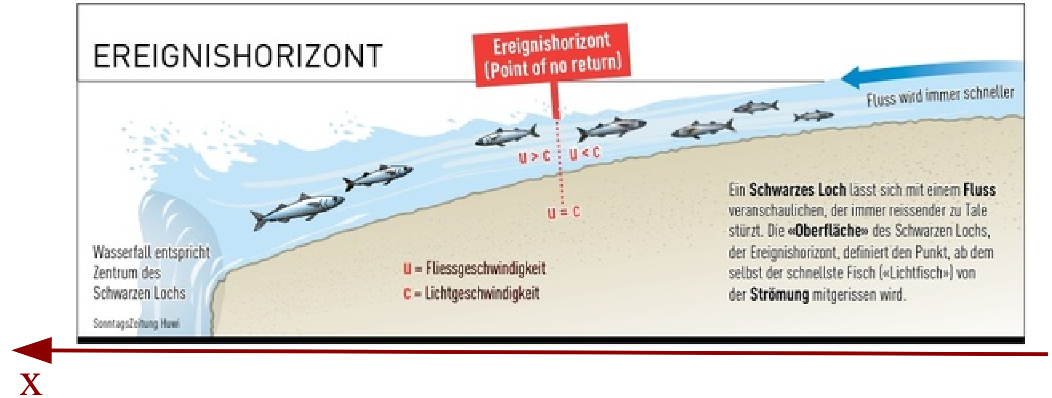
Once quantized → quantum field theory in a curved space time → Hawking emission?

Unruh, PRL 1981; Barceló, Liberati, Visser, Liv. Rev. Relativity **14**, 3 (2011)

Acoustic Black Hole

Simplest analog black hole geometry:

- one-dimensional geometry
- flow in the +x direction
- $v(x)/c_s(x)$ increases along +x direction
- horizon where $v(x_H) / c_s(x_H) = 1$



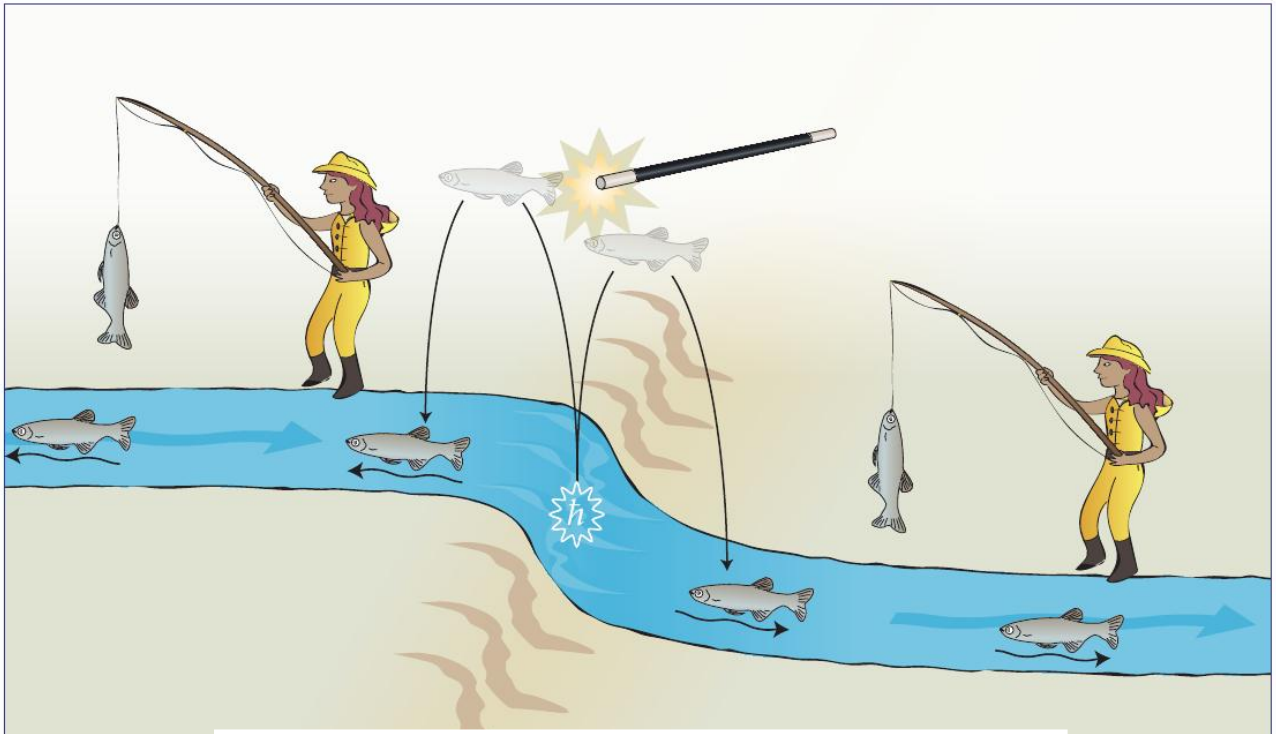
Astrophysical black holes → Hawking emission at $T_H = \frac{\hbar c^3}{8 \pi k_B G M}$

- ≈ fraction of μK for solar mass BHs, even lower for supermassive BH
- to be compared to 2.7K of Cosmic Microwave Background

Analog models → Hawking emission of sound at $T_H = \frac{\hbar}{4\pi k_B v} \frac{d}{dx}(c_s^2 - v^2)\Big|_{x=x_h}$

- $T_H \sim$ nK, to be compared with $T_{BEC} \sim$ nK as well
- but also something new and exciting...

<u>How to detect Hawking radiation?</u>



Picture from IC-Balbinot, Nat. Phys. N&V, Aug. 2016

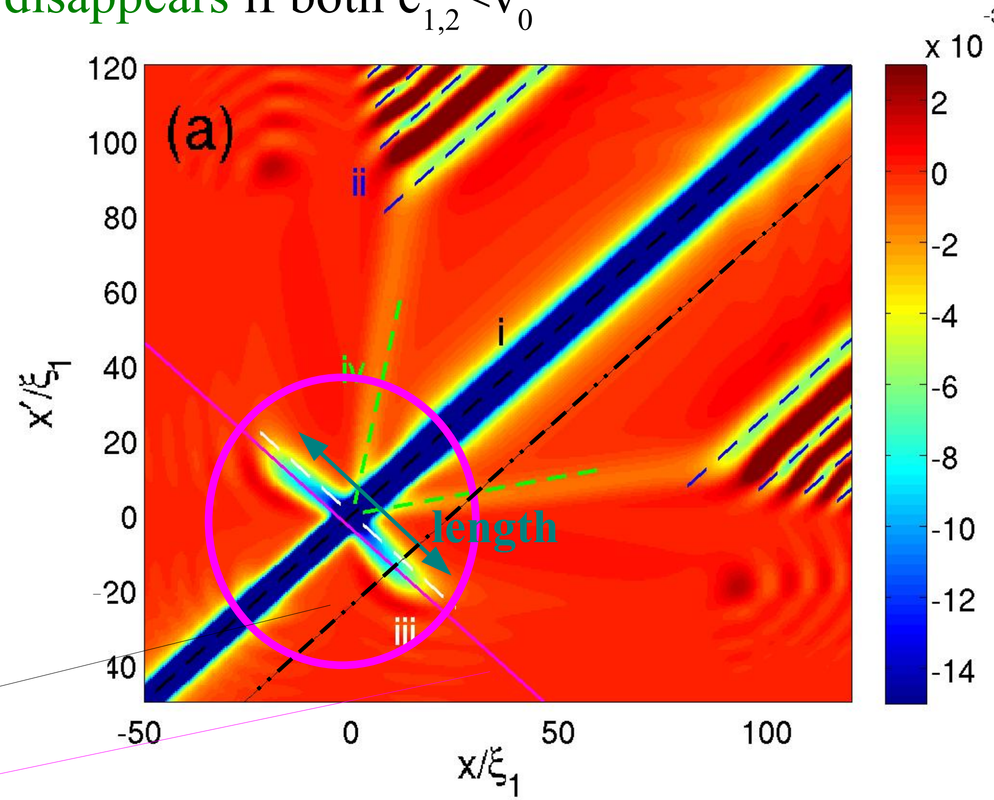
- Hawking radiation → correlated pairs generated simultaneously at the horizon
- Of course not detectable in astrophysics
- In analog models, HR isolated from background of thermal and noise phonons by measuring correlations on opposite sides of horizon
- <u>In the picture:</u> Hawking fish are caught simultaneously by the two fisherwomen!

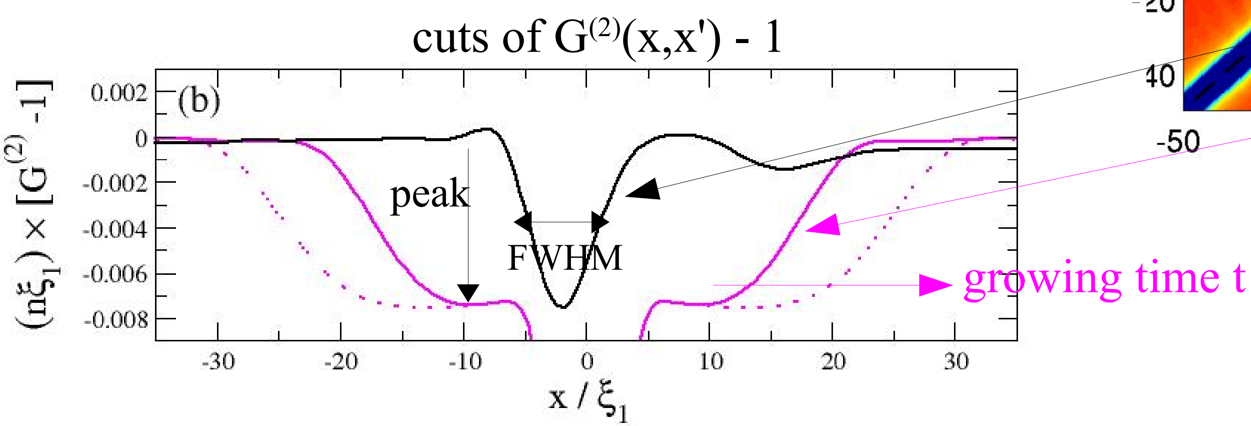
The Hawking signal: theoretical prediction

Wigner Monte Carlo simulations of the quantum fluctuations

Negative correlation tongue extending from the horizon x=x'=0

- long-range in/out density correlation which disappears if both $c_{1,2}<v_0$
- length grows linearly in t
- peak height, FWHM constant in t
- slope $\frac{v_0-c_2}{v_0-c_1}$ agrees with theory
  - pairs emitted at all t from horizon
  - propagate at sound speed



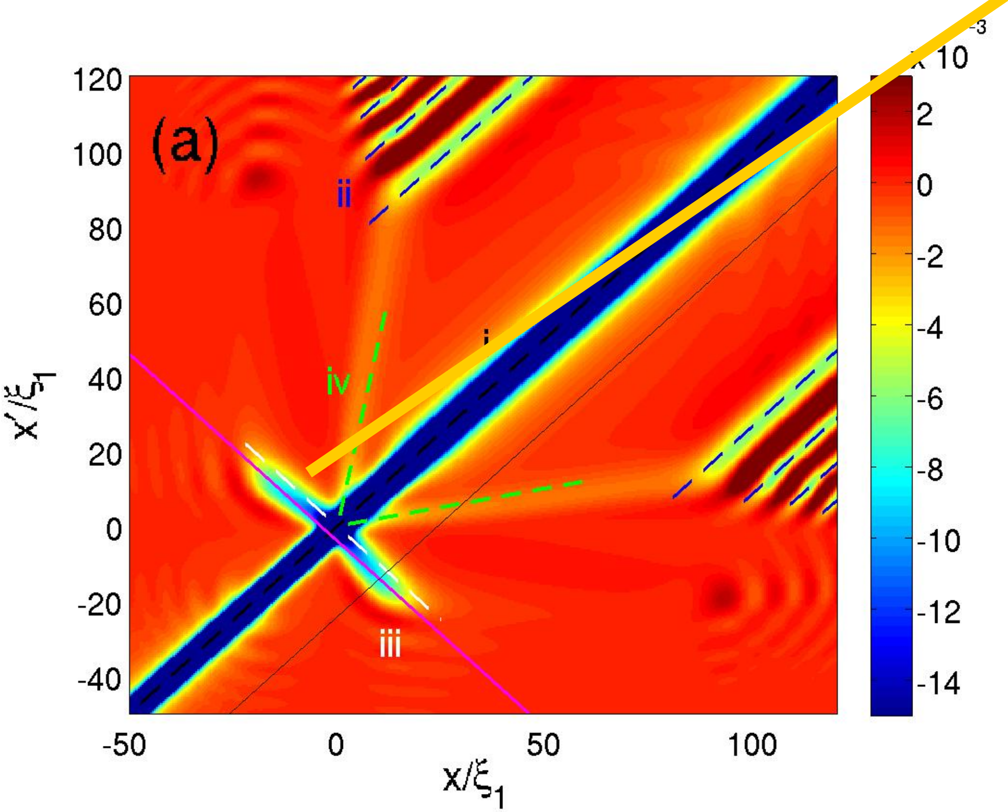


Numerics in: IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)
In agreement with theory in: R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008).

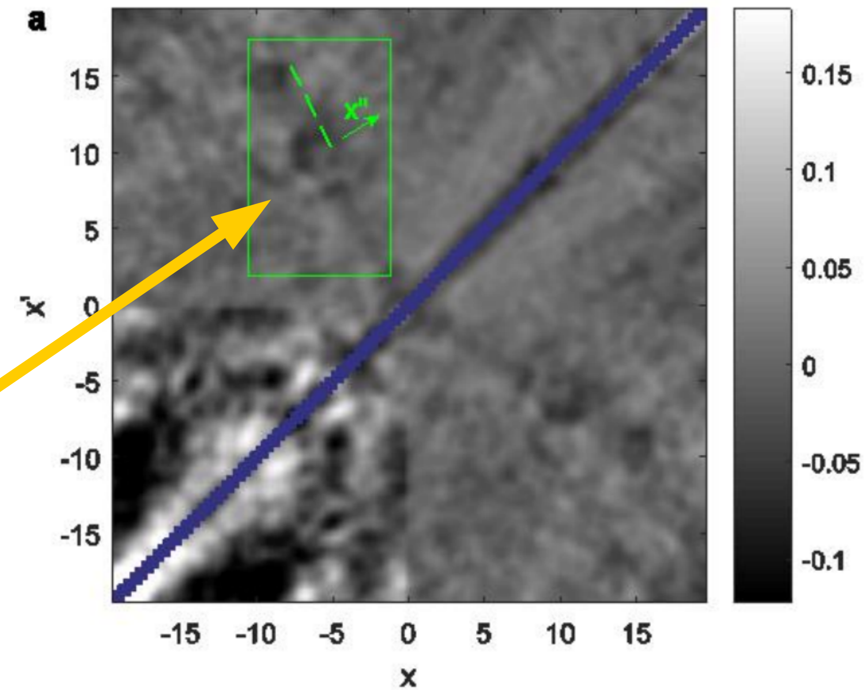
Analog Hawking radiation detected in the lab!

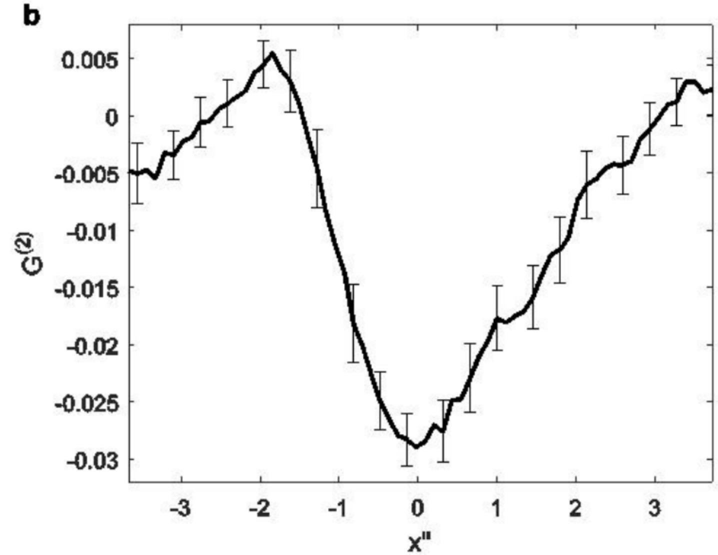
Analog black hole configuration obtained by sending 1D atomic BEC against optical potential

Experimental evidence of HR based on Balbinot-Fabbri moustache in correlation function of density fluctuations



Theory: IC et al., NJP 2008





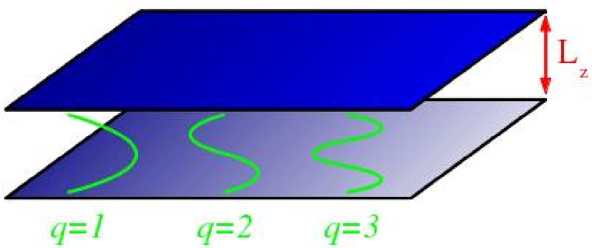
Expt: Steinhauer Nat. Phys. '16
New (better) pictures from De Nova et al., Nature '19

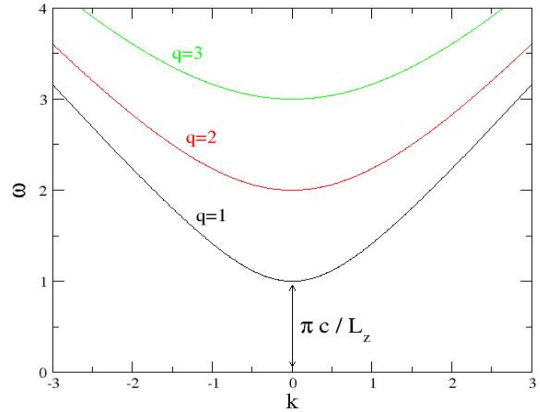
Part 2

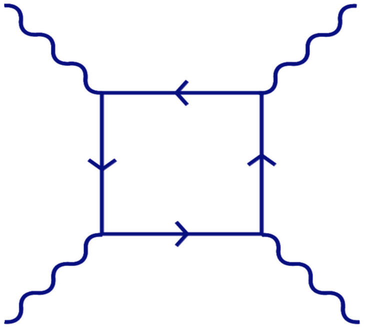
Analog Hawking radiation in quantum fluids of light

An unexpected discovery
on quasi-normal modes

What are quantum fluids of light ?







- Photons confined along z by mirrors; free to propagate along *xy* plane
  → relativistic dispersion: rest mass ω_o & effective mass m_o
- Optical nonlinearity of medium provides binary interactions

  → collective behaviour as a quantum fluid of light

- Laser pump coherently injects photons:
  → radiative losses determine non equilibrium steady state
  → coherence of polariton fluid guaranteed by coherent pump
- All properties of in-cavity photon fluid transferred to secondary emitted light

IC and C. Ciuti, *Quantum fluids of light*, RMP **85**, 299 (2013)

Experimental observation of superfluid behaviour

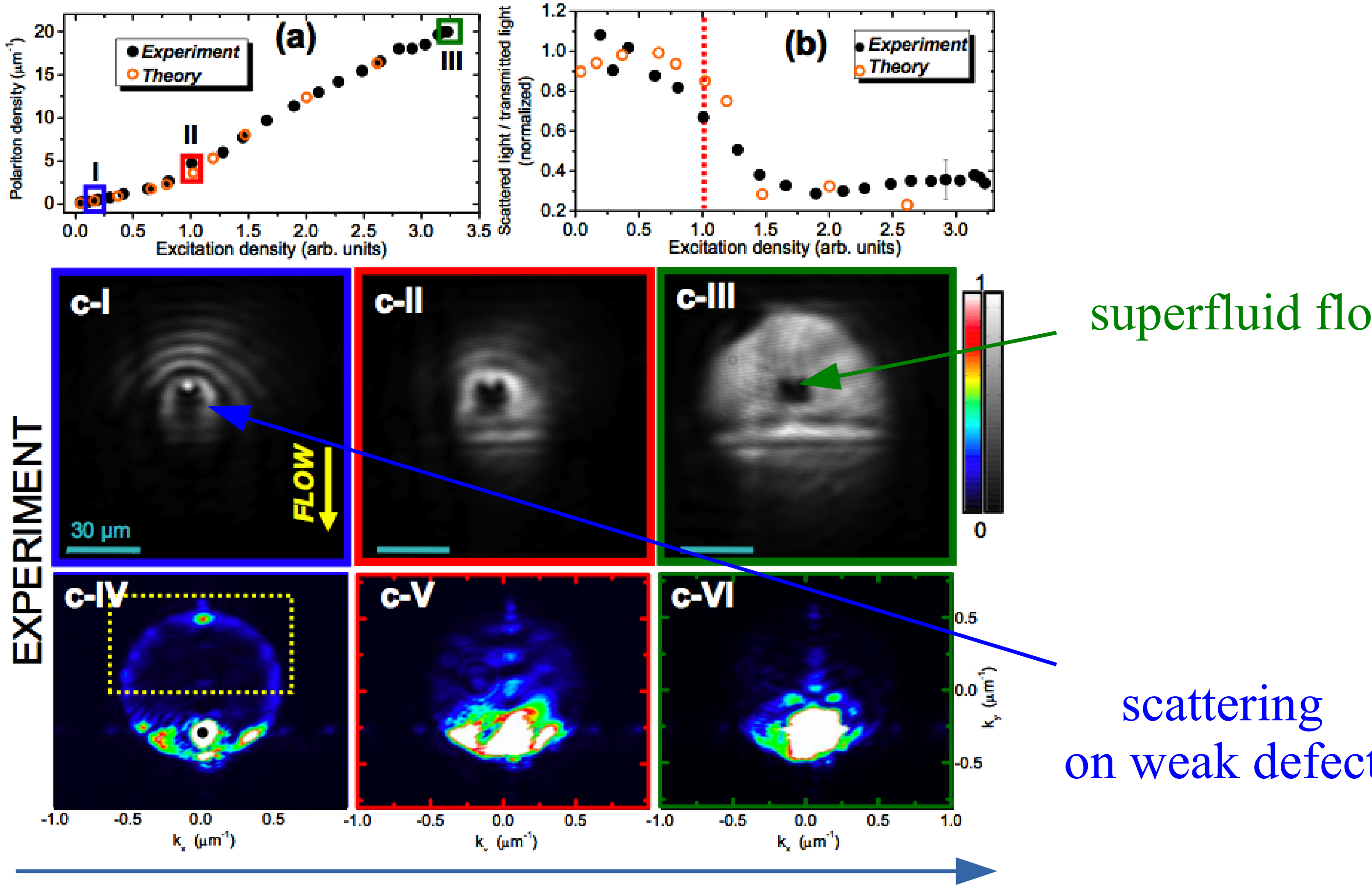


Figure from LKB-P6 group:
A.Amo, J.Lefrère, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

Theory: IC and C. Ciuti, PRL **93**, 166401 (2004)

Acoustic horizons in fluid of light

Photon-photon interactions

- Bogoliubov phonon dispersion on top of photon fluid

Pump at an angle

- finite in-plane wavevector, so condensate is flowing

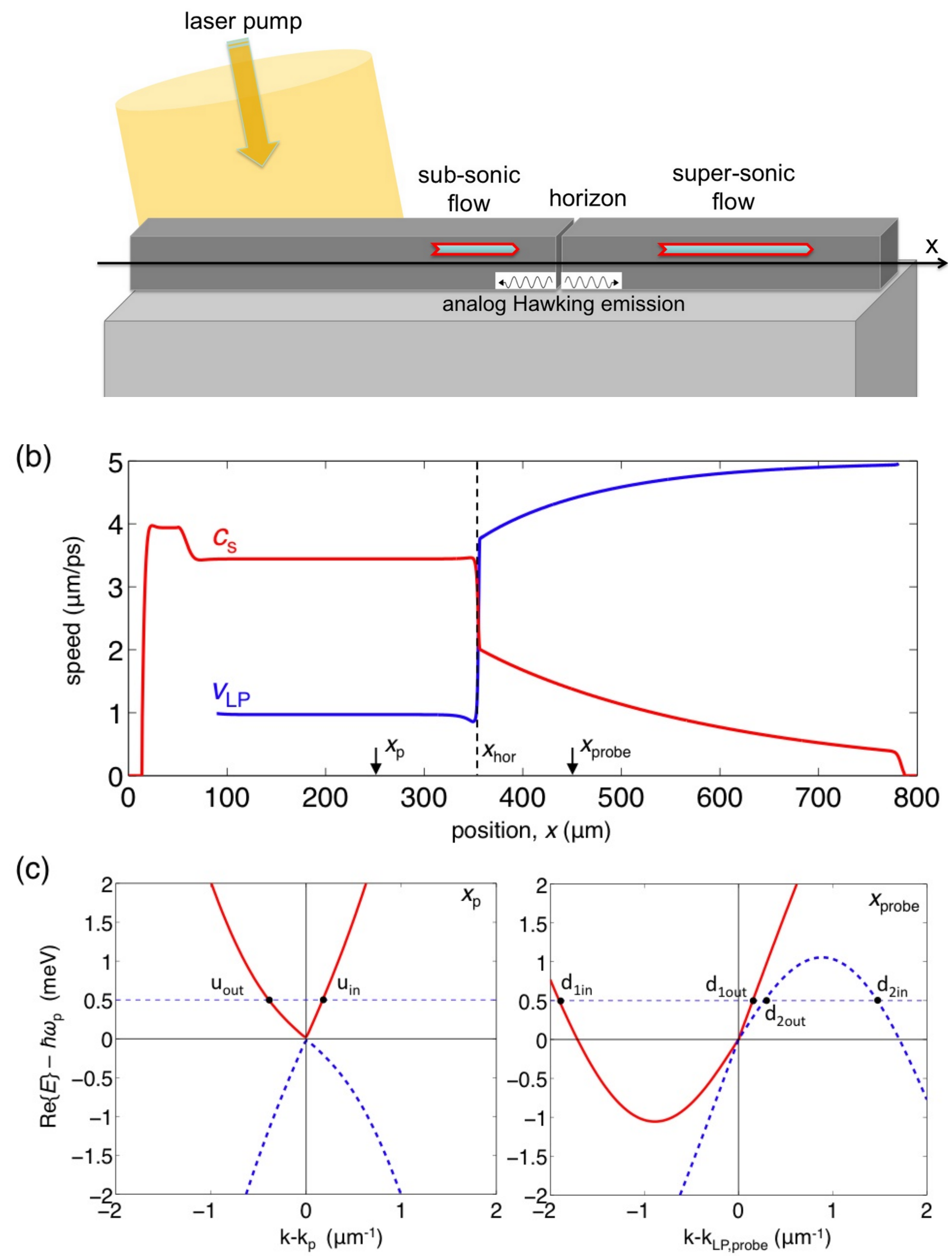
Tailored pump spot + Defect

→ Horizon with large surface gravity

Hawking emission

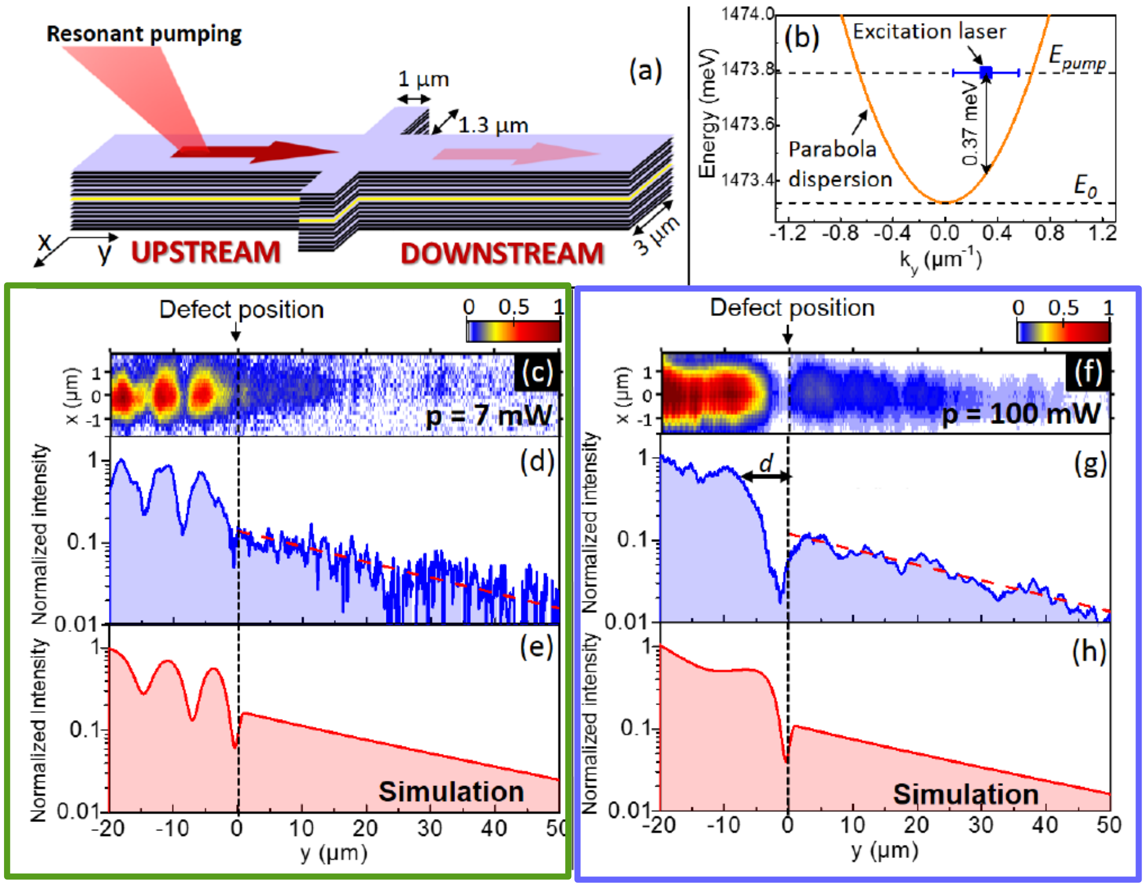
- phonons on photon fluid
- correlations of emitted light
- much higher T_H thanks to small photon mass

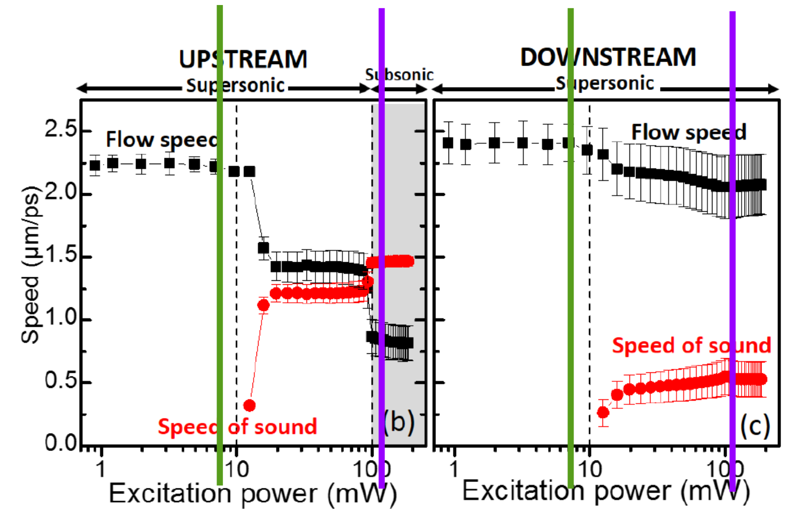
first proposed by F. Marino, PRA **78**, 063804 (2008)



D. Gerace and IC, PRB **86**, 144505 (2012)

Experimental results @ LPN (now C2N)





BH created!

The hunt for
Hawking radiation
is now open!!

H.-S. Nguyen, Gerace, IC, *et al.,* PRL **114**, 036402 (2015)

Hawking emission in photon fluids

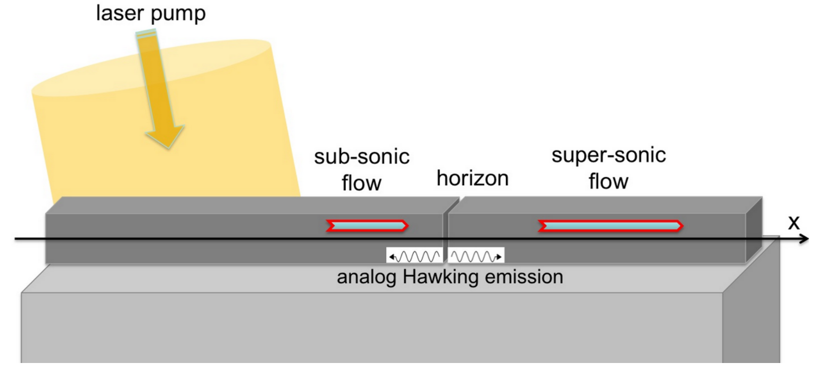
- Wigner-MC simulation with driving/losses:

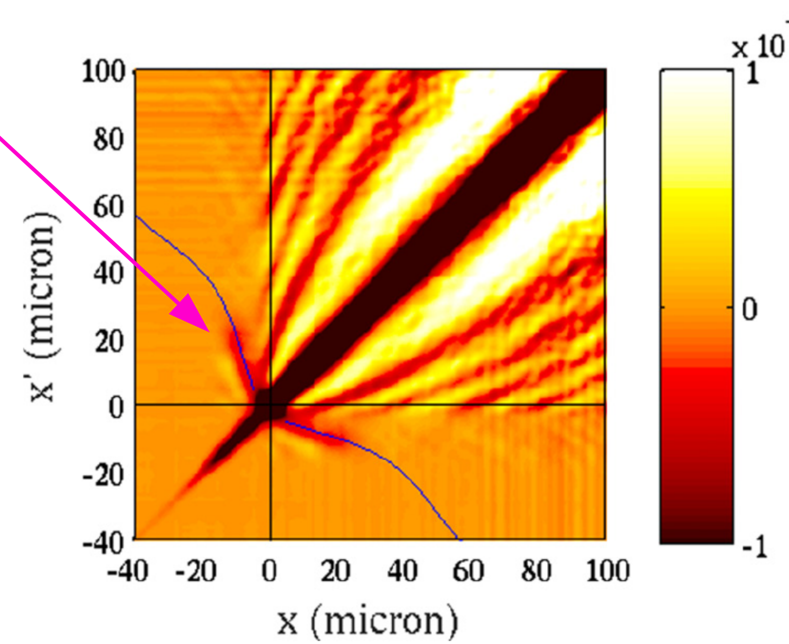
$$i\,dE=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|E|^2-\frac{i}{2}\gamma\right\}E\;dt+F_{ext}(x,t)\;dt+dW$$

- Near-field emission pattern from wire :
  Correlation function of intensity noise
  at different positions (x,x')

- Signature of Hawking radiation processes:
  "Balbinot-Fabbri" correlation tongues
  Conversion of zero-point fluctuations
  into correlated pairs of Bogoliubov phonons
  propagating away from horizon

- In optics language:
  parametric emission of entangled photons
  flow+horizon play role of pump
  photons dressed by fluid into phonons

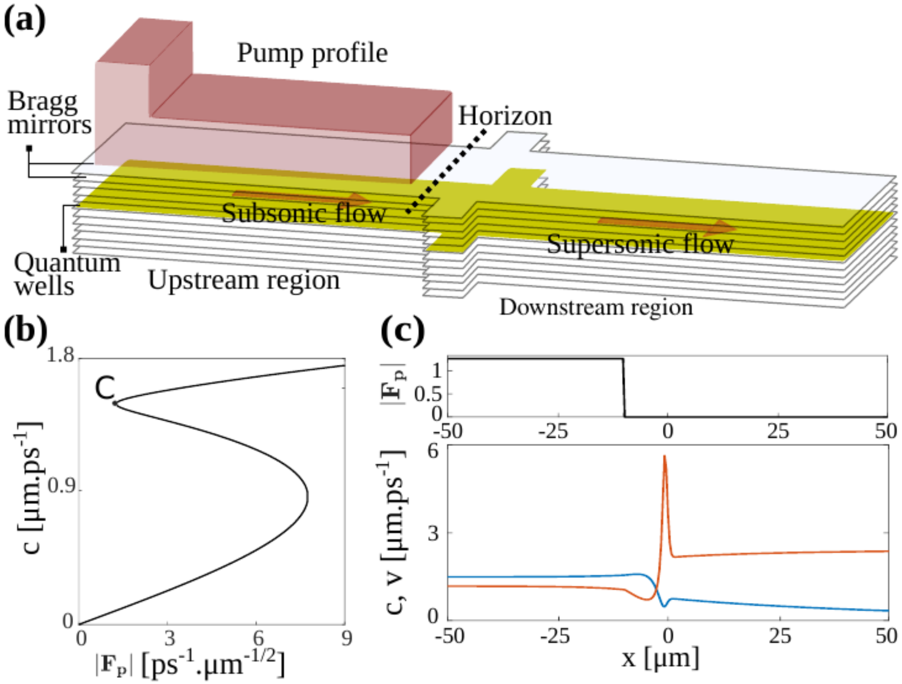


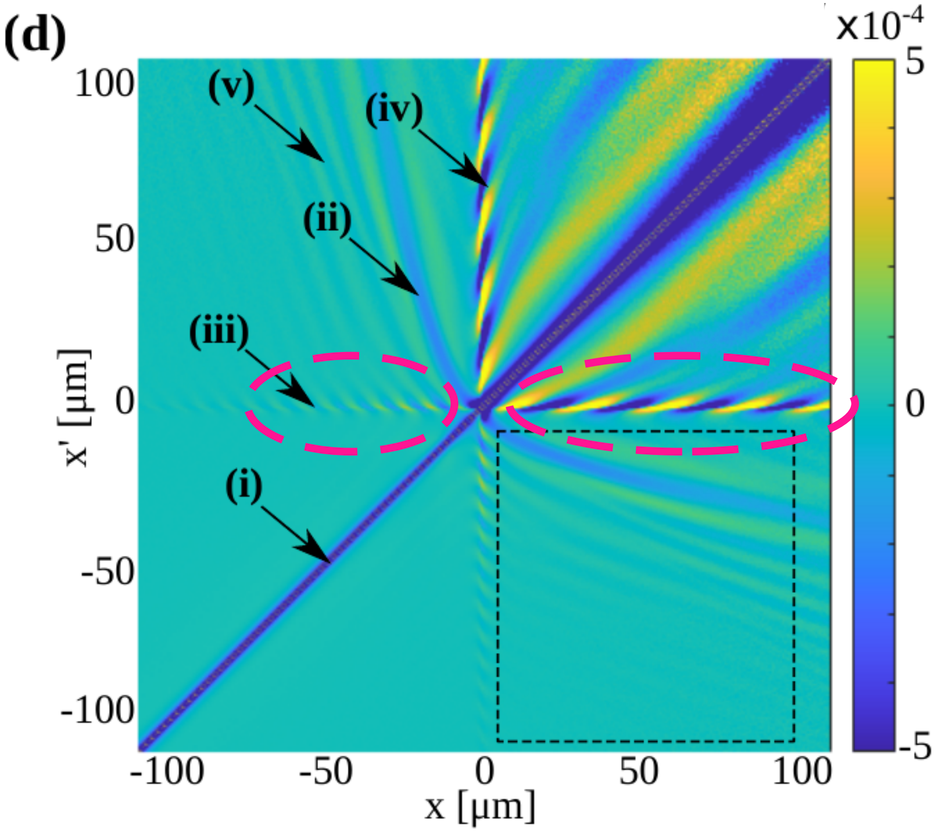


- Proposed experiment:
  - steady state under cw pumping
  - collect near-field emission
  - measure intensity noise
  - integrate over long time to extract signal out of shot noise

D. Gerace and IC, PRB **86**, 144505 (2012); P. Grisins, H.-S. Nguyen, J. Bloch, A. Amo and IC, PRB **94**, 144518 (2016).

An unexpected surprise



A (slightly) different pump configuration:

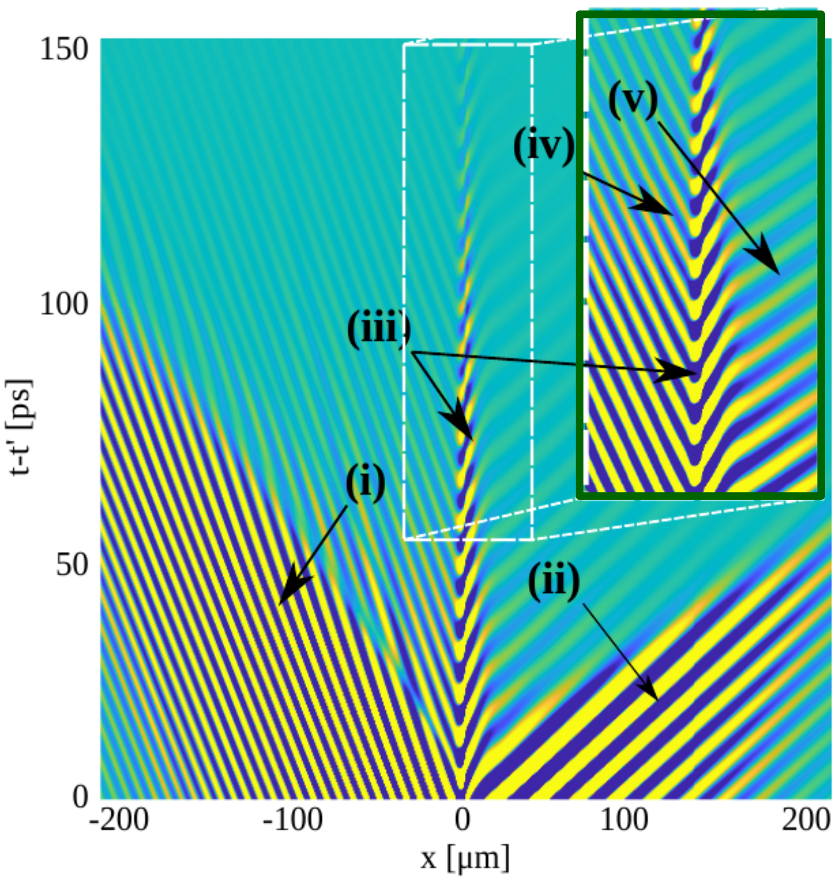
- Flat-top intensity profile
- Next to sonic point C in upstream region

… for a much richer correlation pattern:

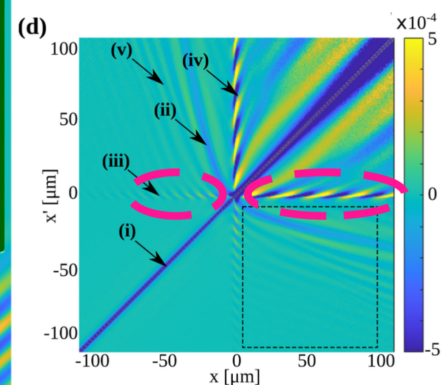
- Striped features along x=0 (and symmetrically x'=0) line
- Indicates correlation between horizon region & emitted radiation

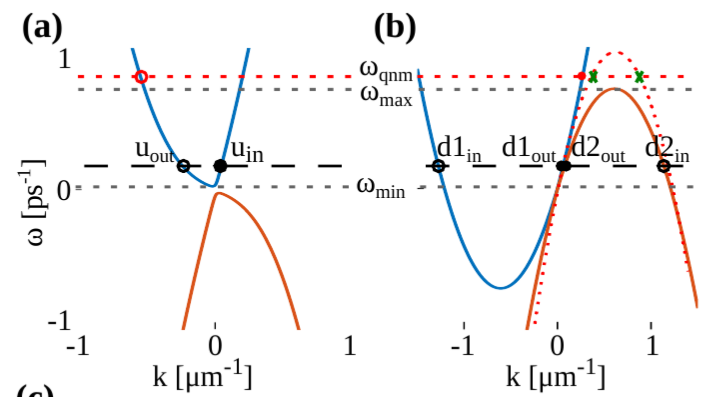
Jacquet, Giacomelli, … IC, Bramati, *Quantum vacuum excitation of the quasi-normal mode of an analog BH in a polariton fluid*
arXiv:2201.02038

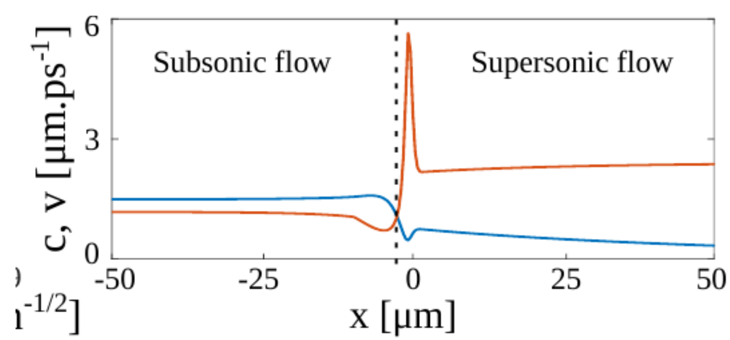
Localized quasi-normal mode next to horizon



Localized features in $g^{(2)}(x,x')$ suggest the presence of localized mode next to horizon
→ quasi-normal mode of black-hole







To verify this:

- Classical dynamics after classical incident wavepacket show localized long-lived mode: (i) reflected; (ii) transmitted; (iii) quasi-normal mode.
- QNM radiatively decays in both directions (iv,v)

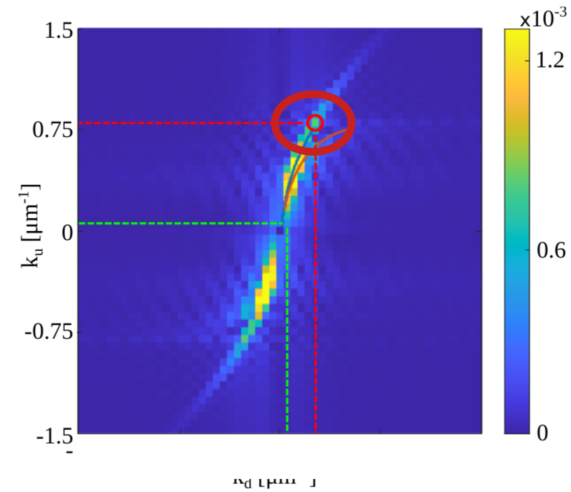
Physical origin:

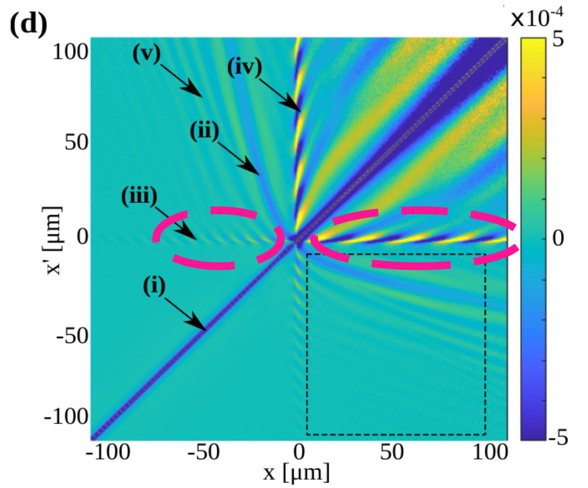
- strongly supersonic region → attractive potential for negative-norm mode → localized state

What can we learn about astrophysical BHs ?

Theory of photon-based analog model

- features a localized QNM
- spontaneous Hawking-like emission displays peak @ QNM
- hosts finite zero-point quantum excitation
  by same mechanism responsible of Hawking emission
- correlation between emission and QNM excitation
- result generic applies to other analog models





QNMs are common feature of astrophysical BHs

- decaying eigenmodes of BH dynamics; classically excited
  during astrophysical processes, e.g. mergers
- radiatively decay into emitted gravitational waves

So… what we may dare to learn about general Bhs?

- Not only BHs are not fully "black" because they emit Hawking radiation...
- ...but also their shape "fluctuates" under the effect of zero-point fluctuations of space-time

Jacquet, Giacomelli, … IC, Bramati, *Quantum vacuum excitation of the quasi-normal mode of an analog BH in a polariton fluid*
arXiv:2201.02038

Part 3

Superradiance

how a naive quantum optician understands it and how analog models could give further insight on fundamental gravitational processes

L. Giacomelli & IC, *Understanding superradiant phenomena with synthetic vector potentials in atomic BECs*, PRA 2021

L. Giacomelli & IC, *Spontaneous quantum superradiant emission in atomic BECs subject to a synthetic vector potential*, PRA 2021

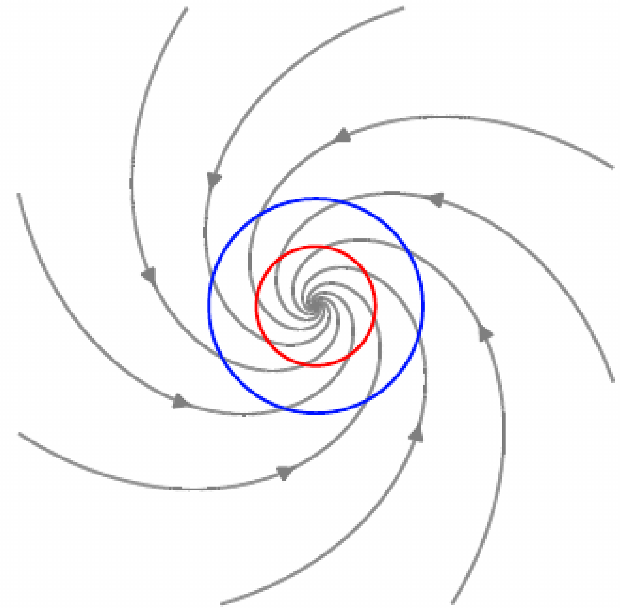
L. Giacomelli & IC, *Interplay of Kelvin-Helmholtz and superradiant instabilities of an array of quantized vortices in a 2D BEC*. arXiv:2110.10588

New geometries and new curved space-time effects

Rotating BH in inspiralling 2D vortex

$$\mathbf{v} = \frac{A}{r}\hat{r} + \frac{B}{r}\hat{\theta}$$

$$g_{\mu\nu} \propto \begin{bmatrix} -\left(c_s^2 - \frac{A^2+B^2}{r^2}\right) & 0 & -B \\ 0 & \frac{r^2 c_s^2}{r^2 c_s^2 - A^2} & 0 \\ -B & 0 & r^2 \end{bmatrix}$$

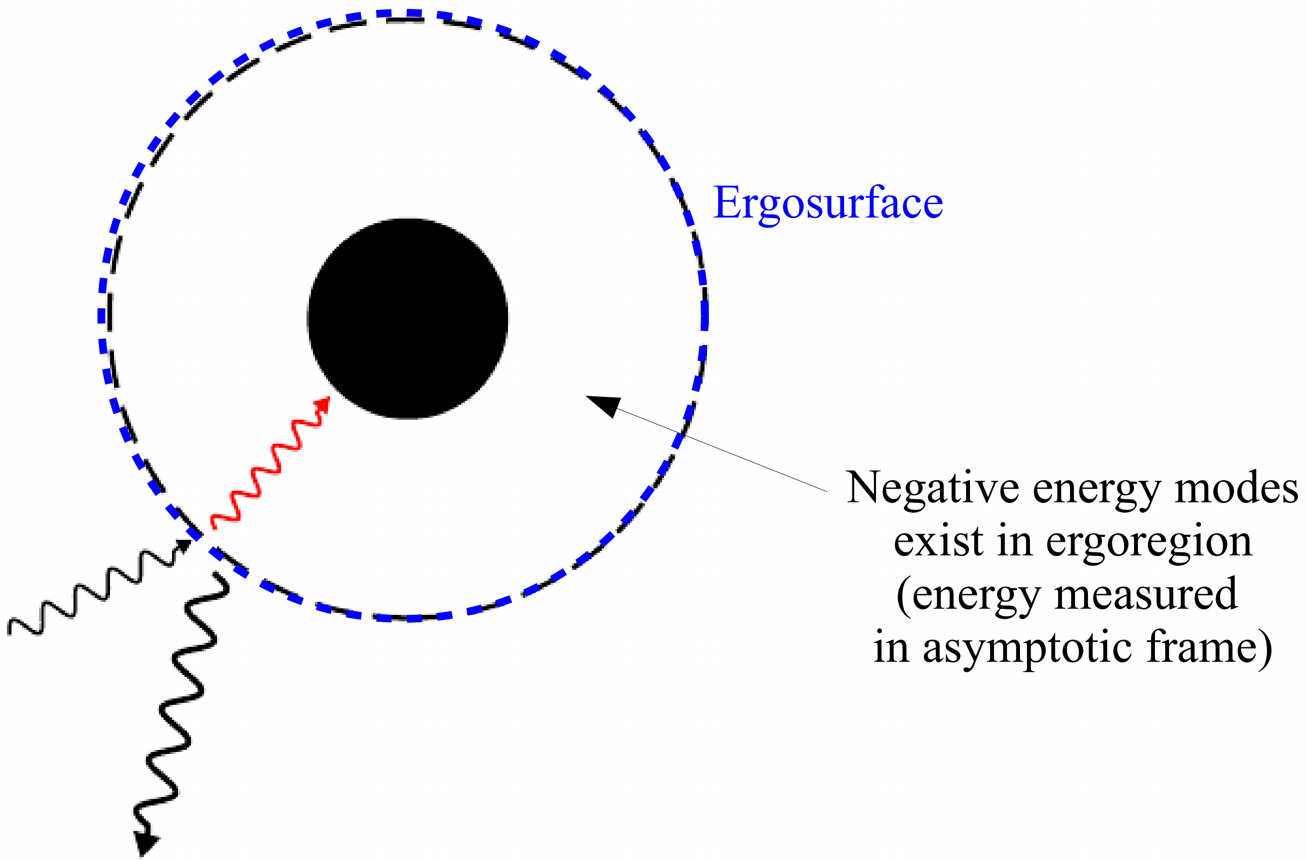


- Acoustic horizon: $r_H^2 = \frac{A^2}{c_s^2}$ $\quad v_r = c_s$ $\quad$ No way of escaping!
- Ergosurface: $r_E^2 = \frac{A^2 + B^2}{c_s^2}$ $\quad |v| = c_s$ $\quad$ No way of staying at rest (in asymptotic frame)

Ref: Visser (1998). Acoustic black holes. Classical and Quantum Gravity, 15(6), 1767

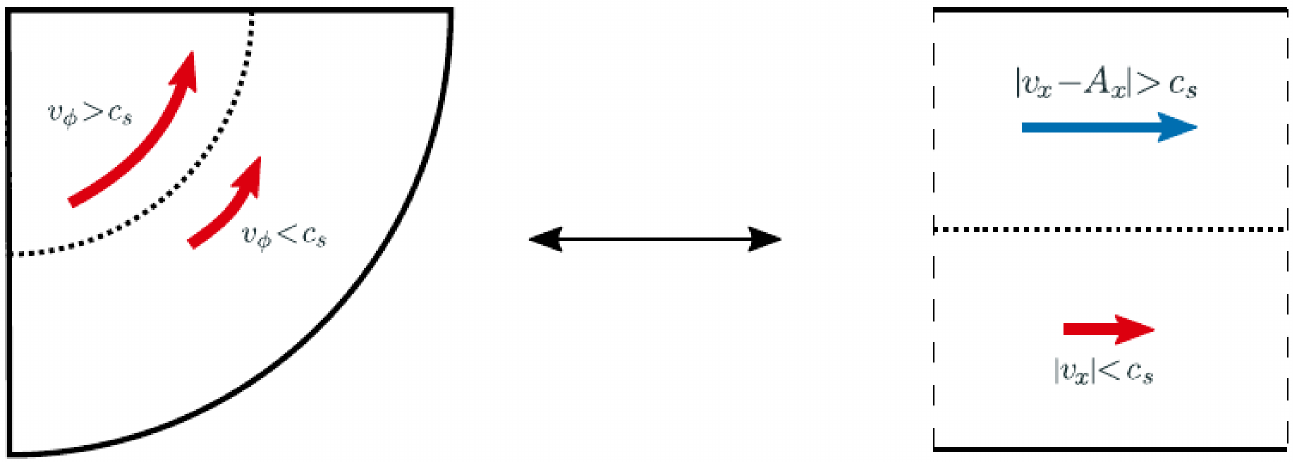
Superradiance

- Amplified reflection by transmission of negative energy



Ref: Brito, Cardoso, Pani (2015). Superradiance. Lect. Notes Phys, 906(1), 1501-06570.

A toy model to understand superradiance (I)



Translational invariance

Arbitrary y-dependence of v_x by removing irrotationality constraint

- Can we really do this?

$$\mathbf{v} = \frac{\hbar \nabla \Theta}{M} \implies \nabla \times \mathbf{v} = 0$$

- We can play with synthetic gauge fields:

$$\mathbf{v} = \frac{\hbar \nabla \Theta - \mathbf{A}}{M}$$

For us it is a trick to get rotational flow.

For an introduction Dalibard et al. RMP 2011

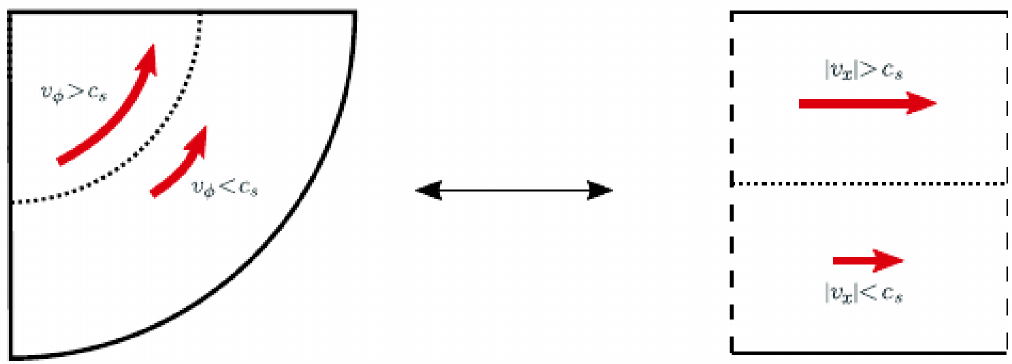
Ozawa et al., RMP 2019

Experimental platforms:
- modulated waveguides in atomic gas
- strained honeycomb lattices
- magnetic polaritons
- …

Similar (yet richer) physics if A_x replaced by string of vortices

L. Giacomelli & IC, *Understanding superradiant phenomena with synthetic vector potentials in atomic BECs*, PRA 2021
L. Giacomelli & IC, *Interplay of Kelvin-Helmholtz and superradiant instabilities of an array of quantized vortices in a 2D BEC*. arXiv:2110.10588

A toy model to understand superradiance (II)



$$g_{\mu\nu} = \frac{n^2}{c^2}\begin{bmatrix} -(c^2 - v_x^2) & -v_x & 0 \\ -v_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Translational invariance along x: $\phi(t,x,y) = e^{ik_x x}\phi(t,y)$
- Klein-Gordon equation becomes:

$$-\left(\frac{1}{c}\partial_t + i\frac{v_x}{c}k_x\right)^2\phi + \partial_y^2\phi - k_x^2\phi = 0$$

- KG for a charged field in an electrostatic potential:

$$-\left(\frac{1}{c}\partial_t + \frac{ie}{\hbar c}A_0\right)^2\phi + \nabla^2\phi - \frac{m^2c^2}{\hbar^2}\phi = 0$$

Reduced to 1D problem

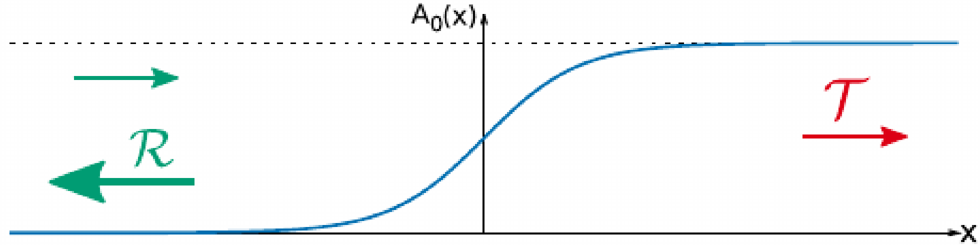
$$m^2 \longleftrightarrow \hbar^2 k_x^2/c^2 \qquad eA_0 \longleftrightarrow \hbar k_x v_x$$

Ref: Fulling (1989). Aspects of QFT in curved spacetime, Cambridge University Press

L. Giacomelli & IC, *Understanding superradiant phenomena with synthetic vector potentials in atomic BECs*, PRA 2021

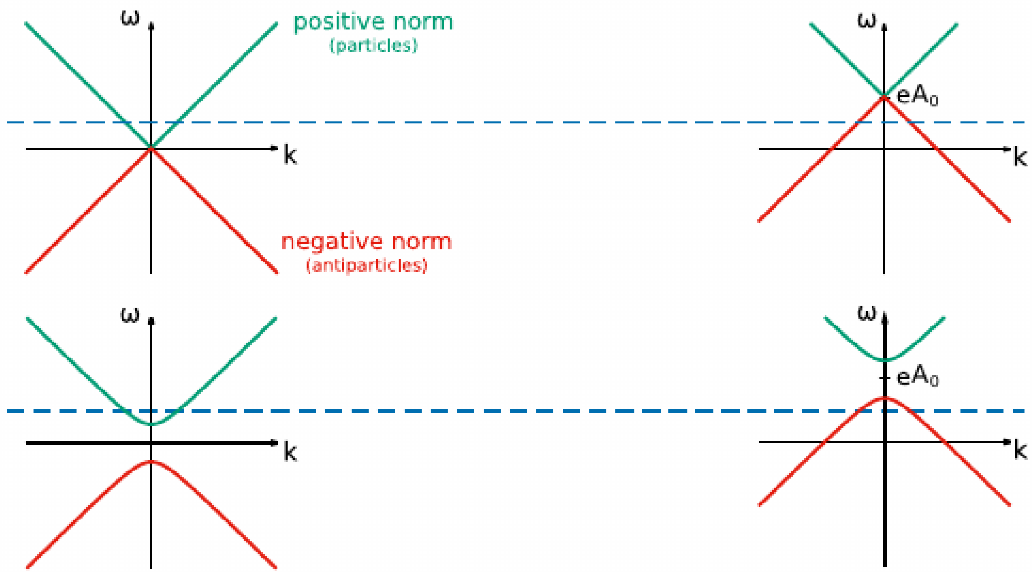
The bosonic Klein paradox

$$-\left(\frac{1}{c}\partial_t + \frac{ie}{c}A_0\right)^2\phi + \partial_x^2\phi - \frac{m^2c^2}{\hbar^2}\phi = 0$$



Transmission of negative norm particles gives amplified reflection

- Massless case:



$\mathcal{R} > 1$

for

$\omega < eA_0$

- Massive case:

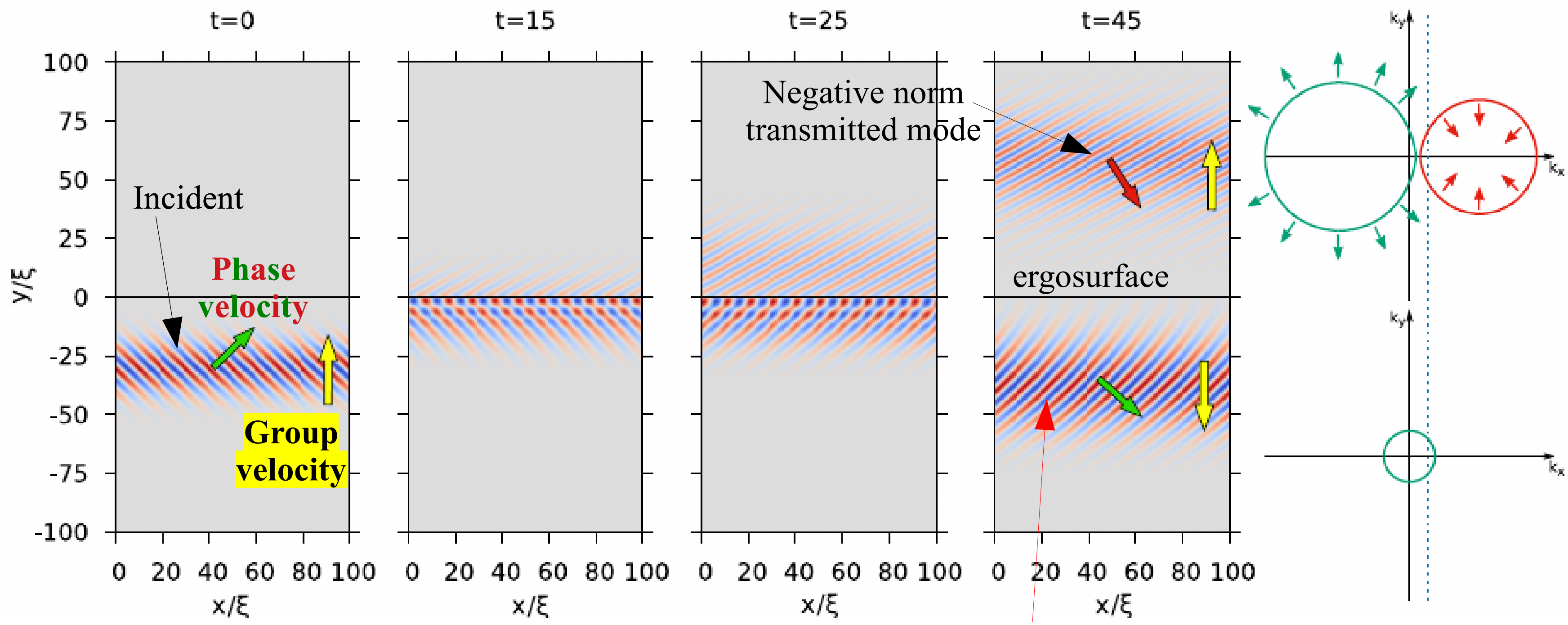
$eA_0 > 2mc^2$

- In our 2D configuration with $A_x(y) = \begin{cases} 0 \text{ for } y < 0 \\ A_x \text{ for } y > 0 \end{cases} \implies |A_x| > 2Mc_s$

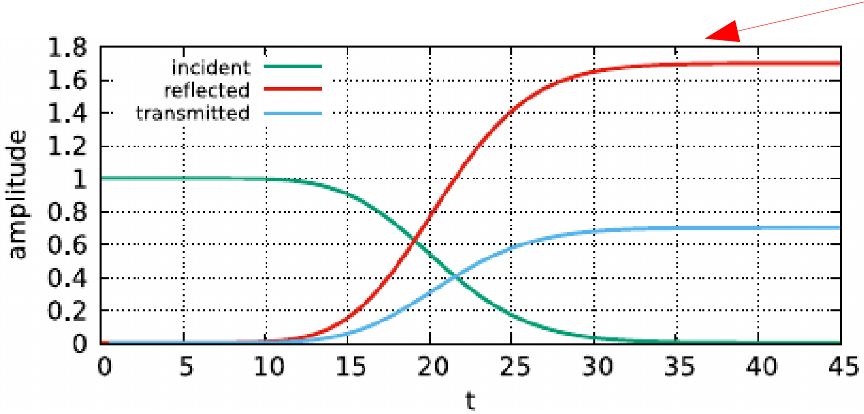
L. Giacomelli & IC, *Understanding superradiant phenomena with synthetic vector potentials in atomic BECs*, PRA 2021

2D GPE simulations for atomic superfluids...

$$i\hbar\partial_t\Psi = \left[\frac{(-i\hbar\nabla - \mathbf{A}(\mathbf{r}))^2}{2M} + V(\mathbf{r}) + g\Psi^2\right]\Psi \; ; \; A_x(y) = \begin{cases} 0 \text{ for } y<0 \\ A_x \text{ for } y>0 \end{cases} ; \; V(y) = \begin{cases} 0 \text{ for } y<0 \\ -\frac{A_x^2}{2M} \text{ for } y>0 \end{cases}$$



Superradiance: amplified reflection of sound scattering onto ergo-surface



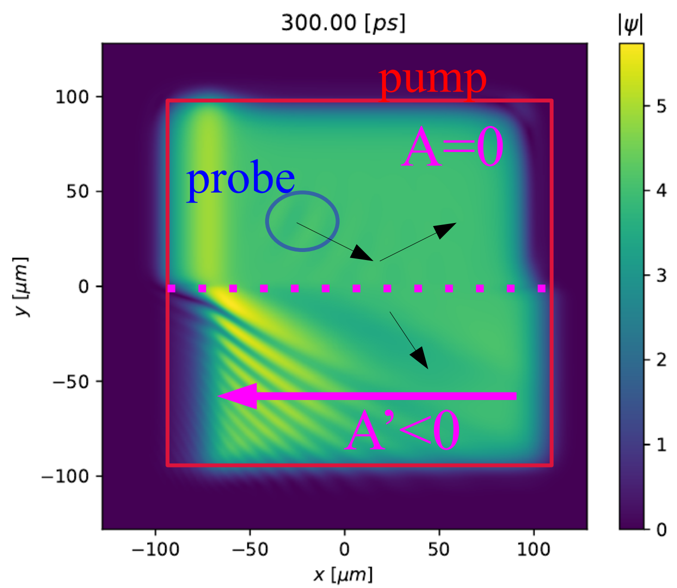
Immediate to build scattering matrix theory linking out- to in-going modes with positive/negative norms (à la Pavloff-Recati-IC, PRA 2009)

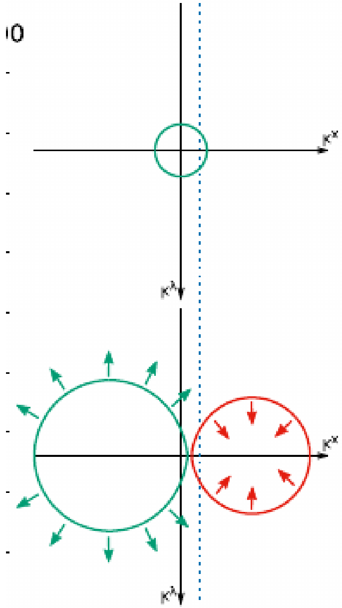
L. Giacomelli & IC, *Understanding superradiant phenomena with synthetic vector potentials in atomic BECs*, PRA 2021

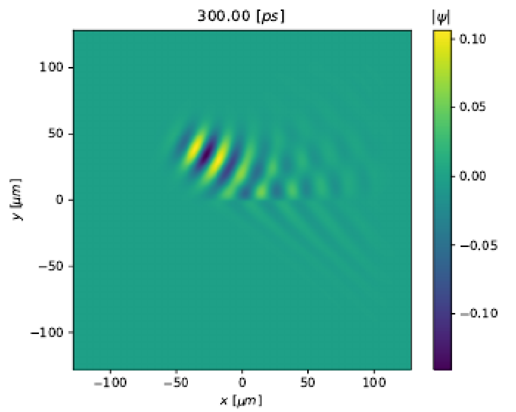
...and for polariton fluids (preliminary data by Alberto & Luca)

Step-shaped synthetic vector potential
(e.g. via crossed E-B fields → Atac's talk)

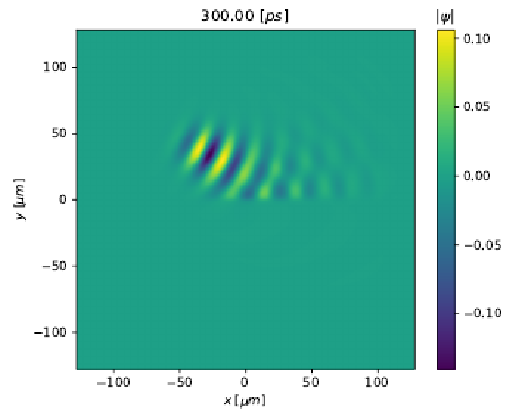
- Positive-norm incident wavepacket
- Amplified positive-norm reflection
- Negative-norm transmission (with unusual phase velocity)



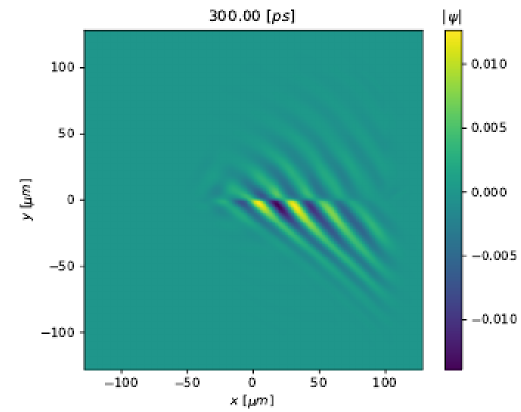




Density perturbation with synth-A



Density perturbation without synth-A

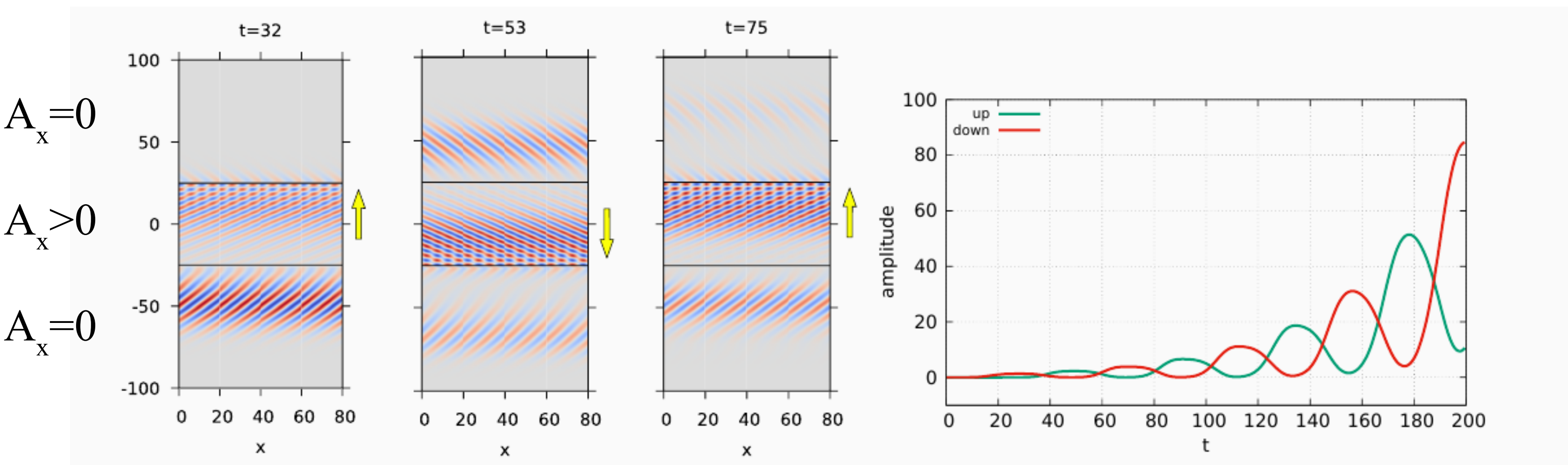


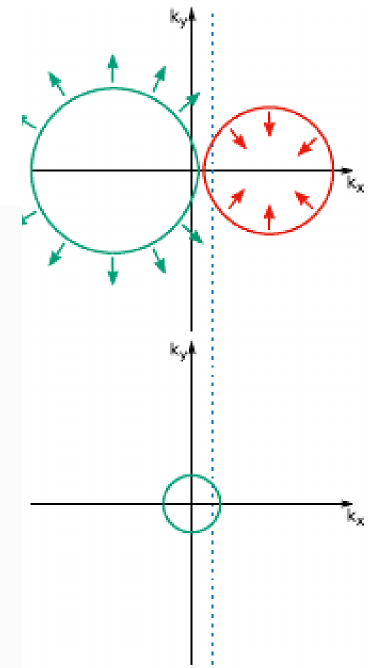
Difference gives evidence for superradiance

A. Canali, MSc thesis @ UniTN, 2021. A. Canali, L. Giacomelli, IC, *Superradiant scattering in polariton fluids*, 2022

Turning superradiance into an instability

- Add second interface
- Negative norm wave continues to bounce in between the interfaces
- Amplification at each bounce





- Negative-norm mode gets dynamically unstable
- Perturbation keeps growing in between interfaces while emitting waves outside
- Similar to lasing and BH lasing
- info on stability & existence of astrophysical objects, bounds on dark matter etc.

L. Giacomelli & IC, *Understanding superradiant phenomena with synthetic vector potentials in atomic BECs*, PRA 2021

Part 4:

(In)stability of quantized vortices

how superradiance allows to understand a classical problem in quantum fluids

L. Giacomelli & IC, *Ergoregion instabilities in rotating two-dimensional Bose-Einstein condensates: Perspectives on the stability of quantized vortices*, Phys. Rev. Res. 2020

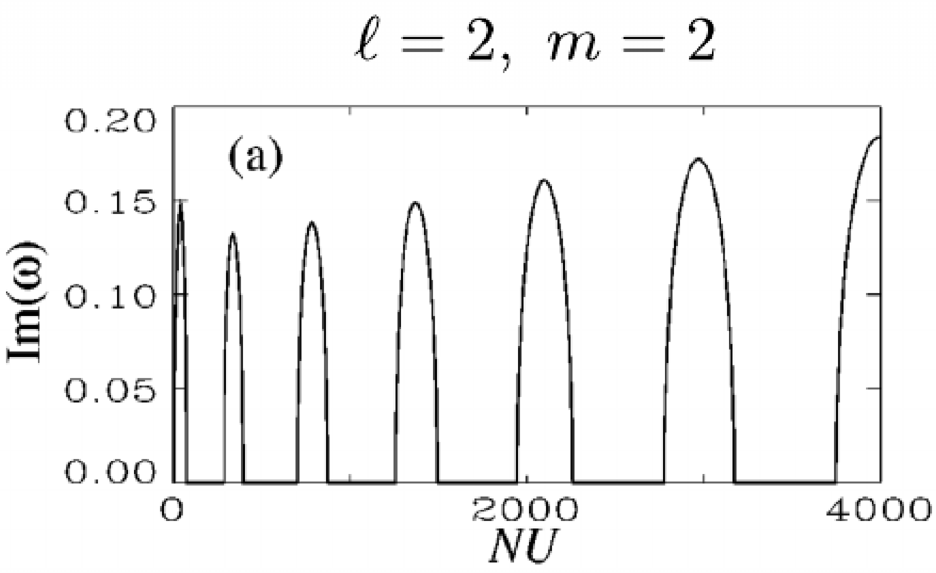
(In)stabilities of quantized vortices in *trapped* fluids

- Multiply charged vortices are energetically unstable
- A trapped singly charged vortex is energetically unstable but dynamically stable

[Rokhsar (1997) PRL, 79(12), 2164]

- Trapped multiply charged vortices can be dynamically unstable

[Pu et al. (1999) PRA, 59(2), 1533]



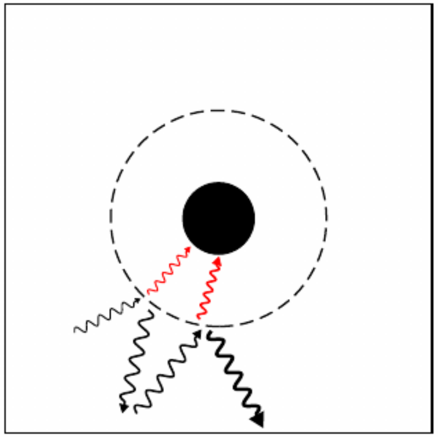
Study the spectrum as

$$\Psi_0(r,\theta) = e^{i\ell\theta} f(r)$$

$$\begin{pmatrix} \delta\psi(r,\theta) \\ \delta\psi^*(r,\theta) \end{pmatrix} = e^{im\theta} \begin{pmatrix} e^{i\ell\theta}\phi(r) \\ e^{-i\ell\theta}\phi^*(r) \end{pmatrix}$$

- What is the mechanism for the instability of multiply quantized vortices?
- Are multiply quantized vortices unstable in spatially infinite geometries?
- What is role of supersonic flow in the vicinity of the vortex core?

Some space-time instabilities



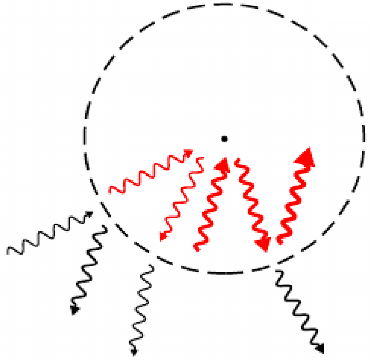
Black hole bomb

Hawking radiation from BH horizon:

- reflected at large distance
- cavity between horizon and external confinement

  $\rightarrow$ self-amplifying radiation

In astrophysics: Hawking emission of light massive particles (Damour-Desruelle-Ruffini, 1976)



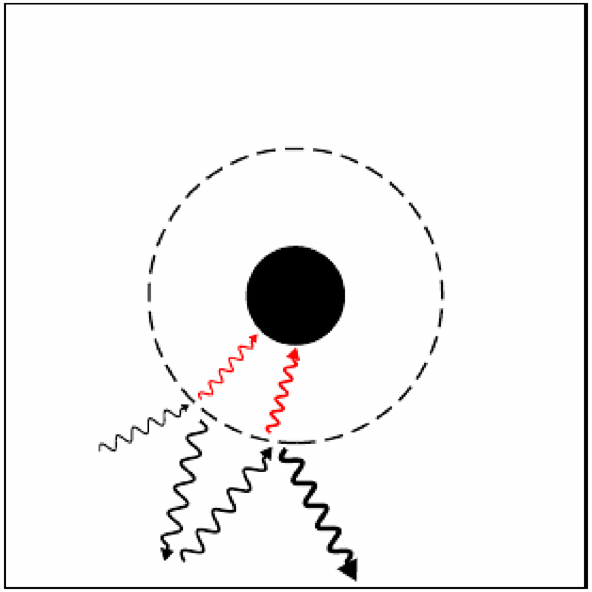
Ergoregion instability

Superradiance at ergosurface:

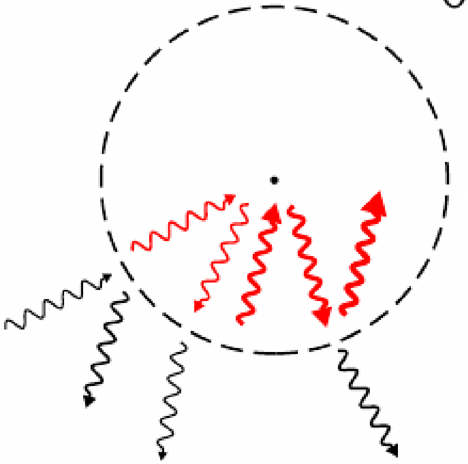
- partner wave does not escape from inner region
- stimulates further superradiance
- laser cavity formed within ergosurface

In astrophysics: compact (non-BH) stellar objects (Friedman 1973)

(In)stabilities of quantized vortices



Black hole bomb



Ergoregion instability

In analogues:
Oliveira et al. PRD 2014

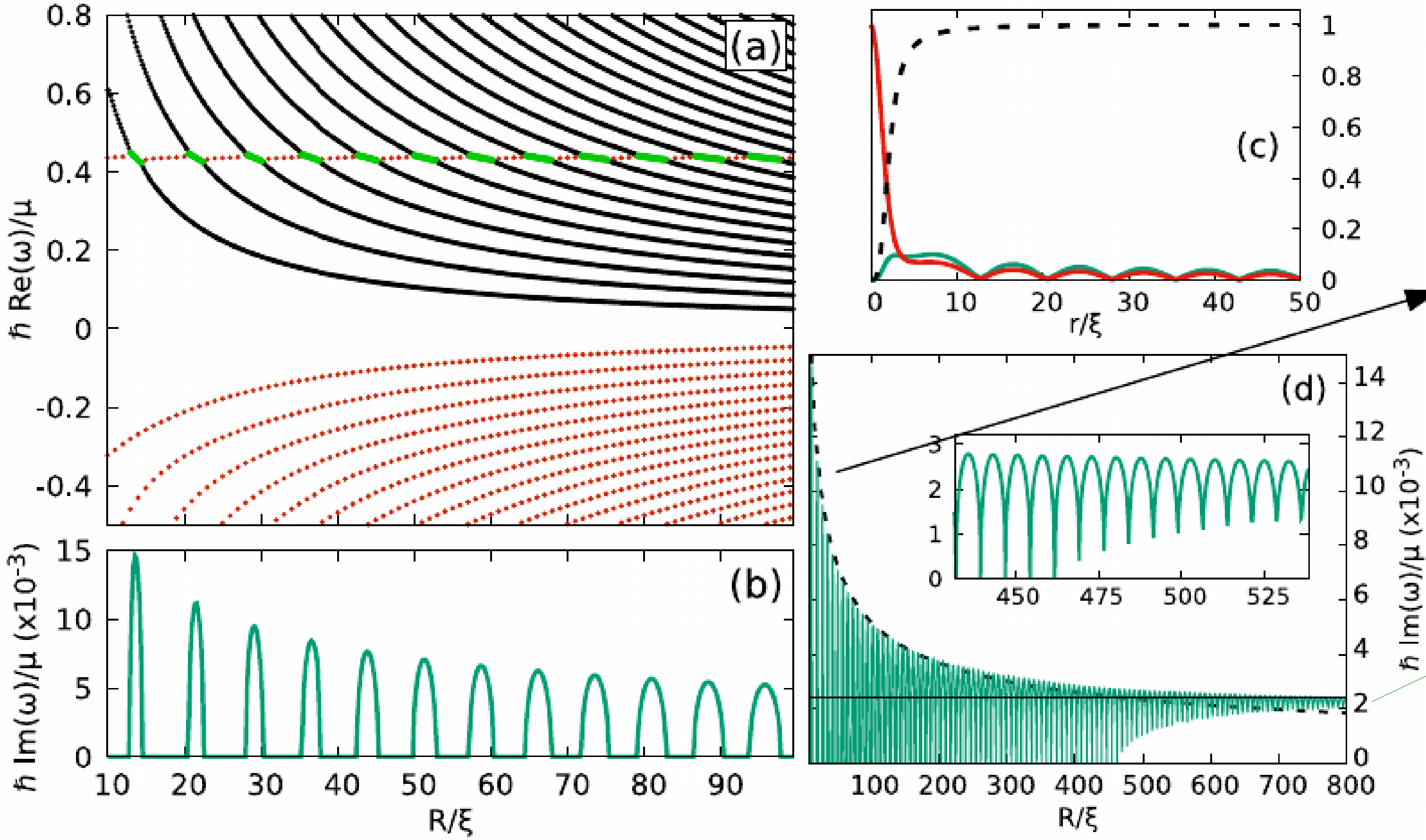
Complex combination of effects:

- In a BEC quantized vortex of charge ℓ ergoregion at $r_E \sim \ell\xi$
- A trap gives a reflecting boundary condition
- Hydrodynamic approximation does not hold!

Large (but finite) system size limit R→ ∞

- Ground state of the radial GPE with Neumann BC at some r=R
- Diagonalize the Bogoliubov problem varying R at fixed m

$$\ell = 2, \ m = 2$$



$1/\sqrt{R}$

As an emitter in a cavity

Tends to constant Instability rate For R→ ∞

L. Giacomelli & IC, *Ergoregion instabilities in rotating two-dimensional Bose-Einstein condensates: Perspectives on the stability of quantized vortices*, Phys. Rev. Res. 2020

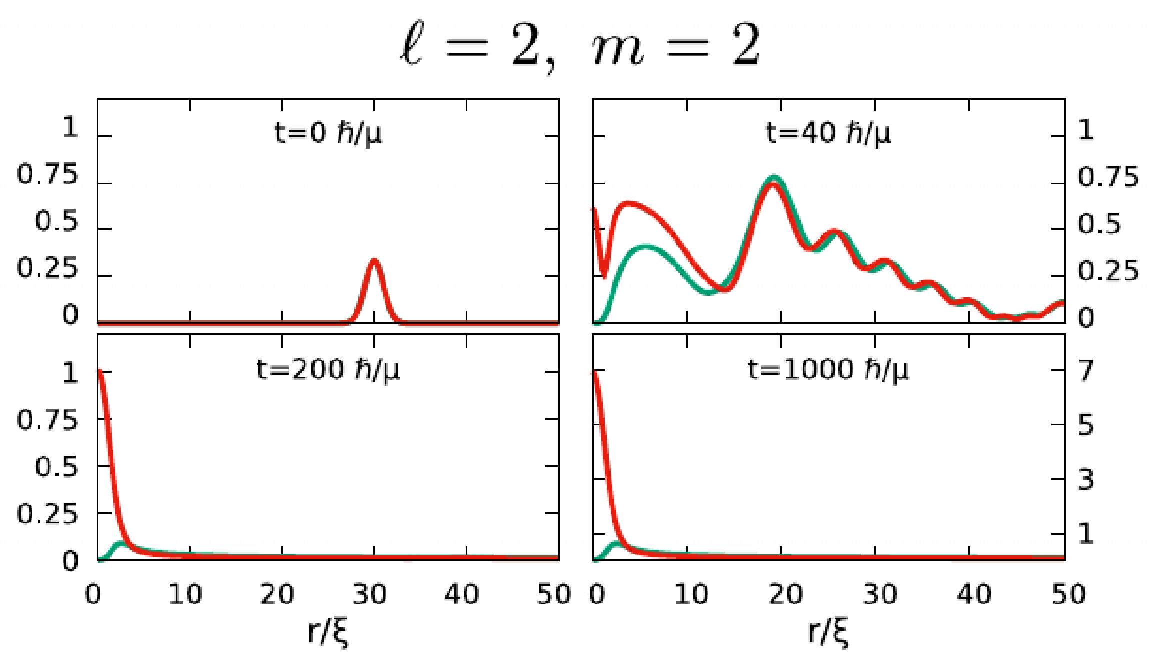
Spatially infinite fluid with a vortex

Impose absorbing boundary conditions at large r: direct simulation of untrapped fluid

In-going phonons: naturally part of stationary waves eigenmodes for finite R, removed with ABC

Time-dependent study of Bogoliubov problem with ABC:

- incident wavepacket scatters on vortex
- triggers dynamical instability of charge-2 vortex



Dynamic instability of a charge 2 vortex is intrinsic and not related to boundary conditions
Ergoregion instability of central core region

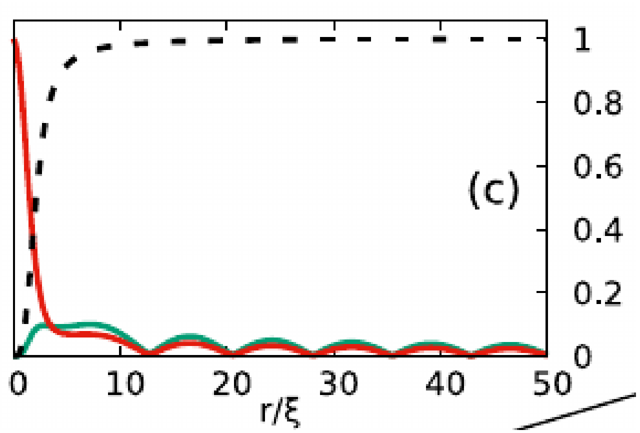
L. Giacomelli & IC, *Ergoregion instabilities in rotating two-dimensional Bose-Einstein condensates: Perspectives on the stability of quantized vortices*, Phys. Rev. Res. 2020

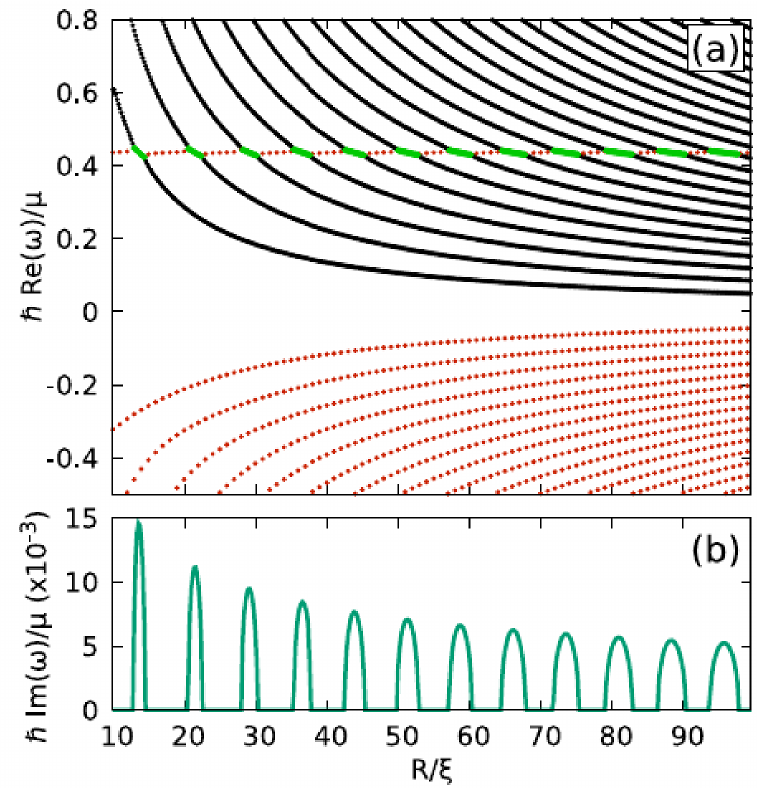
Physical interpretation

Positive frequency, negative norm mode dual to negative frequency, positive norm mode

(Quasi)-bound mode in the core region:

- negative frequency, independent of R
- wavefunction localized in in ergoregion
- mathematically equiv to
  harmonic oscillator with $m<0$ and $k<0$

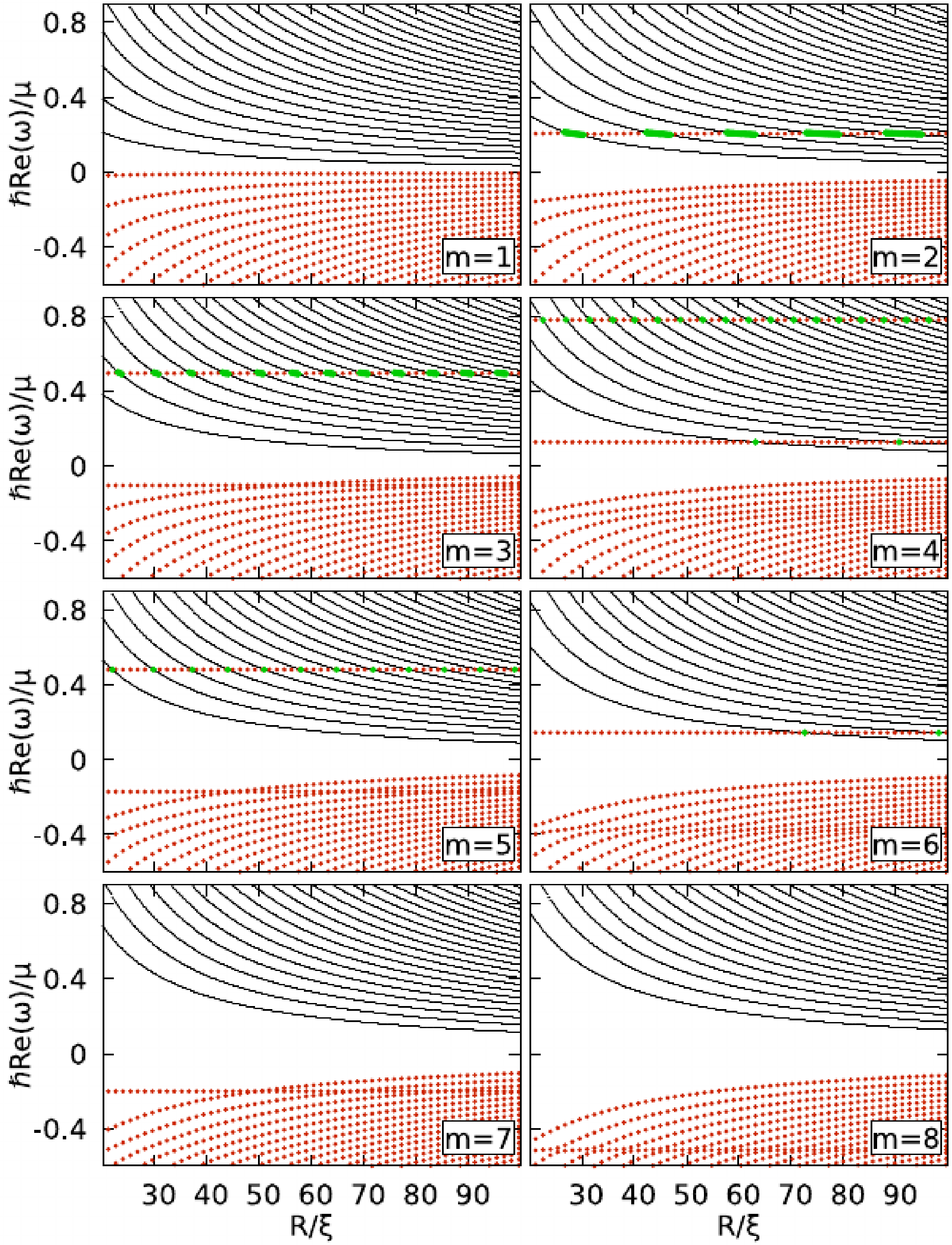




- Localized mode mixed with opposite norm scattering modes
- Band-sticking at every intersection → dynamical instability
- Emission of pairs stimulated by (negative) energy accumulated in ergoregion
  → ergoregion instability
- Instability possibly quenched by destructive interference in spatially finite systems

L. Giacomelli & IC, *Ergoregion instabilities in rotating two-dimensional Bose-Einstein condensates: Perspectives on the stability of quantized vortices*, Phys. Rev. Res. 2020

Higher-charge vortex with l=4

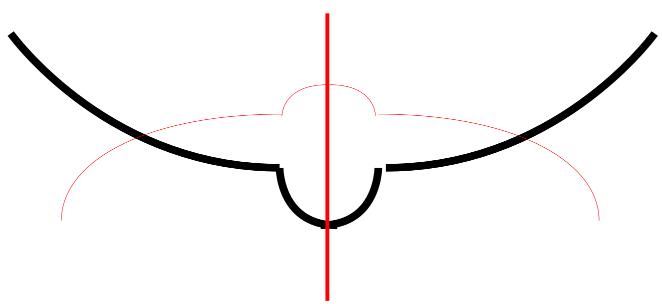


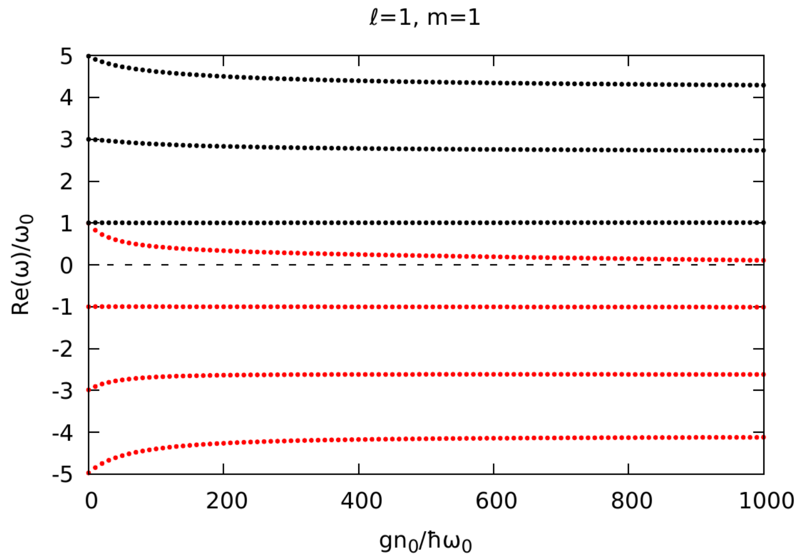
L. Giacomelli & IC, *Ergoregion instabilities in rotating two-dimensional Bose-Einstein condensates: Perspectives on the stability of quantized vortices*, Phys. Rev. Res. 2020

Back to *singly-charged vortex*: always stable?

<u>Harmonically confined cloud</u>

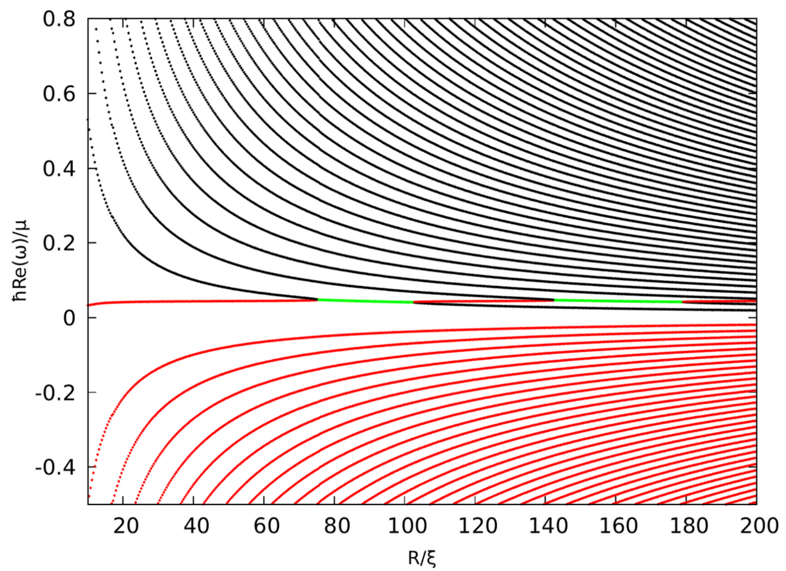
- Same trap frequency ω_0 sets frequency of localized mode and of (discrete) collective excitations
- No intersection → energetic but not dynamic instability
- Is this result fully general ??





<u>Harmonic trap plus localized energy minimum:</u>

- Quasi-continuum of phonon-like collective excitations
- Intersect with localized mode → dynamic instability
- Physically: vortex spirals out while radiating phonons



L. Giacomelli & IC, *Ergoregion instabilities in rotating two-dimensional Bose-Einstein condensates: Perspectives on the stability of quantized vortices*, Phys. Rev. Res. 2020

Part 5:

The new frontier: back-reaction effects

towards BH evaporation

The little I understand about back-reaction in astrophysics, cosmology & quantum gravity

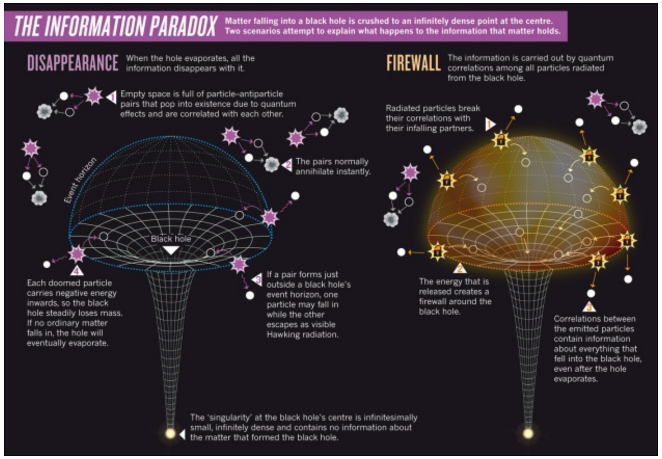


Figure from Nature, 496 (2013)

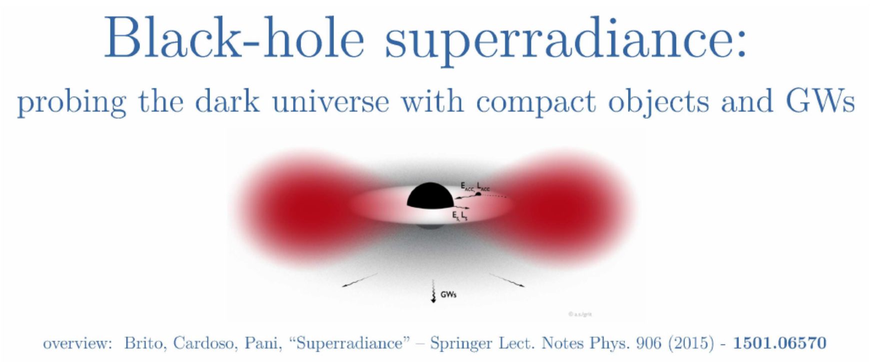
What is the long-term fate of a BH ?

HR carries away energy, so BH horizon must (slowly) shrink to conserve energy/mass

- What is left once BH evaporated?
- Is there any remnant of what has fallen into the BH ?

From cosmology/particle physics perspective:

- what basic physics learnt from BH instabilities?
- BH hairs stabilized by backreaction?
  What GW emission can be observed?



From P. Pani's talk @ Solvay, 2019

Our approach:

- Analog models simulate QFT on curved space-time…
- ...but Einstein eqs. (coupling of matter/energy to metric) not implemented

Still, any hint from higher order couplings of quantum fluctuations to macroscopic flow?

What can a quantum optician's point of view teach on this physics?

A simplified model:

back-reaction of quantum emission onto breathing mode of elongated BEC

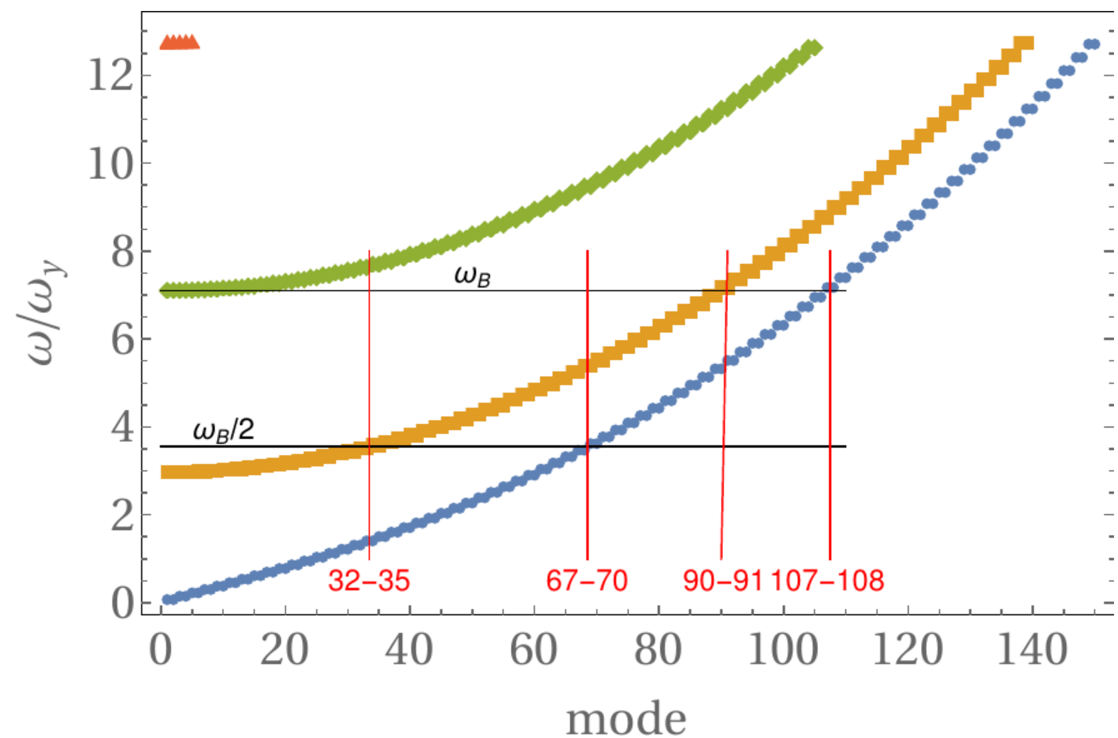
Preliminary results, to appear soon as
S. G. Butera - IC, *Backreaction in an analog model of pre-heating* (2022)

Previous related works on back-reaction in circuit-QED:

- S. G. Butera & IC, *Mechanical back-reaction effect of the dynamical Casimir emission*, Phys.Rev. A 99, 053815 (2019)
- S. G. Butera & IC, *Quantum fluctuations of the friction force induced by the dynamical Casimir emission,* EPL 128, 24002 (2020)

Excitation modes of an elongated BEC

Elongated BEC with $\omega_{xy} >> \omega_z$



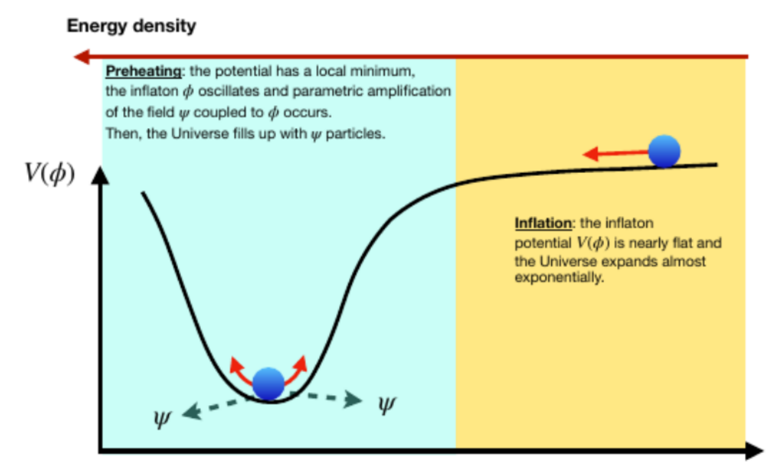
•Breathing mode:
Cylindrically symmetric around axis

•Dipole mode:
Lateral displacement of BEC

•Goldstone zero-sound mode:
Slow axial twist of BEC phase

Each mode → well-defined k_z under PBC

Analog model of cosmological pre-heating:
Inflaton (breathing mode) oscillates around potential minimum, decays via emission of pairs of particles (entangled pair of dipole excitations)



Inspired to: Robertson, Michel, Parentani, *Nonlinearities induced by parametric resonance in effectivel 1D atomic BECs,* Phys. Rev. D 98, 056003 (2018)

Our numerical simulations

Simplifying assumptions:

- effective 1+1D model
- untrapped BEC with periodic boundary conditions along z

Truncated Wigner approximation for dynamics:

$\rightarrow$ GPE with stochastic initial conditions

Start from T=0 ground state

Around $t=t_0$: kick $\omega_{xy} \rightarrow$ excite breathing mode @ $k_z=0$

Observe evolution of population in different modes

Bogoliubov collective eigenmodes

- nonlinear terms give inter-mode coupling
- can be triggered by zero-point quantum fluctuations

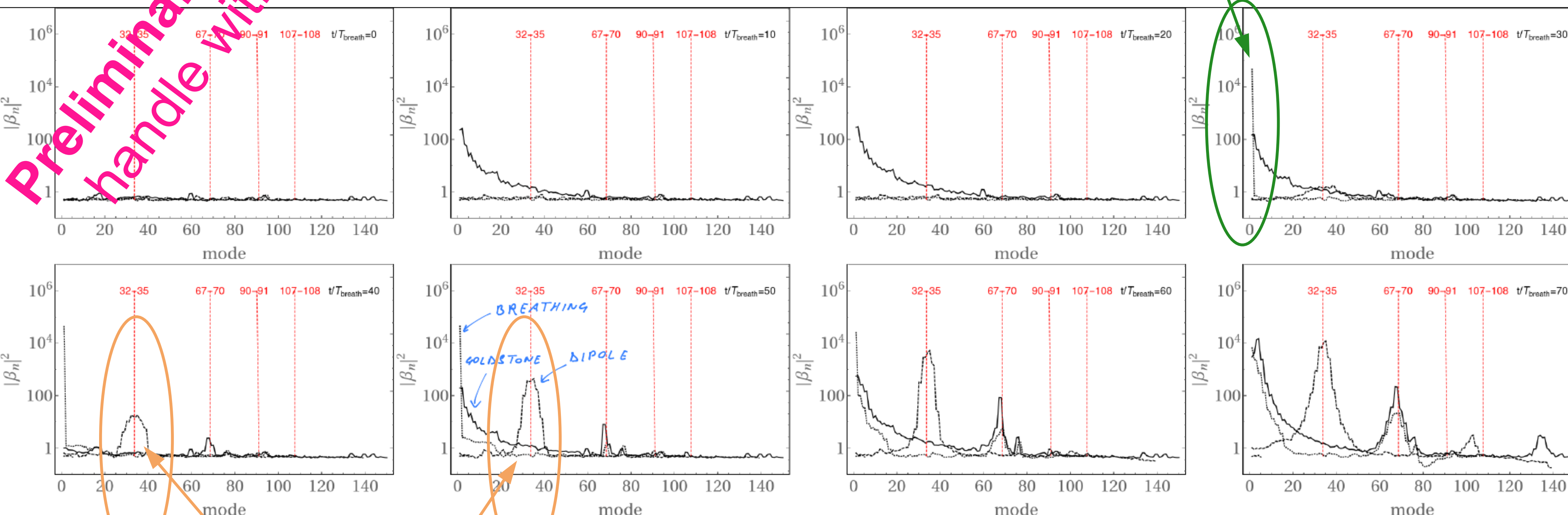
Watch out for pitfalls of Truncated Wigner method: Sinatra-Lobo- Castin 2002; van Regemortel et al., PRA 2017

Parametric excitation of dipole mode

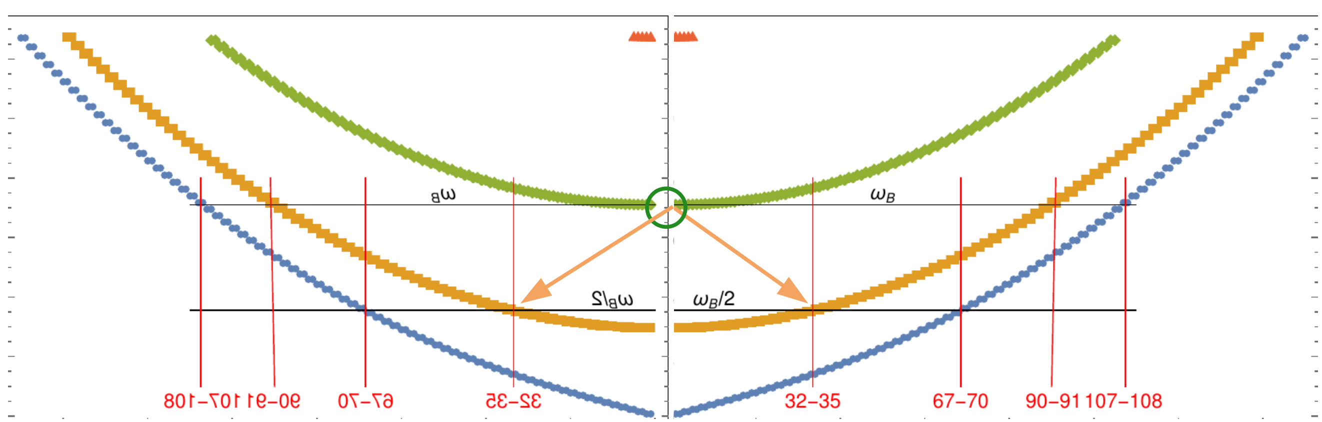
Preliminary results
handle with care

Initial state: BEC at T-0

At $t = t_0$: Excitation of $k_z = 0$ breathing mode

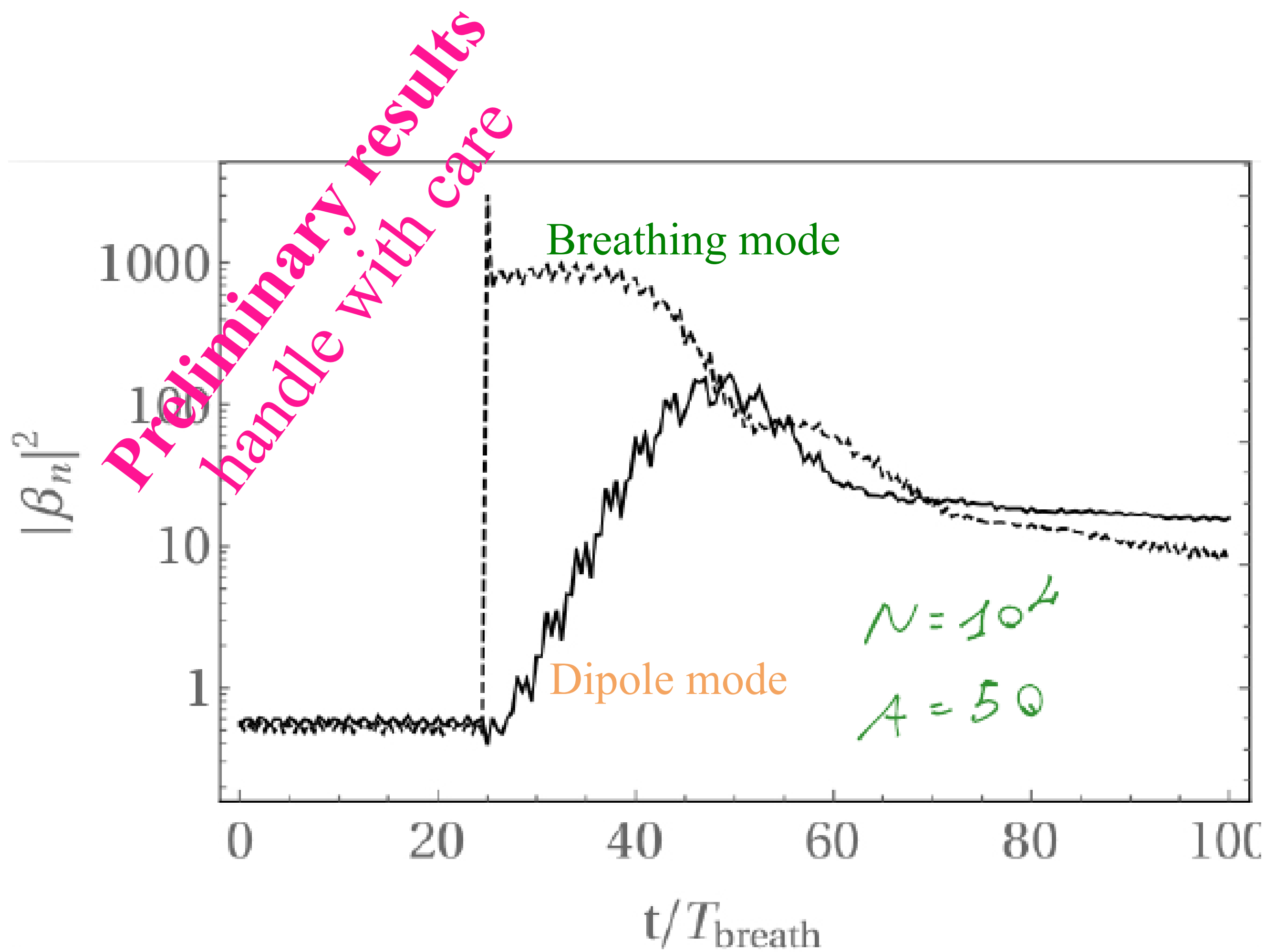


Growing excitation of dipole mode
finite window of k_z modes excited



Backreaction on breathing mode

- Breathing mode slowly decays
- Dipole mode stops growing
- (quasi-) periodic exchange of energy

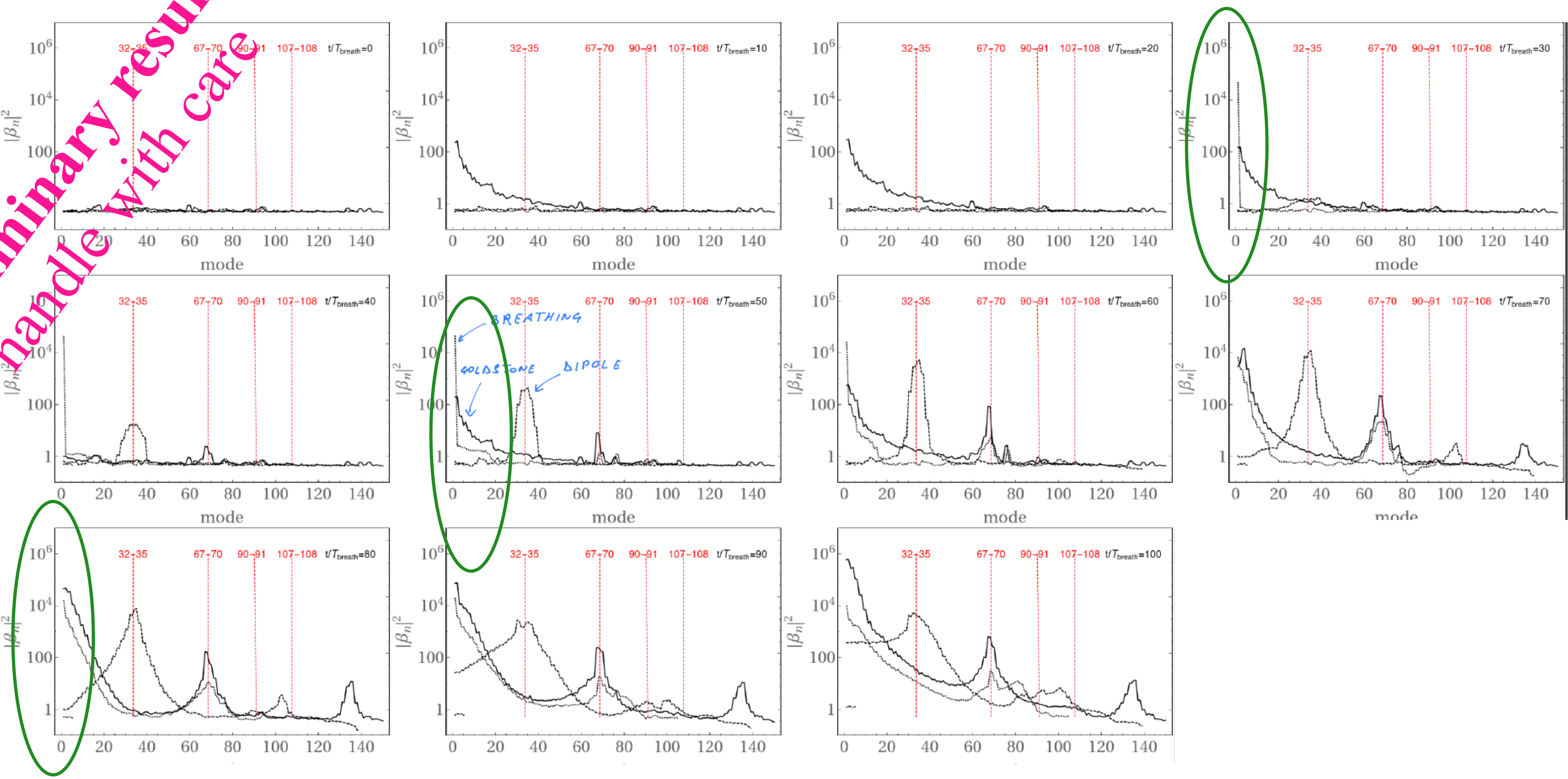


Backreaction of dipole mode on breathing mode

Quantum emission of dipole excitation induces effective friction force on the breathing mode

A subtle feature of backreaction 1/2

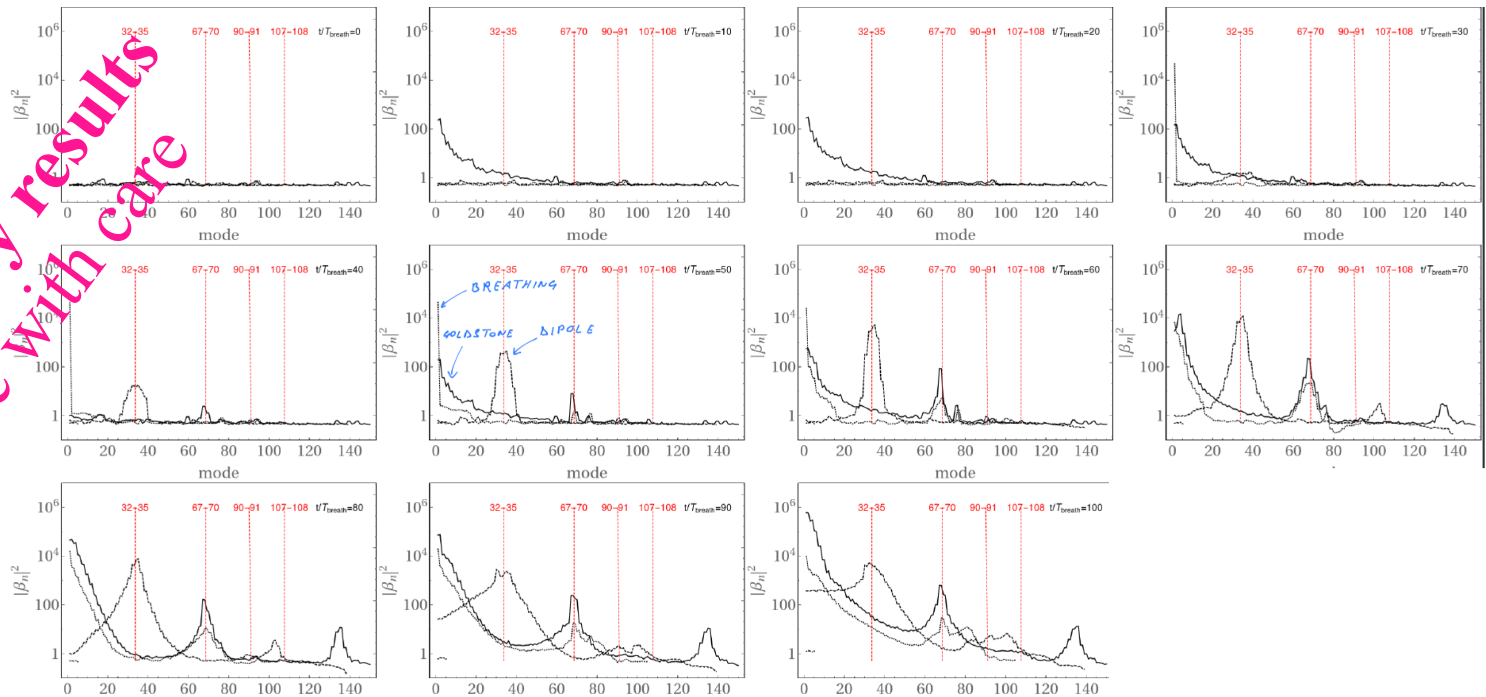
Preliminary results
handle with care



Breathing mode distribution along k_z:

- initially, δ-like excitation, uniform along whole system
- then, loses coherence and acquires strong fluctuations

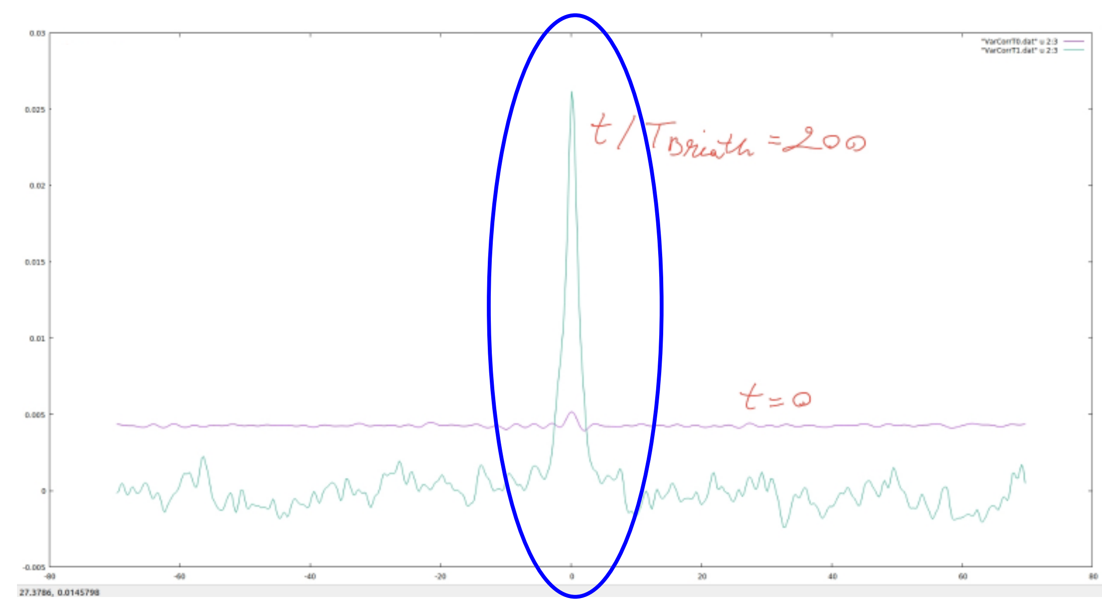
A subtle feature of backreaction 2/2



Preliminary results
handle with care

Breathing mode distribution along k_z:

- initially, δ-like excitation, uniform along whole system
- then, loses coherence and acquires strong fluctuations



- Mechanism goes beyond standard picture of backreaction $R_{\mu\nu} - R\, g_{\mu\nu}/2 = 8\,\pi\, G < T_{\mu\nu} > / c^4$ where only average value of quantum field stress-energy tensor matters

- What are consequences for gravity and cosmology?

Conclusions & perspectives

superfluids of ultracold atoms & light ↔ gravitational phenomena

a fruitful and bidirectional synergy !

Hawking radiation from Black Holes:

- Original theoretical predictions + $g^{(2)}(x,x')$ correlations → experimentally observed
- On-going challenge → robust evidence of quantum correlations in HR → *quantum hydrodynamics*
  "The tale of Navier and Stokes meeting Heisenberg at Hawking's place"

Superradiance:

- Analog models provide microscopic understanding of superradiant emission & instabilities in gravity
- Gravity shines light on classical superfluid hydrodynamics problem of vortex stability

Quantum back-reaction:

- Elongated BEC simulates cosmological dynamics. Back-reaction → friction of transverse modes

Astrophysical/cosmological/quantum gravity consequences:

- Cosmological pre-heating → quantum fluctuation of inflaton field. Observable in CMB ?
- Analog Hawking radiation from BHs → intrinsic fluctuations of space-time around BH ?

We acknowledge generous financial support from:

PROVINCIA AUTONOMA DI TRENTO

Q@TN

JSF Julian Schwinger Foundation

MIR-BOSE

European Commission

Horizon 2020 European Union funding for Research & Innovation

PhoQuS Photons for Quantum Simulation

If you wish to know more...

Superradiant phenomena

Lessons from and for Bose–Einstein condensates

Luca Giacomelli

Ph.D. thesis submitted to Dipartimento di Fisica
Università degli studi di Trento

Under the supervision of
Dr. Iacopo Carusotto
Prof. Massimiliano Rinaldi

Living Reviews in Relativity
December 2011, 14:3 | Cite as

Analogue Gravity

Authors | Authors and affiliations

Carlos Barceló, Stefano Liberati, Matt Visser

Open Access | Review Article
Latest version | View article history
First Online: 11 May 2011

157 Citations 2 Shares

PhD positions hopefully available soon.
Don't hesitate contacting me!
iacopo.carusotto@unitn.it

news & views

QUANTUM HYDRODYNAMICS

Acoustic Hawking radiation

A milestone for quantum hydrodynamics may have been reached, with experiments on a black hole-like event horizon for sound waves providing strong evidence for a sonic analogue of Hawking radiation.

Iacopo Carusotto and Roberto Balbinot

Nat. Phys., Aug.15h, 2016

Come and visit us in Trento!

Dedicated to a friend and a master

Renaud Parentani, July 31, 1962 - May 20, 2020

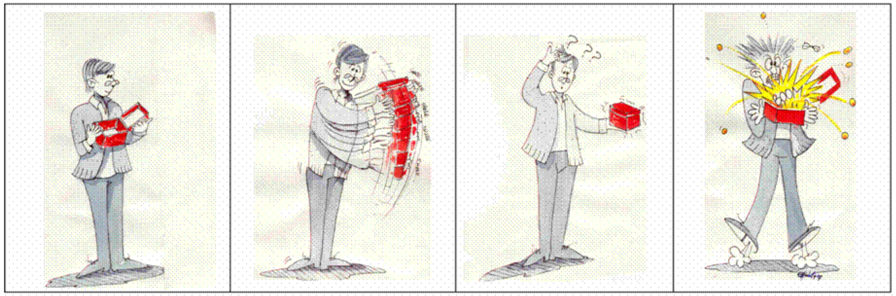
Part 5.a

A simplest toy model:
Dynamical Casimir Emission
in single-mode cavity

S. G. Butera & IC, *Mechanical back-reaction effect of the dynamical Casimir emission*, Phys.Rev. A 99, 053815 (2019)

S. G. Butera & IC, *Quantum fluctuations of the friction force induced by the dynamical Casimir emission,* EPL 128, 24002 (2020).

Back-reaction effect of dynamical Casimir emission



Take an optical cavity in the e.m. vacuum state

Mechanically shake it very fast

Beware when you open it again: (a few) photons may burn you !!

Simplest configuration:

- Half-space slab of refractive index n and mass M
- Mechanically oscillating at frequency Ω

- Prediction for the dissipated energy within 1D scalar model:

$$Q^{-1} = \frac{\tau}{2\pi E_{osc}} \frac{dE_{diss}}{dt} = \frac{1}{6}\left(\frac{n-1}{n}\right)^2 \frac{\hbar\Omega}{Mc^2}$$

(from Barton and Eberlein, Ann. Phys. 227, 222 (1993))

→ value is ridiculously small

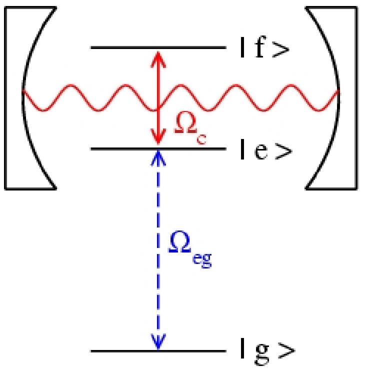
→ experimental observation by mechanical means with bulk objects appears hopeless, but quantum optics gives new hopes...

All-optical back-reaction effect

PHYSICAL REVIEW A **85**, 023805 (2012)

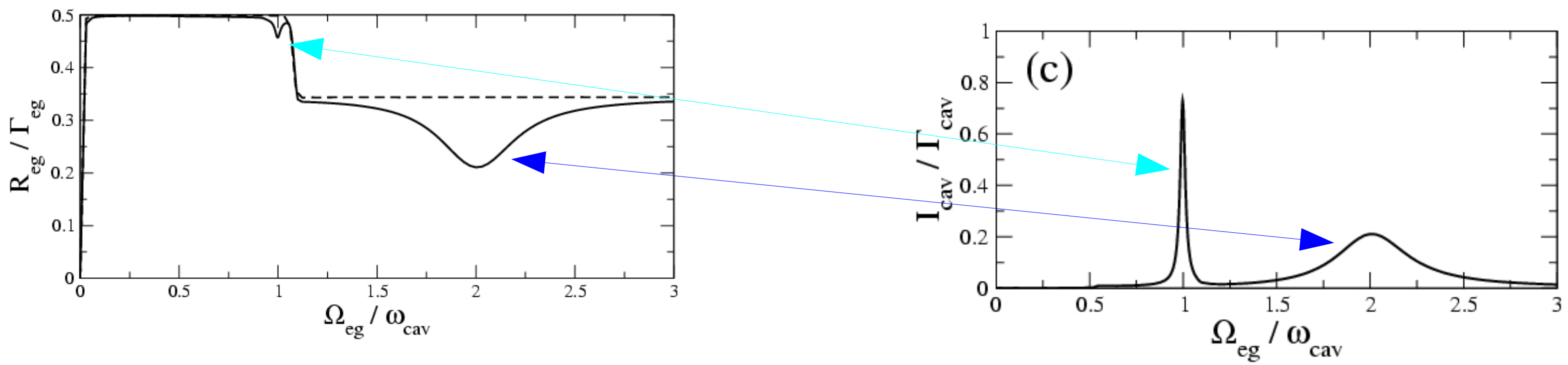
Back-reaction effects of quantum vacuum in cavity quantum electrodynamics

I. Carusotto,[1,*] S. De Liberato,[2] D. Gerace,[3] and C. Ciuti[2]



Coherently-driven 3-level emitter embedded in optical cavity

- Drive laser on g ↔ e transition → Rabi oscillations at Ω_R, cavity periodically modulated
- Generates DCE emission, strongest when Ω_R resonant with cavity
- Absorption of drive laser: $R_{eg} = 2\Omega_{eg}\,\mathrm{Im}\{\mathrm{Tr}[\hat{c}_{eg}^{\dagger}\,\rho_{ss}]\}.$
- Peaks in DCE give dip in absorption: stronger "friction" reduces absorption rate



- Feasible with optical or μ-wave (circuit-QED) techniques

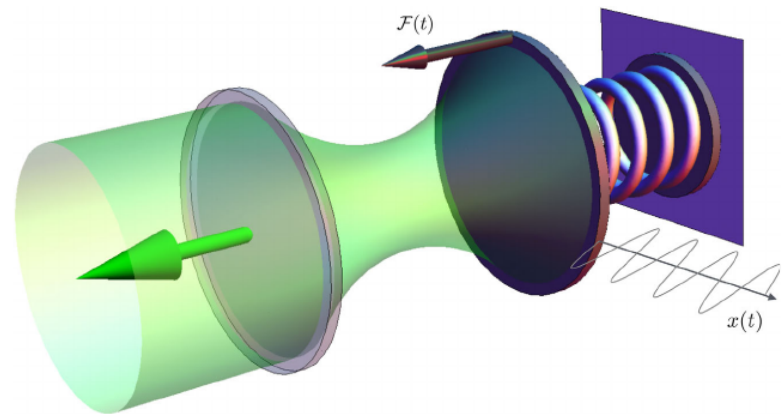
An even simpler toy-model

Single-mode optical cavity *a*

Mirror mounted on mechanically moving part
with harmonic restoring force *b*

Opto-mechanical coupling via radiation pressure
on mirror or length-dependent shift of cavity resonance

$$\hat{H} = \hbar\omega_0 \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} + \hbar\omega_c \left(\hat{a} + \hat{a}^\dagger\right)^2 \left(\hat{b} + \hat{b}^\dagger\right)$$



If $\omega_b \sim 2\omega_a$, dynamical Casimir emission (with time-indep. H)
energy transferred from mechanical to optical field

Simple on paper, a bit harder in experiment:

- generally mechanical frequencies << optical frequencies
- appears feasible in μ-waves with recent GHz acoustics experiments
  (e.g. Schoelkopf's group, Science 2017)

PHYSICAL REVIEW X **8**, 011031 (2018)

Nonperturbative Dynamical Casimir Effect in Optomechanical Systems: Vacuum Casimir-Rabi Splittings

Vincenzo Macrì,[1,2] Alessandro Ridolfo,[2] Omar Di Stefano,[2] Anton Frisk Kockum,[2] Franco Nori,[2,3] and Salvatore Savasta[1,2]

Circuit-QED observation of DCE

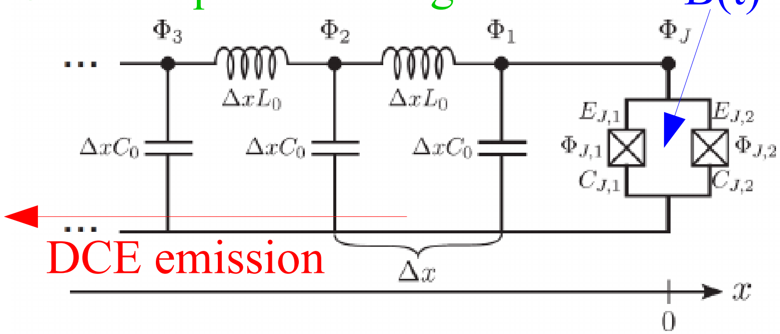
LETTER

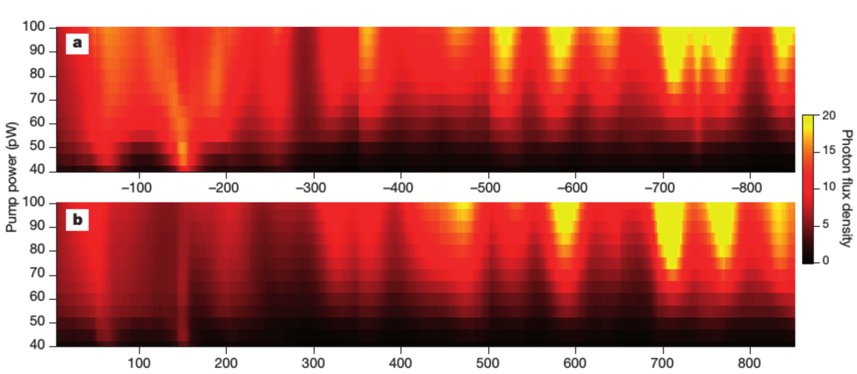
doi:10.1038/nature10561

Observation of the dynamical Casimir effect in a superconducting circuit

C. M. Wilson[1], G. Johansson[1], A. Pourkabirian[1], M. Simoen[1], J. R. Johansson[2], T. Duty[3], F. Nori[2,4] & P. Delsing[1]

Circuit equivalent diagram:





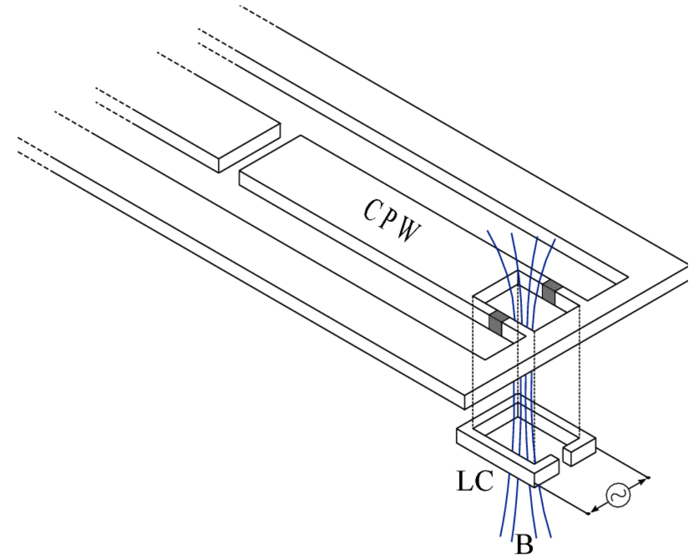
- Co-planar waveguide (CPW) for microwaves terminated on SQUID
- Effective mirror position controlled via B-field threaded through SQUID
- Modulation of B(t) leads to DCE
- Observed in radiation that propagates along CPW, quantum correlations established

- When waveguide closed by second mirror, hard to observe DCE:
  - quickly above parametric threshold (Wilson et al., PRL 2010)
  - classical emission loses quantum features

The mirror as an independent DoF

B-field generated by LC circuit concatenated to SQUID

- LC circuit → mechanical oscillator
- DCE effect → coplanar waveguide



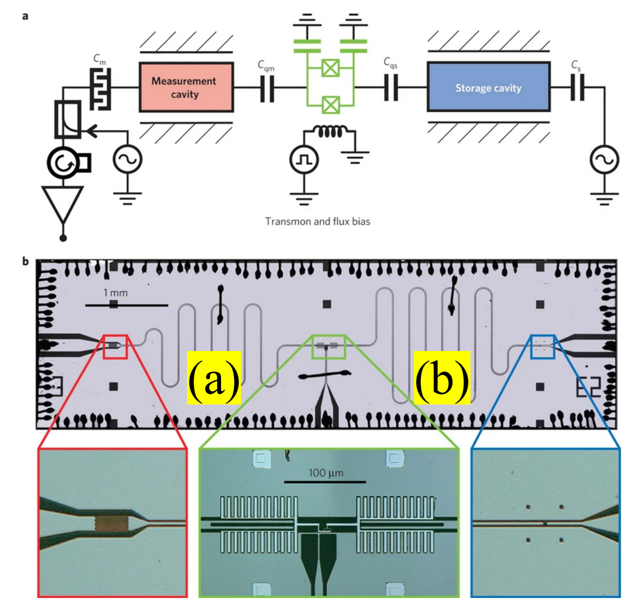
To enhance DCE & back-reaction effect:

- close CPW with second mirror to create cavity and resonantly enhance DCE
- Back-reaction of DCE expected to be visible as additional dissipation on LC circuit
- To be electronically probed on the LC dynamics

- Estimated single-quantum coupling ~10kHz, not far from typical decay

Another useful configuration (to exploit 2nd-hand samples):

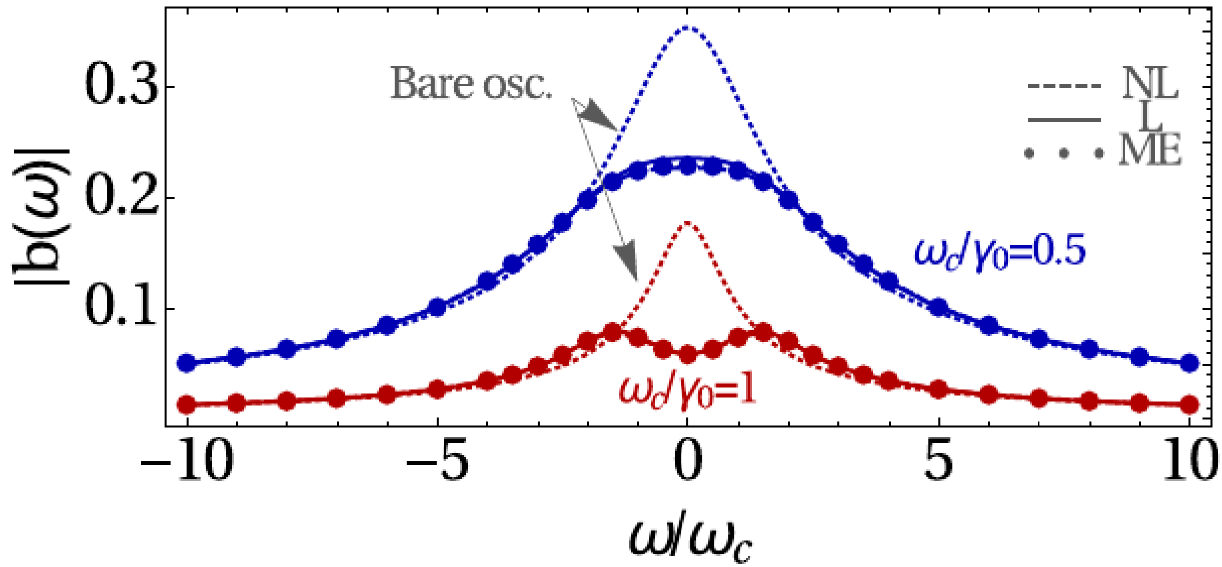
- Two (a,b) cavities, connected by cross-Kerr Josephson element
- Send μw's into (b) to modulate effective length of (a)
- Watch DCE emission into (a), backreaction in (b)



Sketch from Johnson et al, Nat. Phys. 2010

S. G. Butera & IC, *Mechanical back-reaction effect of the dynamical Casimir emission*, Phys.Rev. A 99, 053815 (2019)
Further steps in collaboration with *DartWars* INFN project – P. Falferi, A. Vinante, C. Gatti

Response of LC to external monochromatic drive



DCE results in broadened resonance by $\gamma_{DCE} \sim 2\ \omega_c^{\ 2} / \gamma$

Strong DCE coupling $\omega_c > \gamma$ gives nonlinear Rabi splitting of resonance

$$\hat{H} = \hbar\omega_0 \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} + \hbar\omega_c \left(\hat{b}^\dagger \hat{a}^2 + \hat{b} \left(\hat{a}^\dagger \right)^2 \right)$$

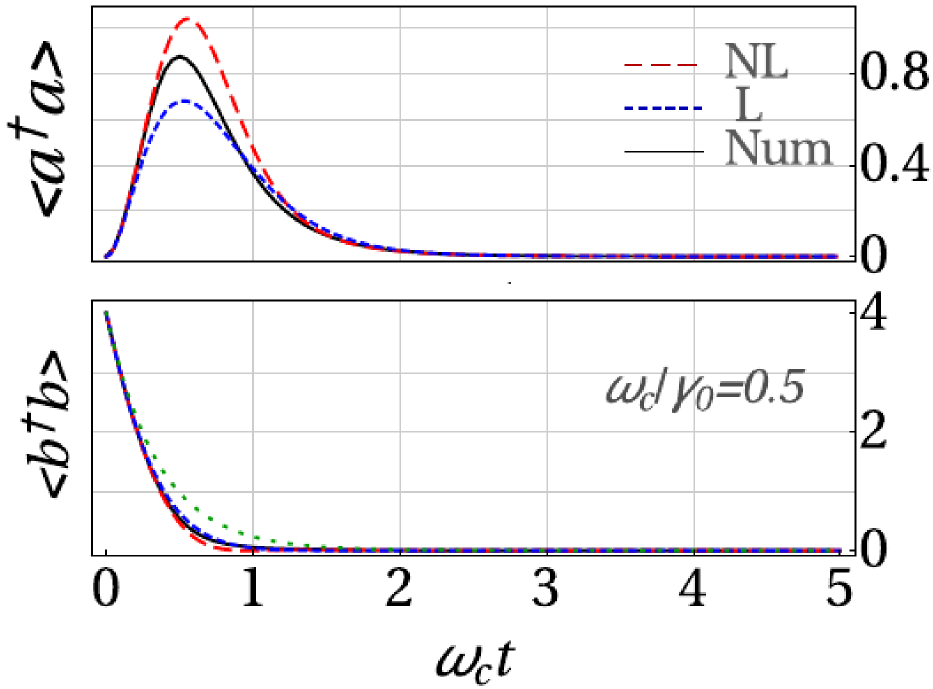
Next steps:

- extend the calculation to open CPW where DCE is broadband and no resonant enhancement of DCE
- design optimal set-up and try the experiment
  (Q@TN collab. A. Vinante, B. Margesin, P. Falferi, G. Casse -Trento)

Q@TN

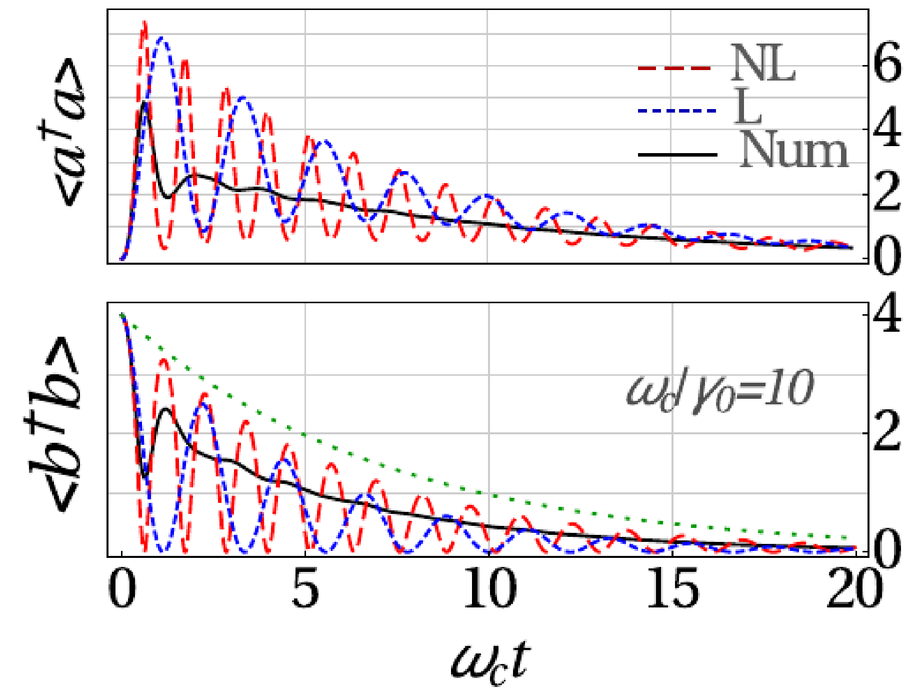
S. G. Butera & IC, *Mechanical back-reaction effect of the dynamical Casimir emission*, Phys.Rev. A 99, 053815 (2019).

Free evolution after initial kick of LC



Weak DCE coupling

reinforced decay due to DCE emission



Strong DCE coupling

periodic exchange of energy

[also in Macrì et al., PRX 2018]

S. G. Butera & IC, *Mechanical back-reaction effect of the dynamical Casimir emission*, Phys.Rev. A 99, 053815 (2019).

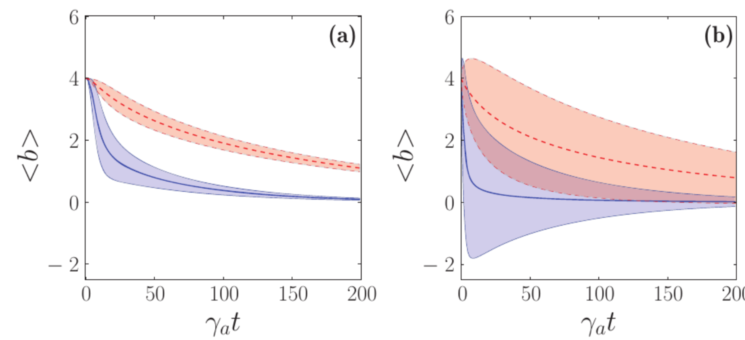
Quantum fluctuation effects

Numerical integration of Master Equation

- Temporal decay of the mechanical oscillations by DCE friction
- Quantum fluctuations (shading) much larger in non-resonant case

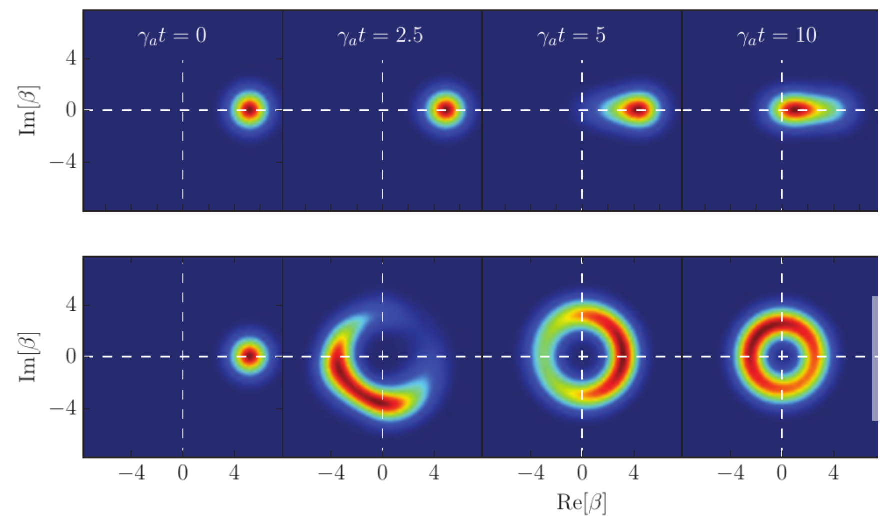
Resonant DCE

Slightly non-resonant DCE



Phase space interpretation:

- Resonant → fluctuations in DCE damping force
- Non-resonant → fluctuations in DCE frequency shift



Resonant

Slightly non-resonant

Fluctuations are experimentally accessible in circuit-QED by measuring quantum state of LC cicuit

S. G. Butera & IC, *Quantum fluctuations of the friction force induced by the dynamical Casimir emission,* EPL 128, 24002 (2020).

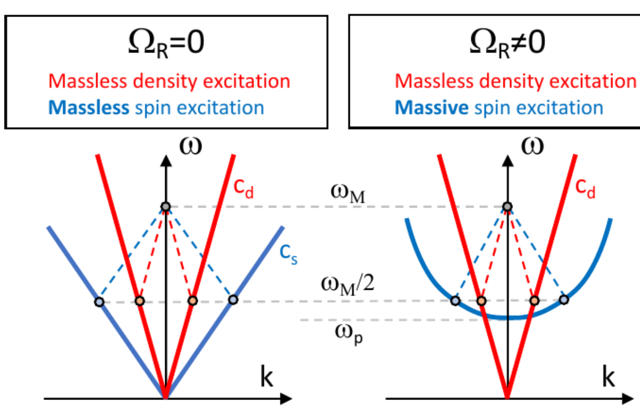
Part 4:

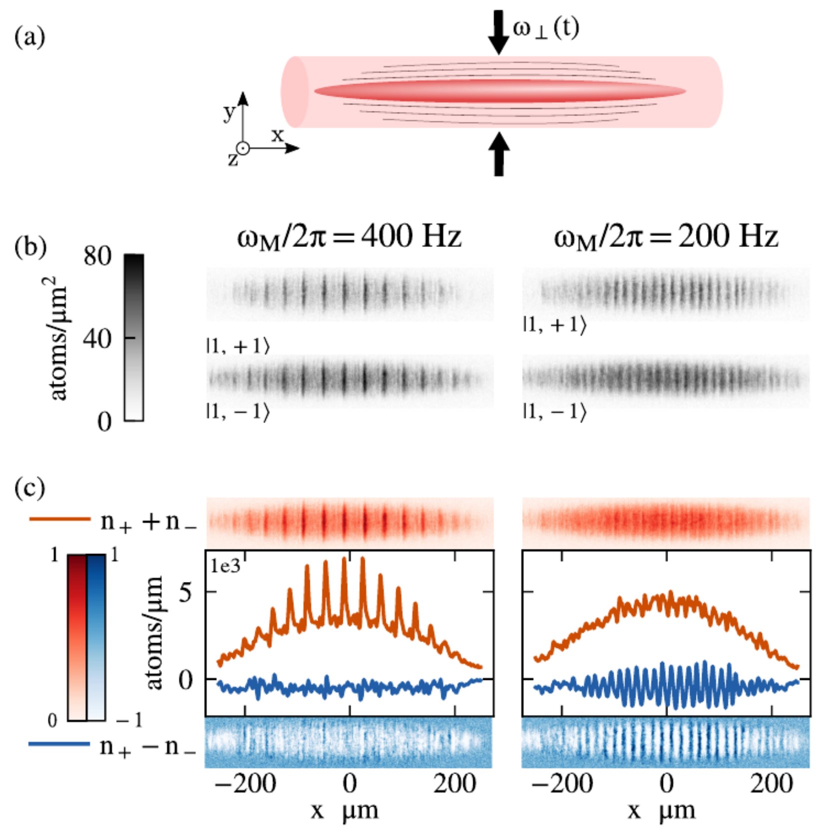
On-going experiments in Trento

On-going experiments in Trento (I)

Spinor BEC of Na atoms:

- density and spin modes:
- different sound speeds
- spin mode gapped with µw coupling Ω_R
- excited by shaking radial confinement, emission of Faraday waves
- measured in density/magnetization

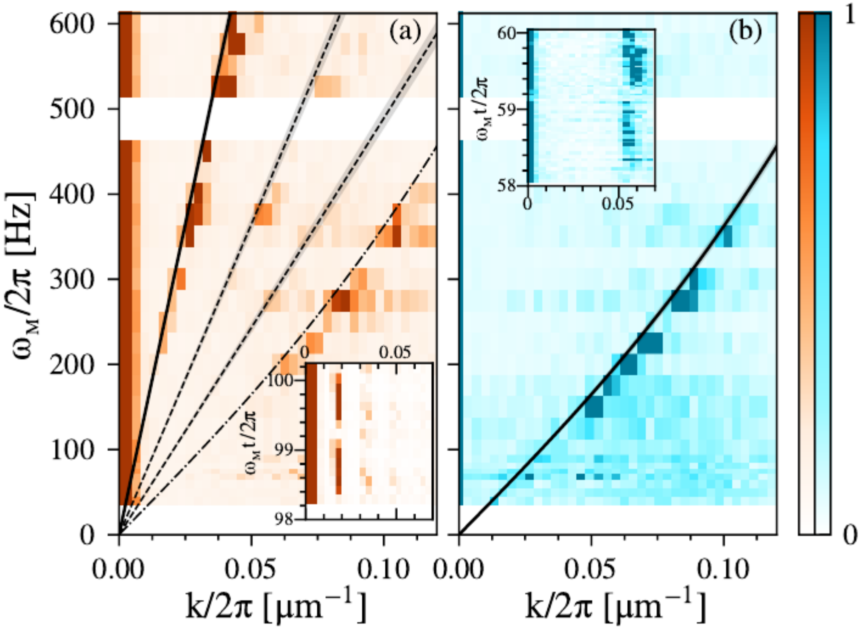




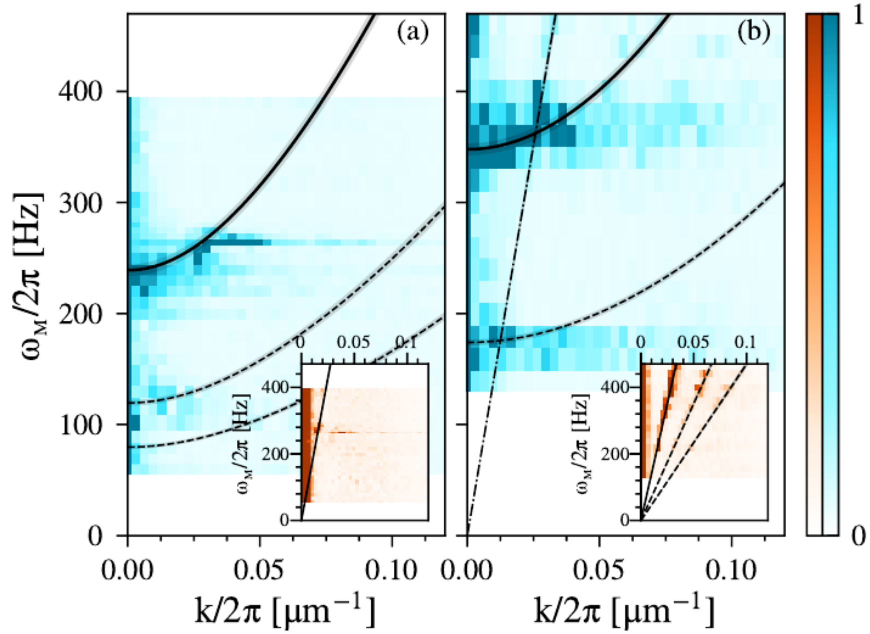
Cominotti, Berti… Lamporesi, IC, Recati, Ferrari, arXiv:2112.09880, accepted on PRL.

On-going experiments in Trento (II)

No coherent coupling $\Omega_R=0$
massless spin mode



Coherent coupling $\Omega_R\neq0$
massive spin mode



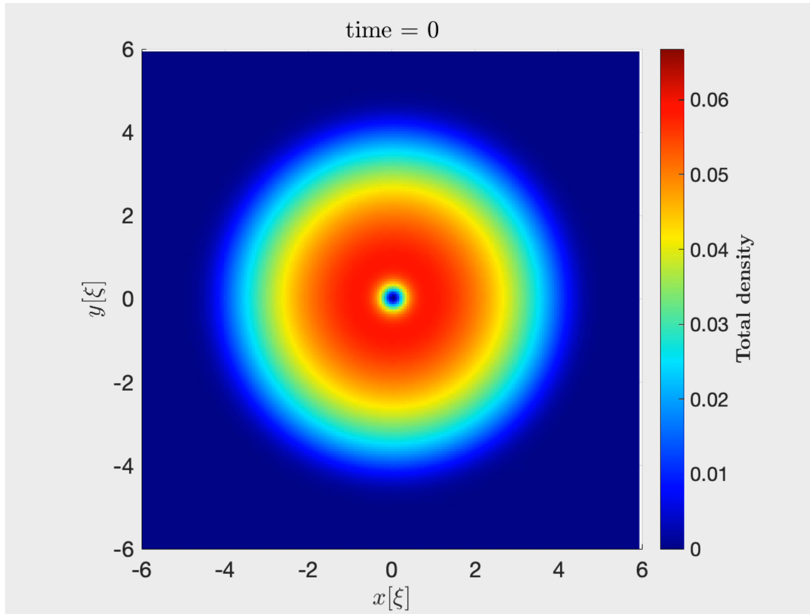
Density- and spin-mode dispersions measured via response to transverse shaking

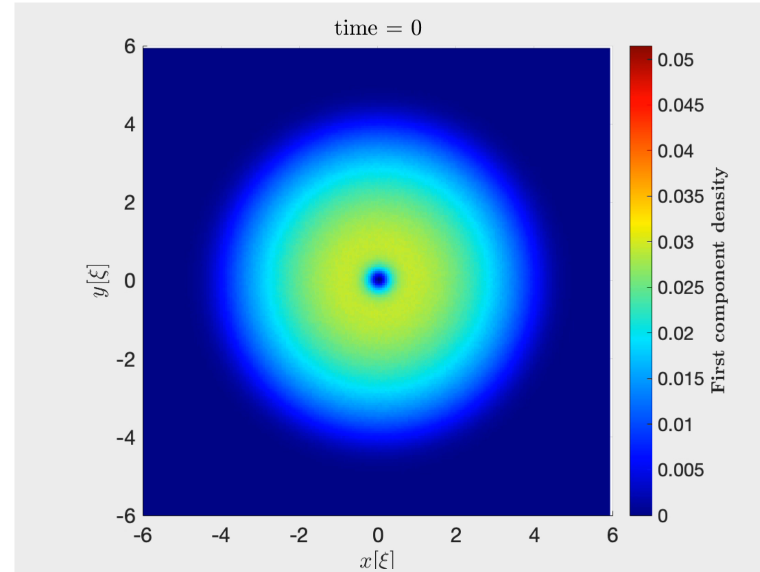
Next steps: quantum features in parametric emission, ergoregion instability of vortices

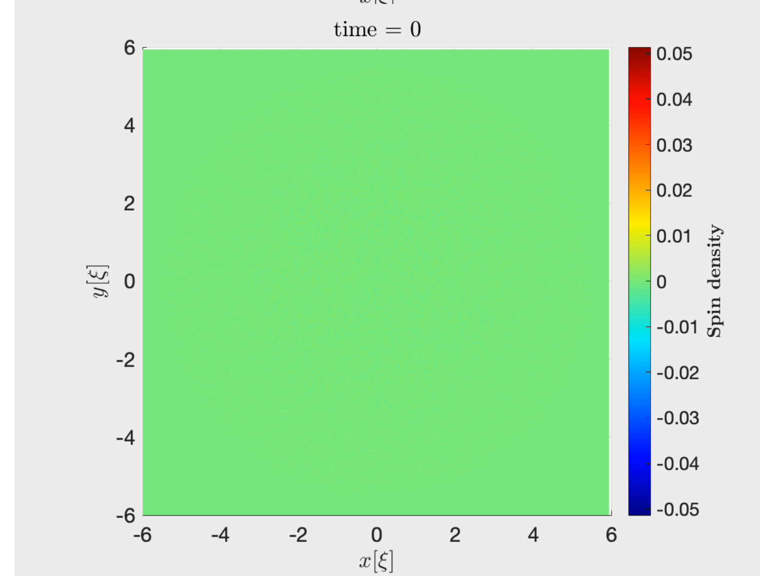
Cominotti, Berti… Lamporesi, IC, Recati, Ferrari, arXiv:2112.09880, accepted on PRL.

Ergoregion instability of vortices in spinor BEC

Anna Berti







Numerical solution of time-dependent GPE:

- Two-component BEC
- Density-sound faster than spin-sound
- Spin-ergoregion >> vortex core
  → ergoregion instability of spin modes
- Q=1 vortex splits into a pair of vortices, one per spin component

A. Berti and IC, in preparation (2022)

I can not refrain myself… quasi-BEC & KPZ effects in non-eq BEC

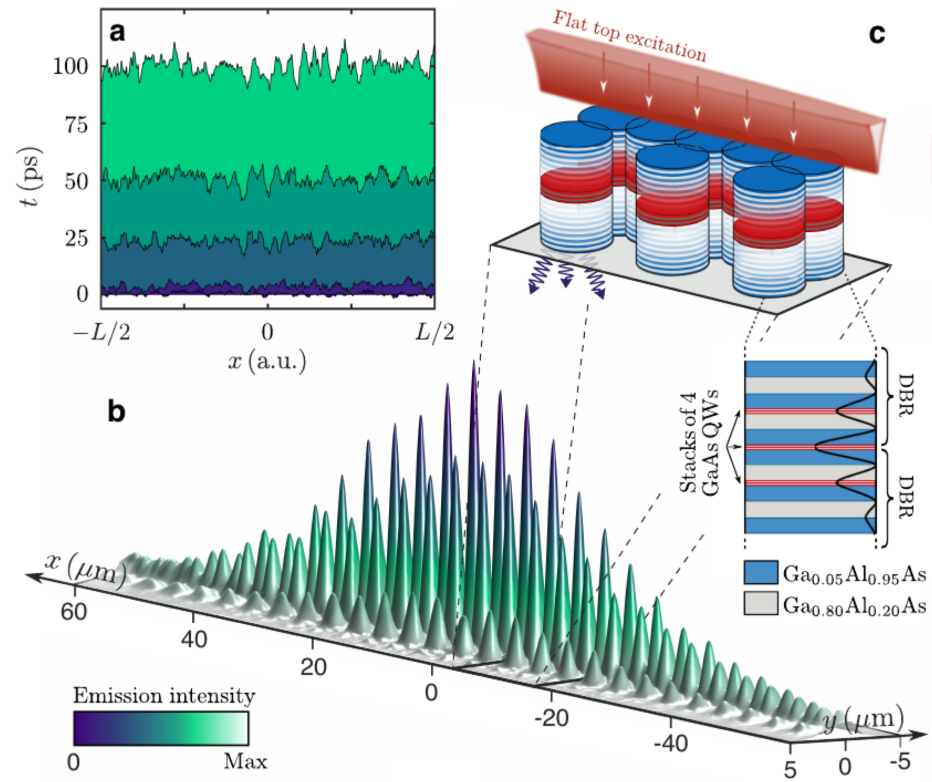
Quasi-condensation features:

Hohenberg-Mermin-Wagner theorem:

- at equilibrium, no BEC in d<3

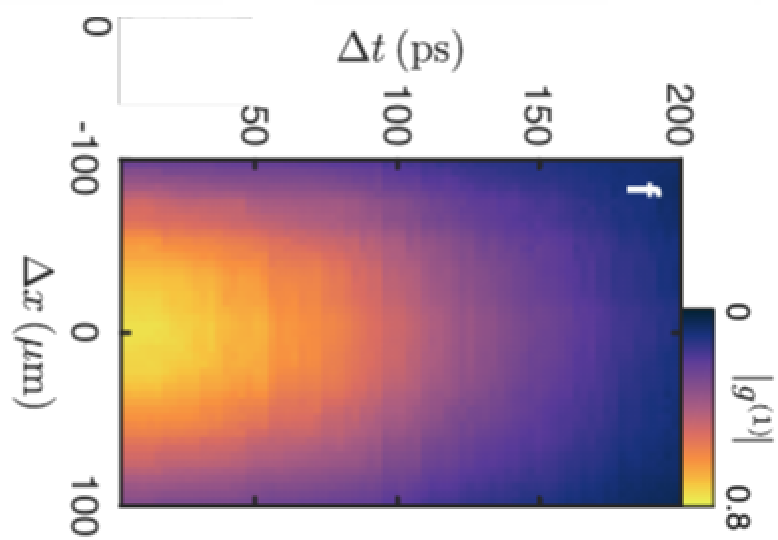
Non-equilibrium condensation or lasing:

- no BEC in 1D: exponential decay of $g^{(1)}(x)$
  (Graham-Haken, 1970; Wouters-IC, PRB 2006)
- debate in 2D: KPZ nonlin. destroy BKT phase?
  (Dagvadorj et al., PRX 2015; Altman et al., PRX 2015;
  Zamora et al., PRX 2017)



Experiment @ C2N Palaiseau:

- 1D lattice with array of semiconductor micropillars
- measure space-time coherence function $g^{(1)}(\Delta x,\Delta t)$
- Coherence only in limited space/time,
  then smoothly decays → quasi-BEC effect



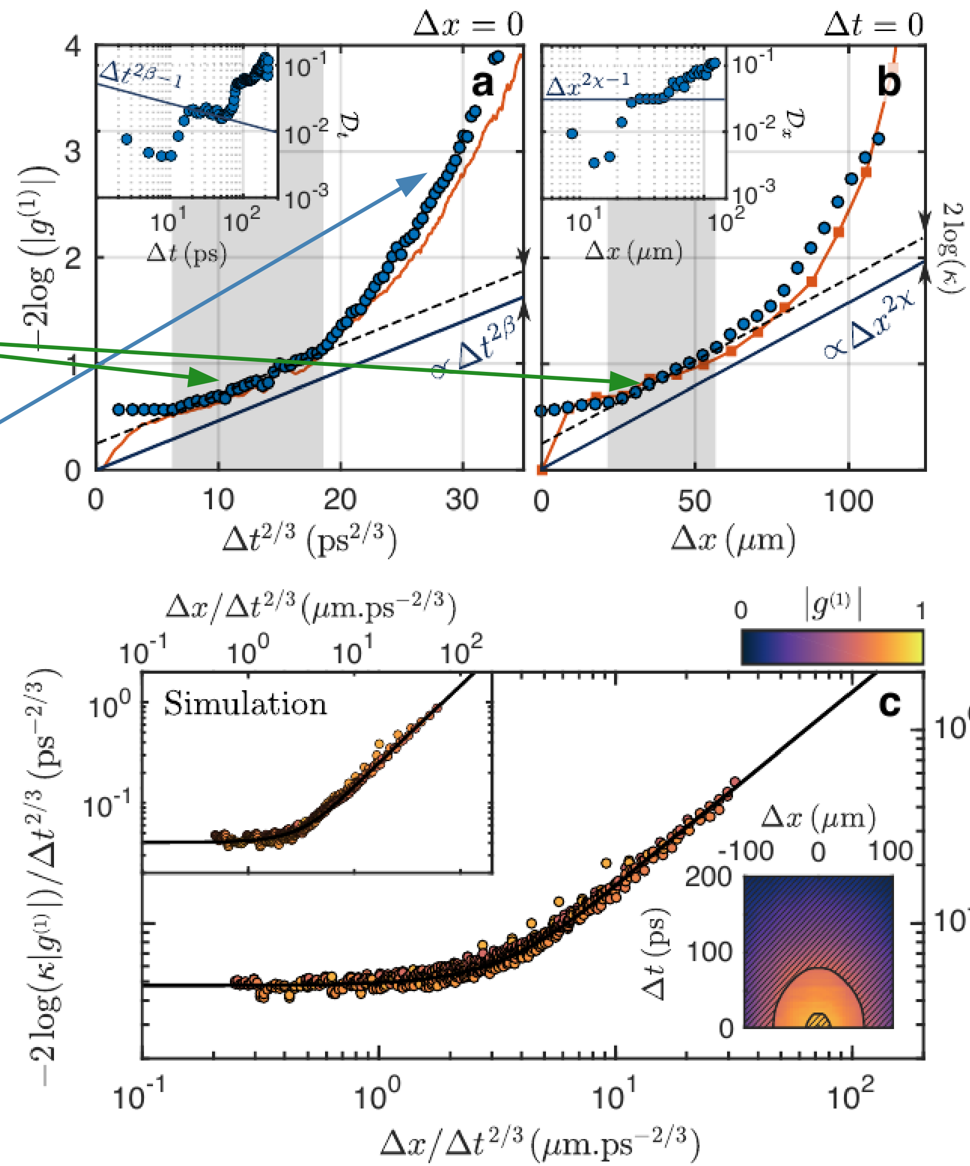
Fontaine, Squizzato, Baboux, Amelio, Lemaître, Morassi, Sagnes, Le Gratiet, Harouri, Wouters, IC, Amo, Richard, Minguzzi, Canet, Ravets, Bloch, *Observation of KPZ universal scaling in a 1D polariton condensate*, arXiv:2112.09550, accepted for publication on Nature

KPZ features

Power laws in the decay of $g^{(1)}(\Delta x, \Delta t)$

KPZ exponent

Schawlow-Townes-like decay



Collapse of cuts of $g^{(1)}(\Delta x, \Delta t)$
on Kardar-Parisi-Zhang universal curve

→ Quasi-BEC dynamics involves non-equilibrium effects &
interactions between Bogoliubov excitations

Fontaine et al., *Observation of KPZ universal scaling in a 1D polariton condensate*, arXiv:2112.09550.

Experimental evidence of entanglement ??

Peres-Horodecki criterion for non-separability

$$\Delta \equiv \langle \hat{b}^\dagger_{k_{HR}} \hat{b}_{k_{HR}} \rangle \langle \hat{b}^\dagger_{k_P} \hat{b}_{k_P} \rangle - |\langle \hat{b}_{k_{HR}} \hat{b}_{k_P} \rangle|^2 < 0$$

Population Anomalous correlation

Assumption of uncorrelated initial fluctuations
→ simplified Finazzi-IC PRA 2014 protocol to extract anom. corr. from density fluct.

Entanglement visible in intermediate k-range
→ HR from zero-point fluctuations
→ produces entangled phonon pairs

Are data statistically significant?
Any other alternative explanation?

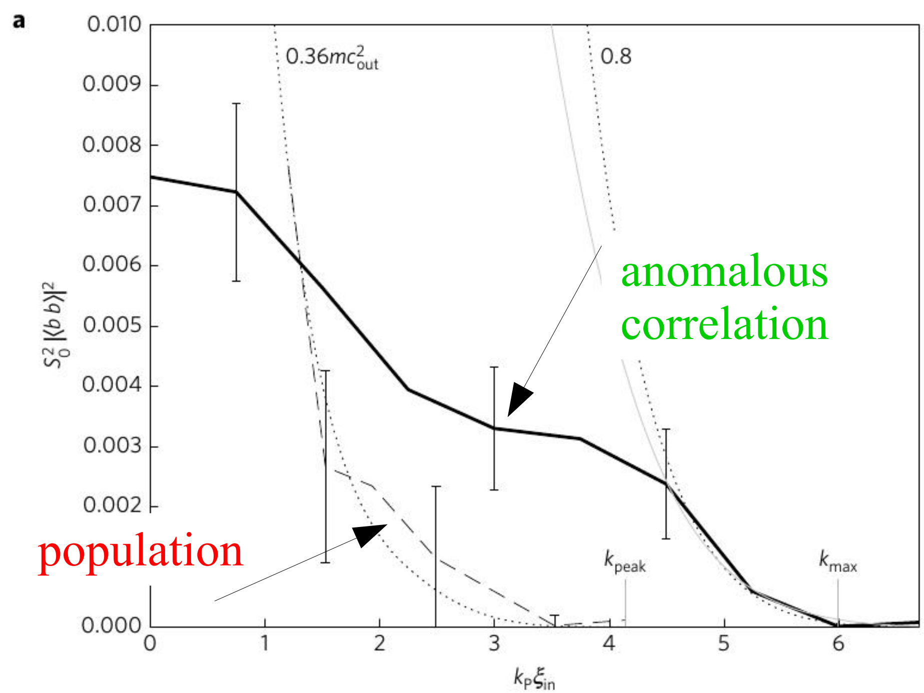


Figure from Steinhauer, Nat. Phys. '16

Much more theoretical work by de Nova, Sols, Parentani, Bruschi, Fuentes, etc.

Long-term perspectives

- Quantum Hydrodynamics: Navier-Stokes eqs. with hats on macroscopic hydrodynamic variables
- Entangled states of a macroscopic fluid

Part 4:

Analog two-level emitters in curved space-time QFTs

impurity atoms in flowing BECs

J. Marino, A. Recati, IC, *Casimir Forces and Quantum Friction from Ginzburg Radiation in Atomic Bose-Einstein Condensates*, PRL 2017
J. Marino. G. Menezes, IC, *Zero-point excitation of a circularly moving detector in an atomic condensate and phonon laser dynamical instabilities*, preprint arXiv:2001.08646

Dressed two-level atom interacting with BEC

Curved space-time → inhomogeneous flow of BEC

Quantum field → phonons on top of BEC

Two-level atom:

- Coherent resonant dressing |g> ↔ |e>
  Rabi freq. Ω_R
- Dressed eigenstates split by Ω_R

Ground |g> and excited |e> states
with opposite interaction with BEC $a_g=-a_e$.

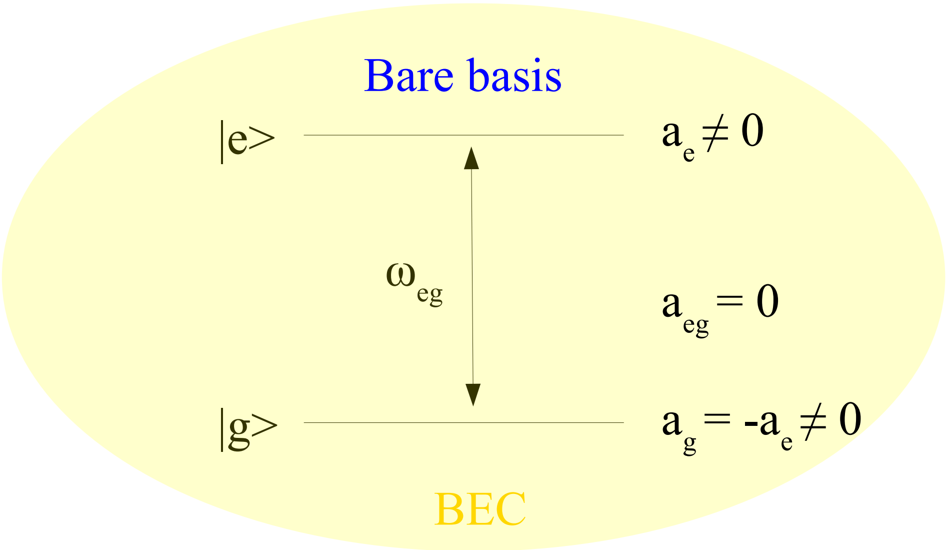
In dressed eigenstate basis:

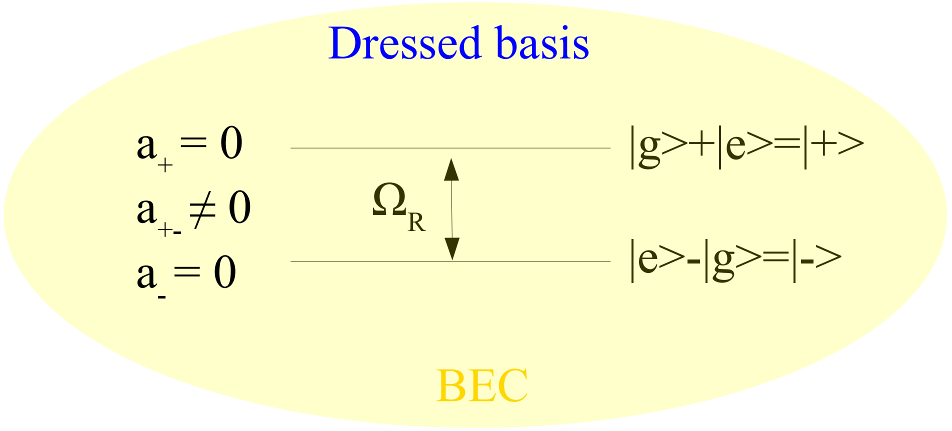
- No diagonal coupling to BEC
- Off-diagonal coupling to density fluctuations $\delta\rho(\mathbf{r})$:

$$H_A = \hbar\Omega_R\sigma_z + a_{\pm}\,\delta\rho(\mathbf{r})\,\sigma_x$$

→ simulates electric-dipole coupling $-d\,E(\mathbf{r})$
of 2-level atom to e.m. field

Motion of atom → ultrarelativistic (and beyond)





J. Marino, A. Recati, IC, *Casimir Forces and Quantum Friction from Ginzburg Radiation in Atomic Bose-Einstein Condensates*, PRL 2017

Ginzburg emission from moving atoms in ground state

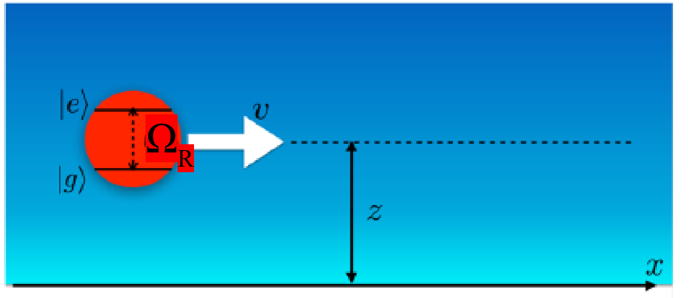
Atom moving supersonically across BEC

Prepared in lower state |->

Phase space opens for Ginzburg process

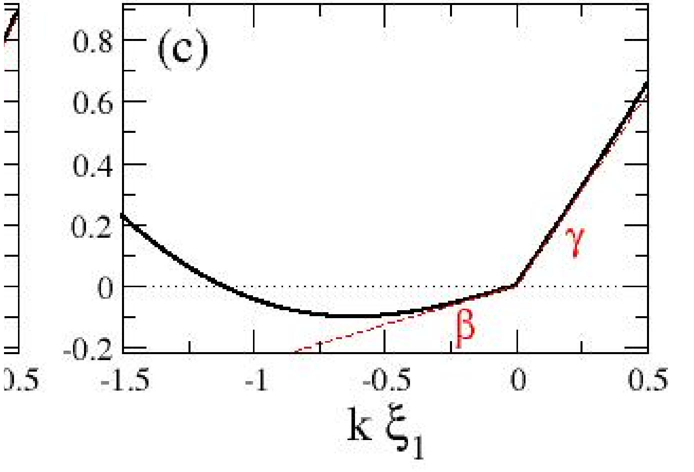
| - > → | + > + phonon k

$$\hbar\Gamma^g = 2\pi g_-^2 \rho_0 \int \frac{d^3k}{(2\pi)^3} (u_k + v_k)^2 \delta(\hbar\omega_0 + \hbar\Omega_R - \hbar\mathbf{k}\cdot\mathbf{v})$$



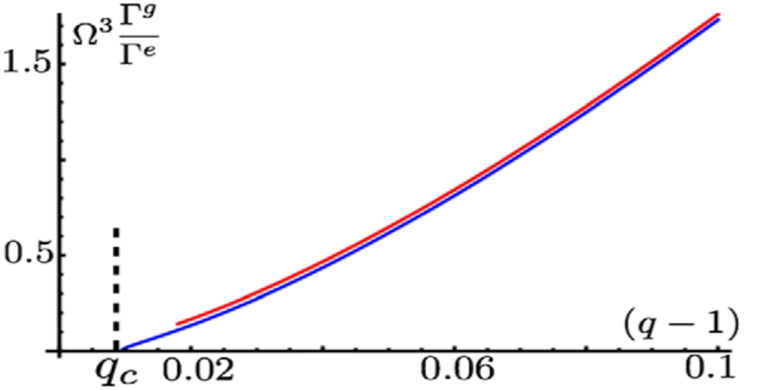
Physical interpretation:

- Doppler shift by supersonic speed gives negative-energy phonon modes
- Emission + excitation conserve energy/momentum
- Different from Cherenkov emission by charge:
  here "neutral" object, spontaneous dipole emission



Many other complex phenomena (J. Marino):

- Casimir forces via 0-point fluctuations of phonons
- Unruh effect: accelerated particle feels finite-T bath
- Peculiar out-of-time order correlations

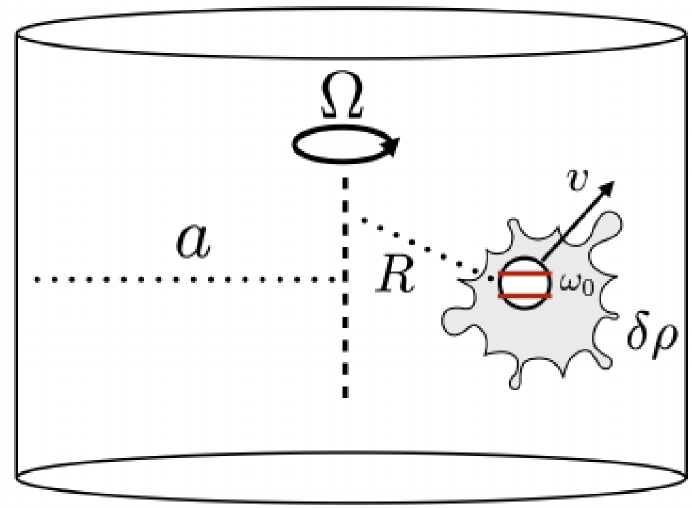


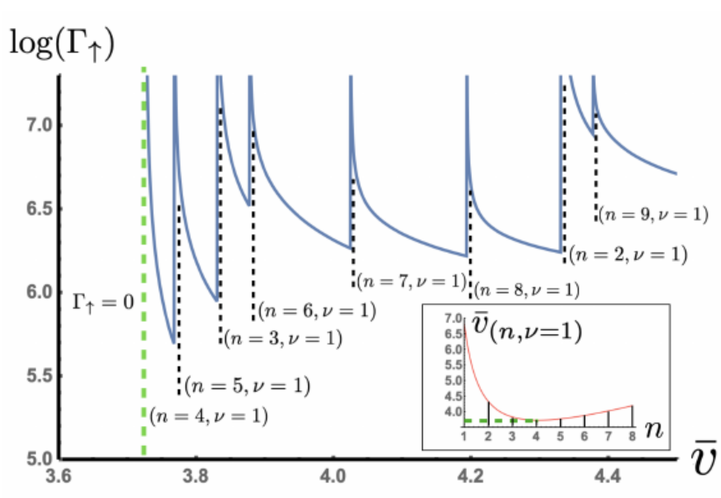
J. Marino, A. Recati, IC, *Casimir Forces and Quantum Friction from Ginzburg Radiation in Atomic Bose-Einstein Condensates*, PRL 2017
G. Menezes, J. Marino, *Slow scrambling in sonic BHs*, EPL 2018

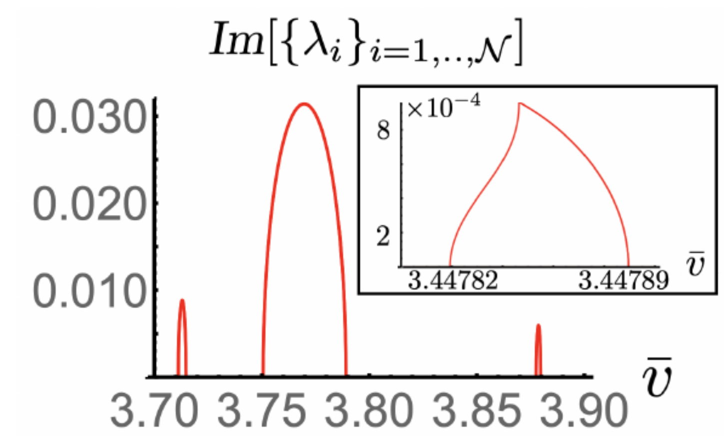
Impurity in circular motion: superradiance effects

Two level atom in circular motion in BEC at rest

- Spontaneous transition ground → excited state via emission of phonons into Bogoliubov modes
- Energy conservation imposes $(\Omega - m\,\omega_0) < 0$ for Bogo mode of angular momentum m
- Geometric suppression if $v = \Omega R < c_s$
- Superradiant instability for fully confined modes and "harmonic" atom







J. Marino. G. Menezes, IC, *Zero-point excitation of a circularly moving detector in an atomic condensate and phonon laser dynamical instabilitie*s, preprint arXiv:2001.08646

Part 3:

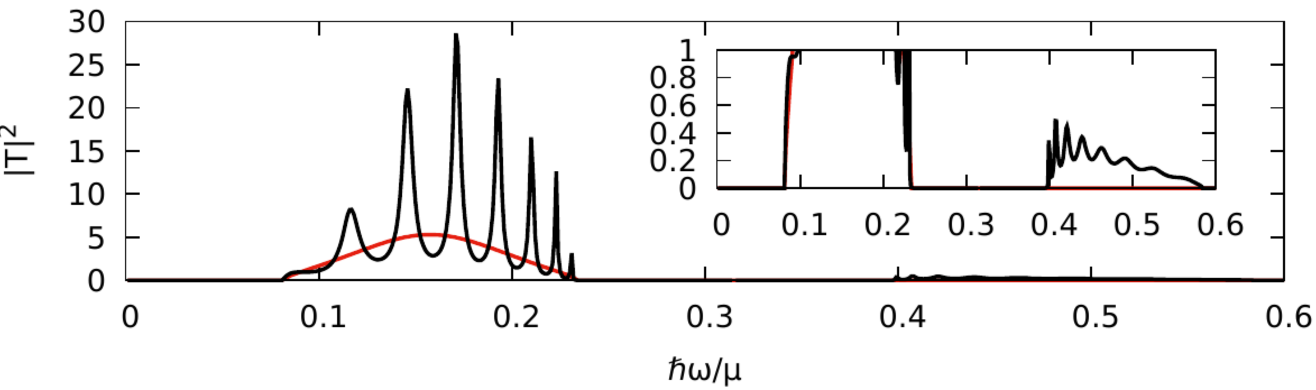
Quasi-Normal modes

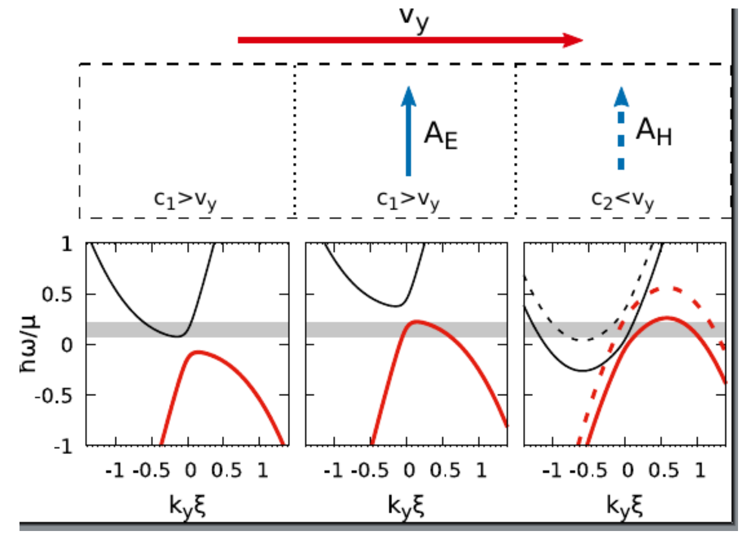
their impact on quantum emission and on BH entropy

Emission spectrum from complex BH configurations

Hawking emission spectrum:

- discrete peaks related to quasi-normal modes
- corresponding features in correlation functions
- are they populated by Hawking processes?
- General question: do they have a role in BH entropy?



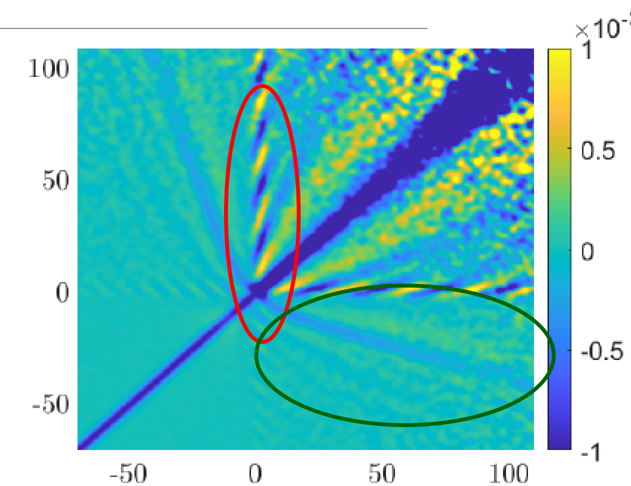


L. Giacomelli, in preparation 2021

Numerics for BH in polariton fluid:

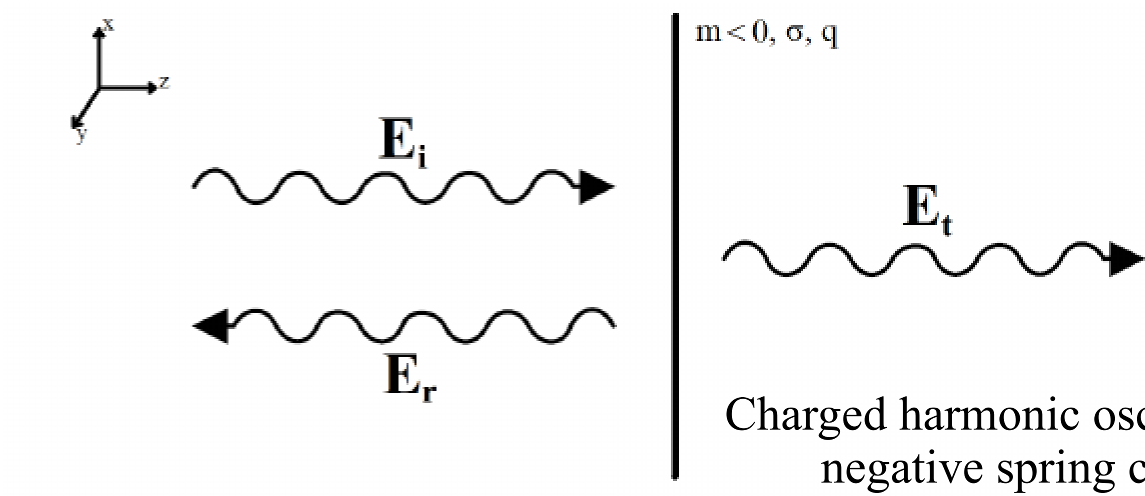
- Hawking signal in intensity correlations
- New feature along vertical/horizontal: correlation between inner and horizon regions

- is it related to QNMs ??
- does it signal some population of them?



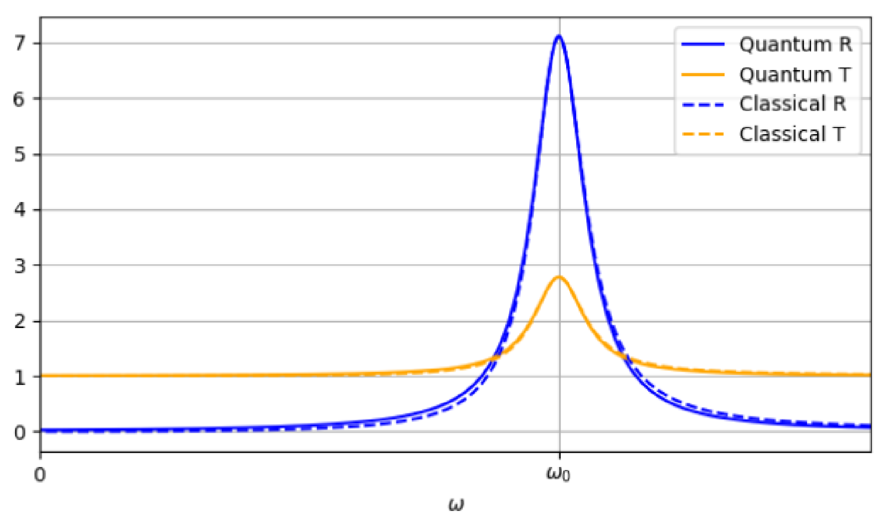
Numerical simulations @ LKB
Malo Joly + Maxime Jacquet

A simple model for superradiance (I)



Charged harmonic oscillator with negative mass and negative spring constant → negative energy mode (sort of population-inverted atom)

Can emit/scatter e.m. waves by reducing its energy

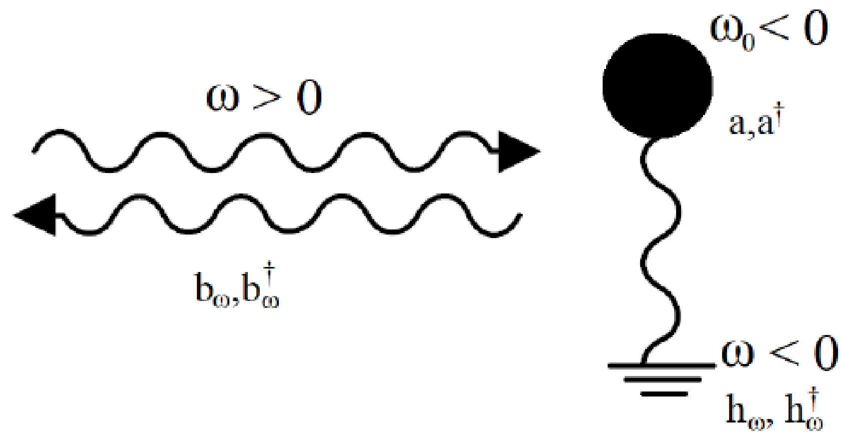


E.m. scattering calculation:
amplified reflection and transmission

A simple model for superradiance (II)

Quantum description:

- Harmonic oscillator $\omega_0<0$
- Coupled to $\omega>0$ radiative bath
- Coupled to $\omega<0$ non-radiative bath

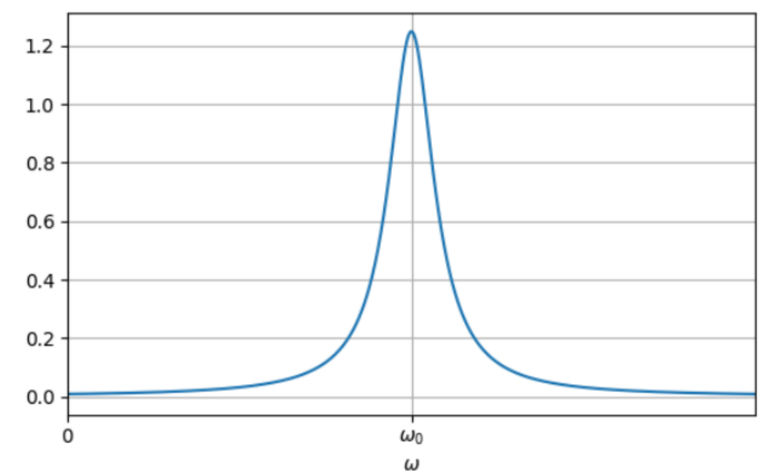


Input-output theory recovers classical calculation of reflection/transmission amplitudes

Quantum emission peaked at $\omega\sim|\omega_0|$

Stationary quantum state

$$\rho_{ss} = \left(1 - \frac{\beta}{\alpha}\right)\left(\frac{\beta}{\alpha}\right)^{a^\dagger a}$$



- finite population of h.o. & finite entropy

$$S = -k_B\left(\log\left(1 - \frac{\gamma^R}{\gamma^{NR}}\right) + \frac{1}{1 - \frac{\gamma^R}{\gamma^{NR}}}\log\left(\frac{\gamma^R}{\gamma^{NR}}\right)\right)$$

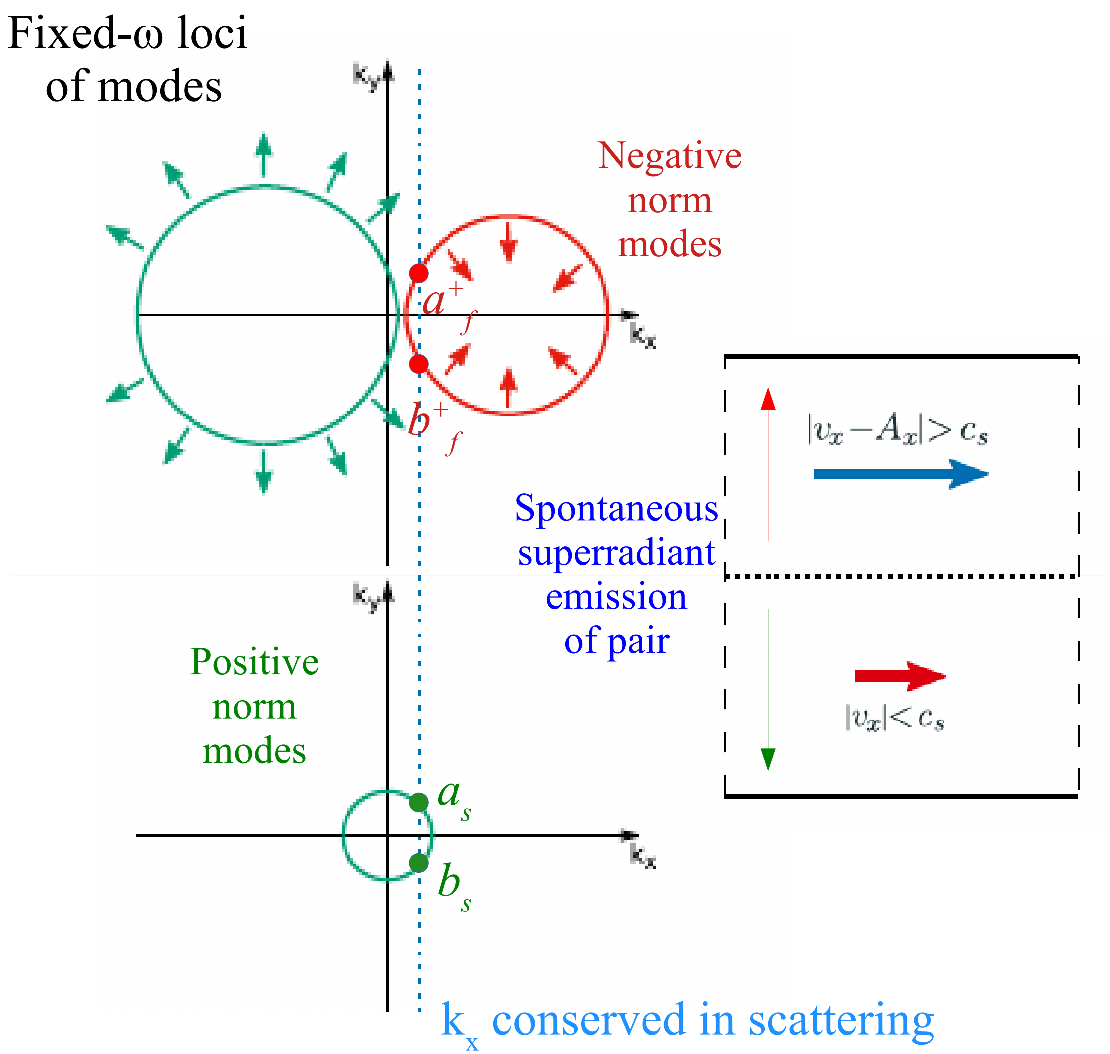
- Mechanism different from back-reaction

What does this entropy means in astrophysics???

Calculation for BHs full of technical issues… but no surrender, Luca is brave...

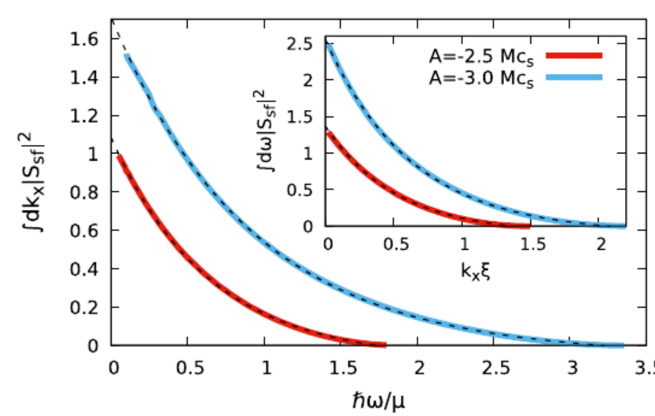
Q. Lamouret, stage report @ ENS-Paris, 2021

Quantum superradiant emission



- 2 x 2 scattering matrix mixes creation/destruction operators
- Pair of positive/negative norm modes on opposite sides
- Superradiant process seeded by zero-point fluctuations
- Zero-point emission similar to Hawking emission
- Interpreted as positive/negative energy quanta emitted on opposite sides
- Only possible if ergoregion present

$$\begin{pmatrix} \hat{b}_s(k_x,\omega) \\ \hat{b}_f^\dagger(k_x,\omega) \end{pmatrix} = S(k_x,\omega) \begin{pmatrix} \hat{a}_s(k_x,\omega) \\ \hat{a}_f^\dagger(k_x,\omega) \end{pmatrix}$$



L. Giacomelli & IC, *Spontaneous quantum superradiant emission in atomic BECs subject to a synthetic vector potential*, PRA 2021