Numerical experiments of Hawking radiation from acoustic black holes in atomic Bose-Einstein condensates

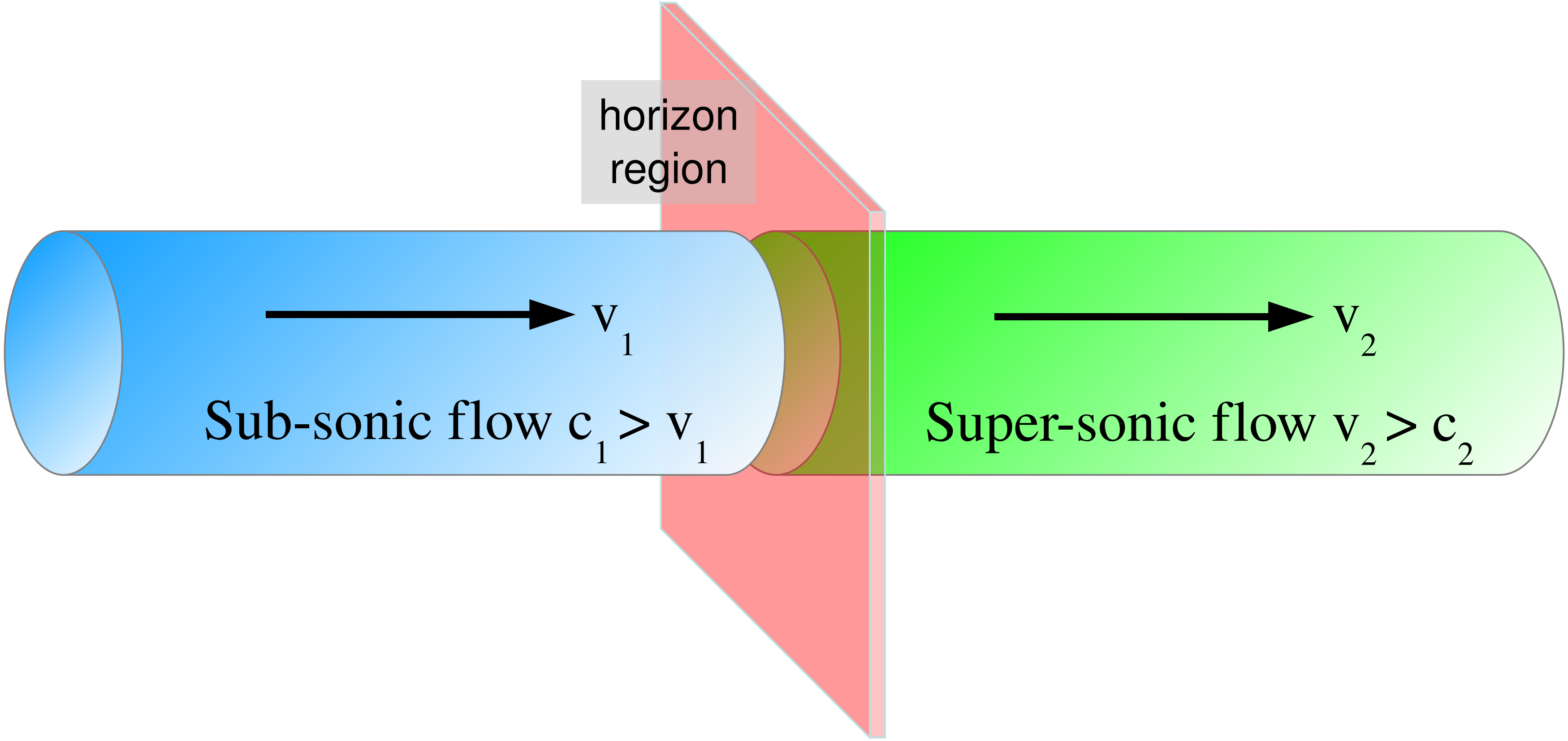
Iacopo Carusotto

BEC CNR-INFM and Università di Trento, Italy

In collaboration with:

Alessio Recati and Davide Sarchi	(BEC CNR-INFM, Trento, Italy)
Serena Fagnocchi and Roberto Balbinot	(Università di Bologna, Italy)
Alessandro Fabbri	(IFIC - Univ. de Valencia and CSIC, Spain)
Nicolas Pavloff	(Université Paris-Sud, Orsay, France)

What is an acoustic black hole ?



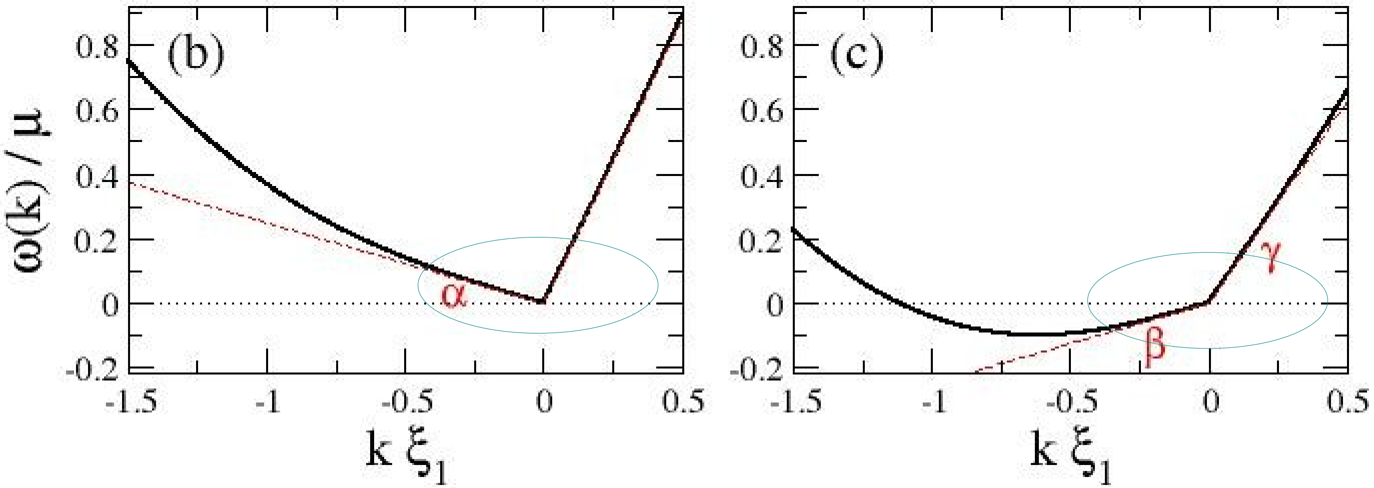
Steady but non-uniform flow in the atomic condensate

Horizon separates region of sub-sonic and super-sonic flow

No sonic perturbation can propagate back from super-sonic region

W. G. Unruh, PRL **46**, 1351 (1981). *"Artificial Black Holes"*, eds. Novello, Visser, Volovik (2002)

The analogy with QFT in curved space-time



Low-k, hydrodynamic region: linear phonon dispersion $\omega = c_s |k|$

Mathematical analogy with light propagation in curved metric

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = \frac{n(x)}{c_s(x)} \left[-c_s(x)^2 dt^2 + (d\vec{x} - \vec{v}(x) dt)(d\vec{x} - \vec{v}(x) dt) \right]$$

Wave equation for BEC phase

$$\frac{1}{\sqrt{-G}} \partial_\mu \left[\sqrt{-G}\, G^{\mu\nu} \partial_\nu \right] \phi(x,t) = 0$$

Hawking radiation in black-hole geometries

Astrophysical black-holes

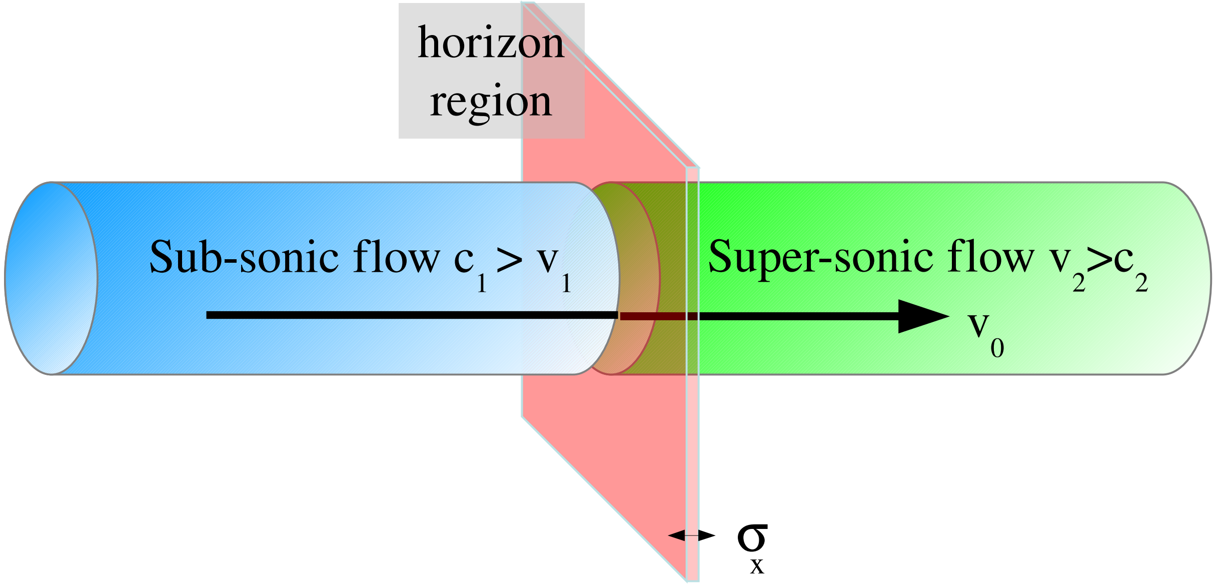
- emit Hawking radiation at $T_H = \frac{\hbar c^3}{8\pi G M k_B}$
- solar mass BH: $T_H = 0.4\ \mu\mathrm{K}$
- hardly visible if compared to cosmological background at 2.73 K

Acoustic black holes:

- emit Hawking radiation of phonons at $T_H = \frac{\hbar}{4\pi k_B c_s}\left[\frac{d}{dx}(c_s^2 - v^2)\right]_H$
- in nK range for µm-sized ultracold atomic BECs (not so bad...)

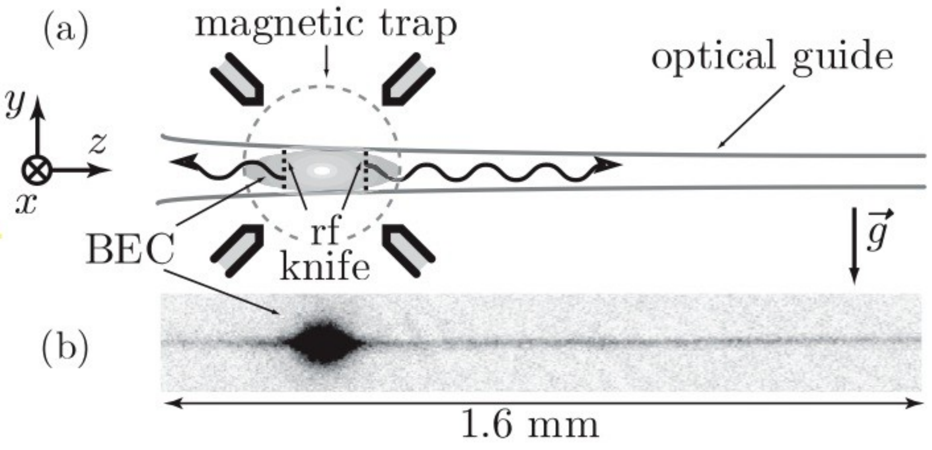
The “experimental” protocol

How to generate and study an acoustic black hole ?

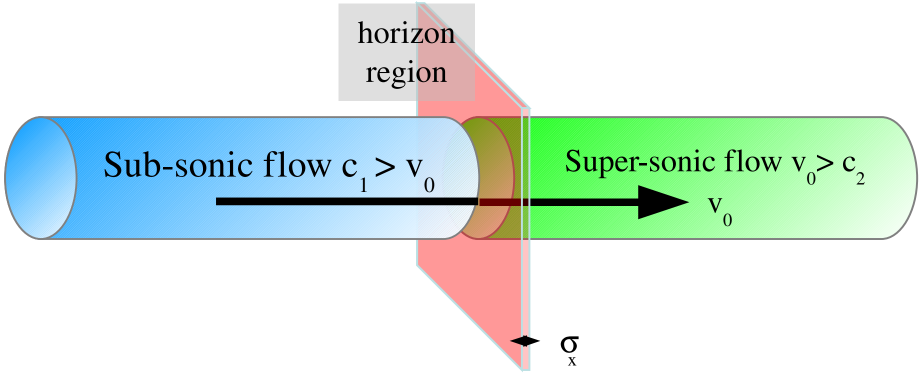


- start from a uniform sub-sonic flow
- switch on horizon at t=0 and go to black-hole regime $c_1 > v_1$, $v_2 > c_2$
  - minimize deterministic disturbances, e.g. Landau processes (in super-sonic region) and soliton shedding during and after switch-on
- concentrate on quantum fluctuations
  - isolate (thermal) Hawking emission from background phonons (also thermal)

(i) How to create a "clean" black hole?



From: W. Guerin *et al*., PRL **97**, 200402 (2006)



- Out-coupled atom laser beam: uniform density and velocity v_0
- Atom-atom interaction constant initially uniform and equal to g_1
- Within σ_t around t=0: modulation $g_1 \rightarrow g_2$ and $V_1 \rightarrow V_2$ in $x>0$ region only

  via: Feshbach resonance (g depends on applied B) or modify transverse confinement
- Step in nonlinear coupling constant g $\Rightarrow$ step in sound speed c.
- Black-hole formed if $c_1 > v_0 > c_2$, thickness σ_x of crossover region determines surface gravity
- Chemical potential jump to be compensated by external potential $V_1 + ng_1 = V_2 + ng_2$

  allows to avoid Cerenkov-Landau phonon emission, soliton shedding

(ii) How to detect Hawking radiation?

Density-density correlation function
$$G^{(2)}(x,x')=\frac{\langle : n(x)\, n(x') : \rangle}{\langle n(x)\rangle \, \langle n(x')\rangle}$$

Prediction of gravitational analogy:

→entanglement in Hawking pairs gives long-range in/out correlations

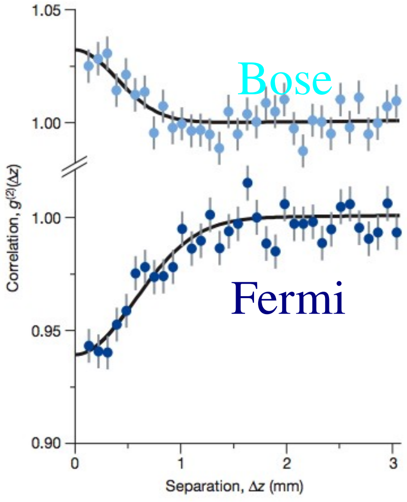
$$G_2(x,x') = 1 - \frac{\xi_1 \xi_2}{16\pi c_1 c_2} \, \frac{k^2}{\sqrt{n^2 \xi_1 \xi_2}} \, \frac{c_1 c_2}{(c_1 - v)(v - c_2)} \, \cosh^{-2}\left[\frac{k}{2}\left(\frac{x}{c_1 - v} + \frac{x'}{v - c_2}\right)\right]$$

→allows to isolate Hawking phonons from background of incoherent thermal phonons

R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008).

(iii) Why to use atomic BECs ?

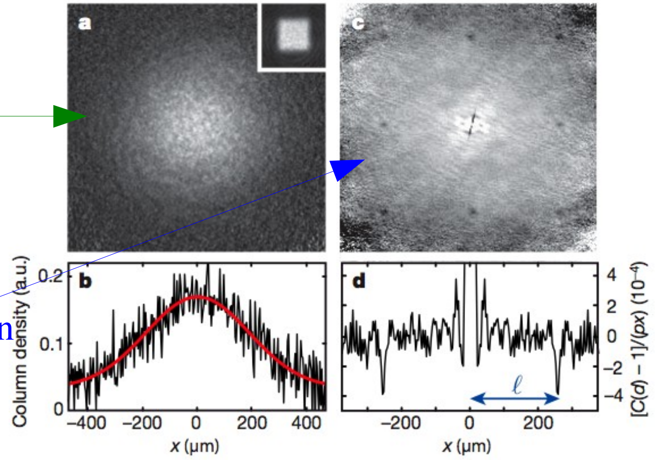
- Minimize thermal background starting from almost T=0
- Microscopic control in space and time of sound speed and external potentials
- Measurement of density correlations experimentally demonstrated:
  - **Atomic HB-T**: positive correlation due to thermal Bose atoms (negative for fermions)
  - Noise correlations in **TOF picture after expansion from lattice**
- Microscopic quantum calculations independent from gravitational analogy



Jeltes *et al.*, Nature 445, 402 (2007)

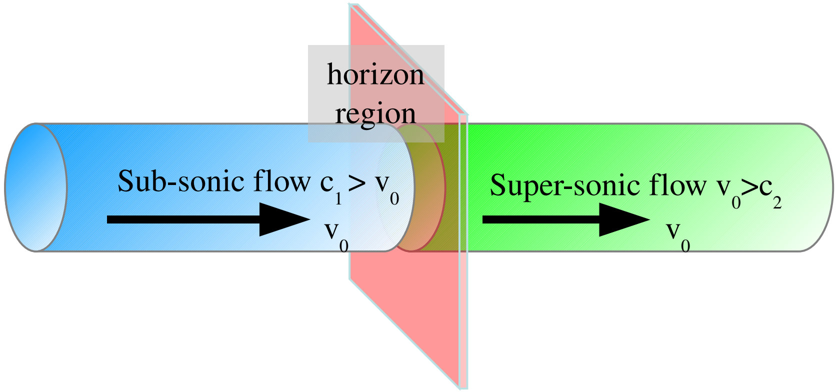
Single-shot image

Spatial correlation of noise



Rom *et al.*, Nature 444, 733 (2006)

A significant detail...



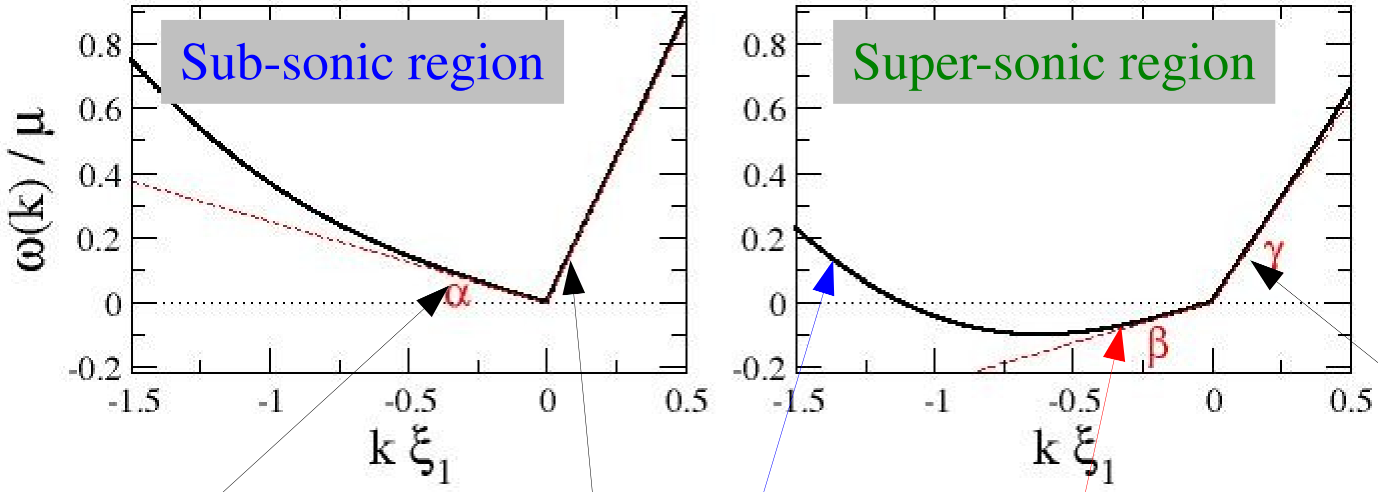
horizon region

Sub-sonic flow $c_1 > v_0$

v_0

Super-sonic flow $v_0 > c_2$

v_0



$v_g = v_0 - c_1 < 0$

$v_g = v_0 + c_1 > 0$

$v_g = v_0 + c_2 > 0$

$v_g = v_0 - c_2 > 0$

Sound can not propagate back !

New feature of atomic BEC: single particle excitations **can emerge** from black hole !!

This raises interesting fundamental questions...

Standard derivations of Hawking radiation often assume:

- linear dispersion $\omega(k) = c\,|k|$ at all length scales
- infinite blue shift of modes at horizon
- relativity and QFT valid up to arbitrary energies

These assumptions violated in BEC-based analogs:

- is HR robust w/r to deviation from hydrodynamic dispersion?
- what is role of single particle nature of high-k excitations?

➔ thermal HR spectrum modified by “Planck-scale” physics?

➔ does this provide new features in BH signal in LHC ?

W. G. Unruh, Phys. Rev. D 51, 2827 (1995); R. Brout et al., Phys. Rev. D 52, 4559 (1995)
T. A. Jacobson and R. Parentani, An echo of Black Holes, Scientific American, Dec. 2005.

The numerical observations

The numerical method: Wigner-Monte Carlo

At t=0, homogeneous system:

- Condensate wavefunction in plane-wave state
- Quantum + thermal fluctuations in plane wave Bogoliubov modes
- Gaussian α_k, variance $\langle|\alpha_k|^2\rangle = [2\tanh(E_k / 2k_B T)]^{-1} \to ½$ for $T \to 0$.

$$\psi(x, t=0) = e^{i k_0 x}\left[\sqrt{n_0} + \sum_k \left(u_k e^{i k x} \alpha_k + v_k e^{-i k x} \alpha_k^*\right)\right]$$

At later times: evolution under GPE

$$i\hbar\ \partial_t \psi(x) = -\frac{\hbar^2}{2\mathrm{m}}\ \partial_x^2 \psi(x) + V(x)\psi(x) + g(x)\,|\psi(x)|^2 \psi(x)$$

Expectation values of observables:

- Average over noise provides symmetrically-ordered observables

$$\langle \psi^*(x)\ \psi(x')\rangle_W = \frac{1}{2}\langle \hat{\psi}^\dagger(x)\ \hat{\psi}(x') + \hat{\psi}(x')\ \hat{\psi}^\dagger(x)\rangle_Q$$

Equivalent to Bogoliubov, but can explore longer-time dynamics

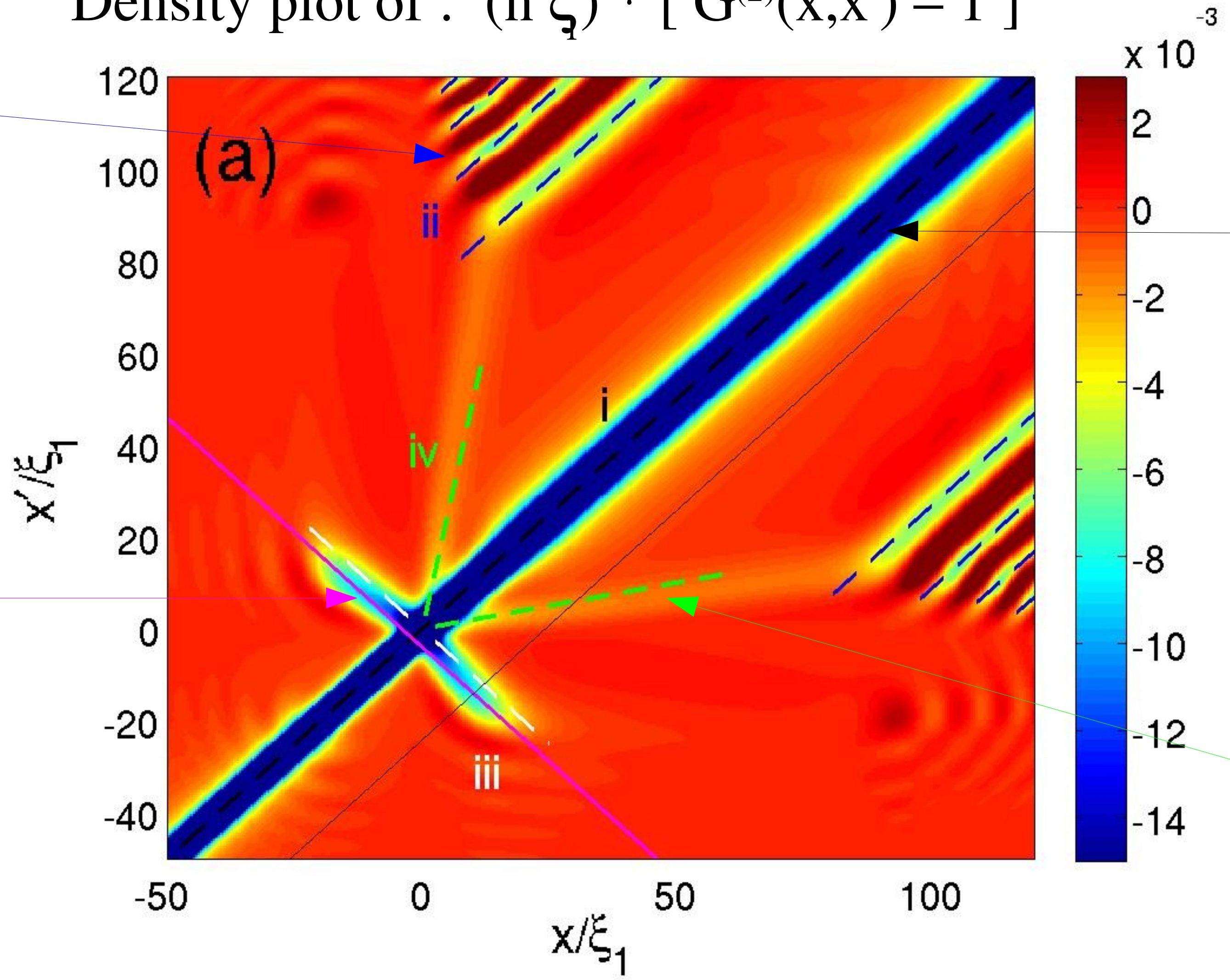
A. Sinatra, C. Lobo, Y. Castin, J. Phys. B 35, 3599 (2002)

Density correlations: the movie

IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)

A snapshot of density correlations

Density plot of : $(n\,\xi) * [\,G^{(2)}(x,x') - 1\,]$



(ii)
Dynamical
Casimir
emission

(i)
Many-body
antibunching

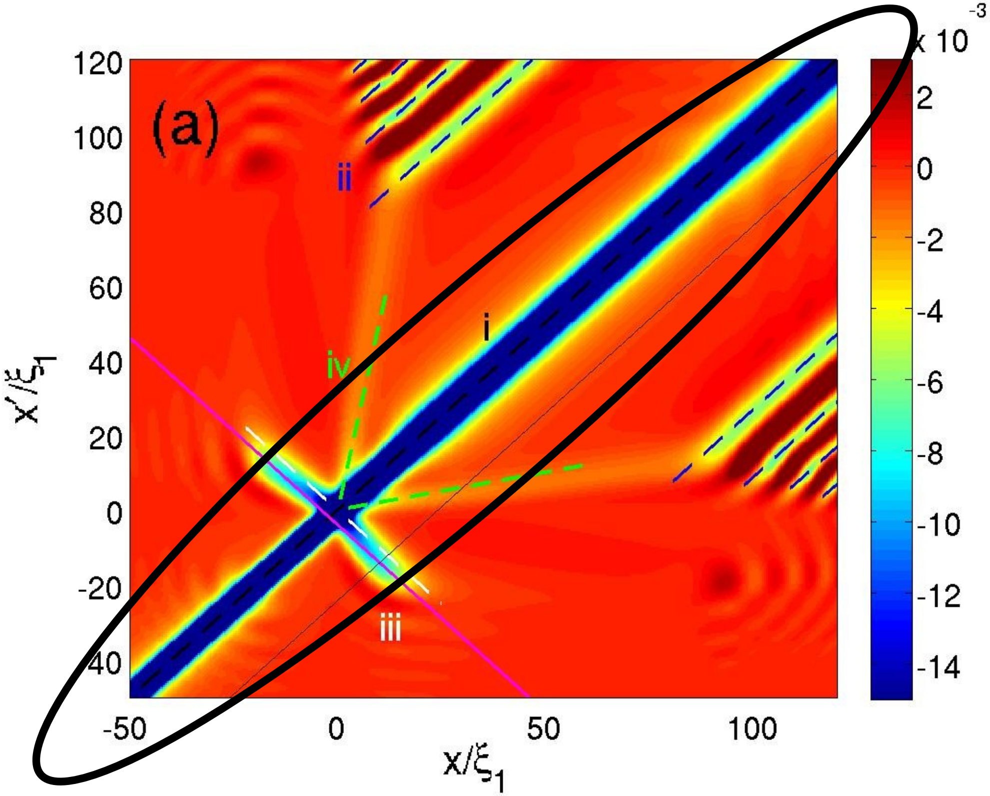
(iii)
Hawking
in / out

(iv)
Hawking
in / in

IC, S.Fagnocchi, A.Recati, R.Balbinot, A.Fabbri, New J. Phys. 10, 103001 (2008)

Feature (i) : Many-body antibunching

- present at all times
- due to repulsive interactions
- almost unaffected by flow



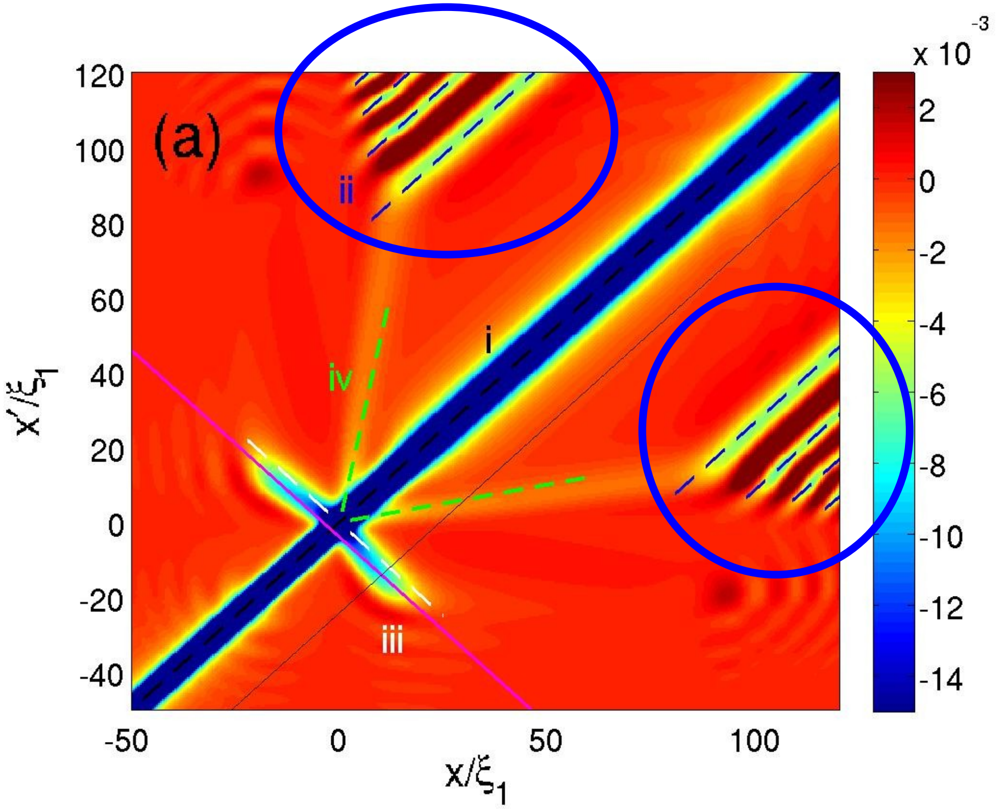
See e.g.: M. Naraschewski and R. J. Glauber, PRA 59, 4595 (1999)

Feature (ii): Dynamical Casimir emission of phonons

Fringes parallel to main diagonal

- intensity depends on speed of switch-on
- only in x>0 region, move away in time
- do not depend on flow pattern,

  also present in homogeneous system



Physical interpretation:

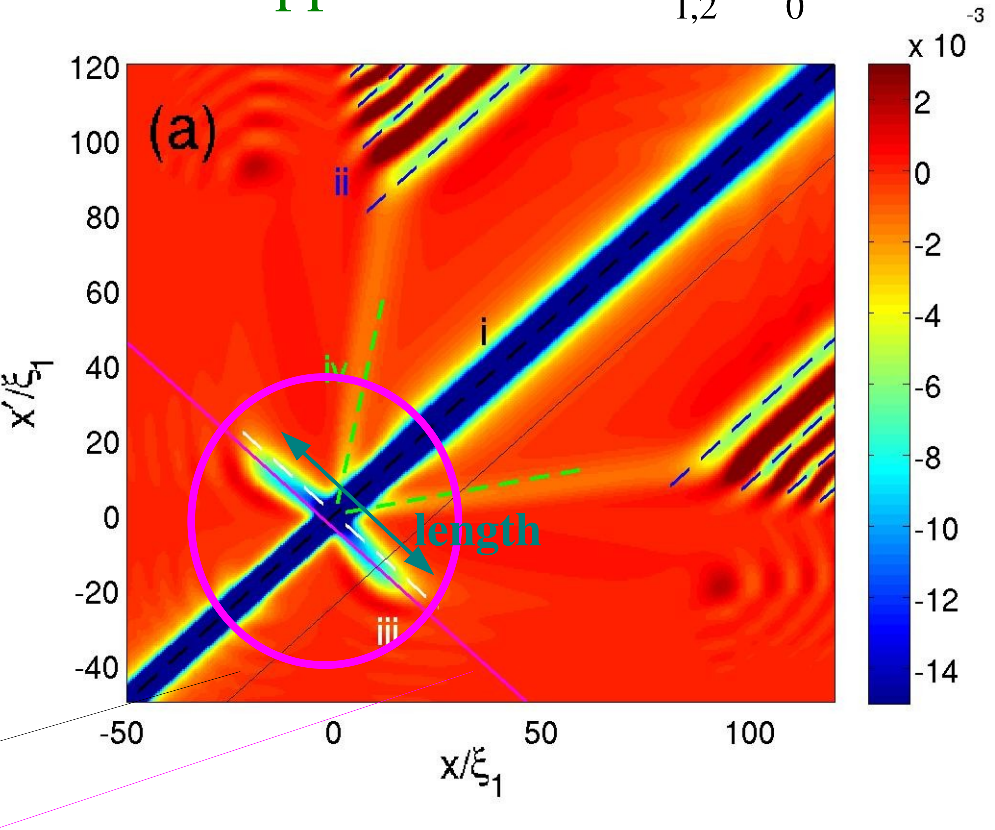
- in x>0 region $g_1 \to g_2$ within short time σ_t:
- non-adiabatic modulation of Bogoliubov vacuum
- fringes depend on |x-x'|: counter-propagating correlated pairs emitted at t=0 at all points x>0
- correlations propagate away at speed $\geq 2c_s$
- model of amplification of metric fluctuations during cosmological inflation period

<u>See also:</u> M. Kramer *et al.* PRA **71**, 061602(R) (2005); K. Staliunas *el al.*, PRL **89**, 210406 (2002);
C. Ciuti, IC, G. Bastard, PRB **72**, 115303 (2005).

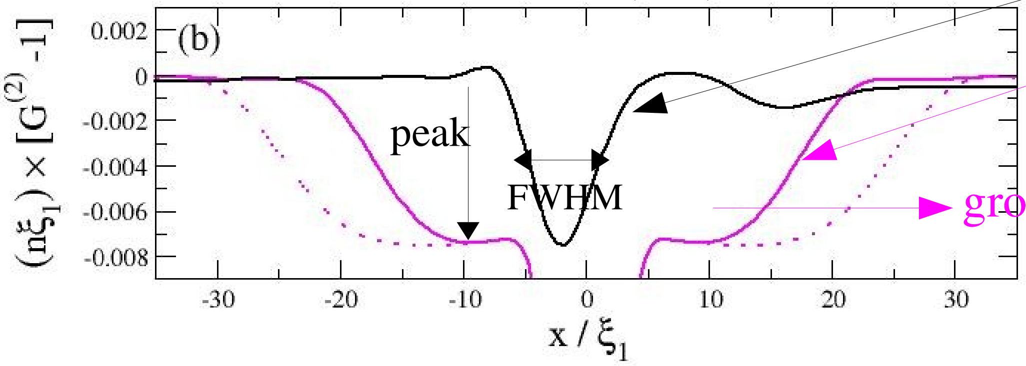
Feature (iii) : The Hawking signal

Negative correlation tongue extending from the horizon x=x'=0

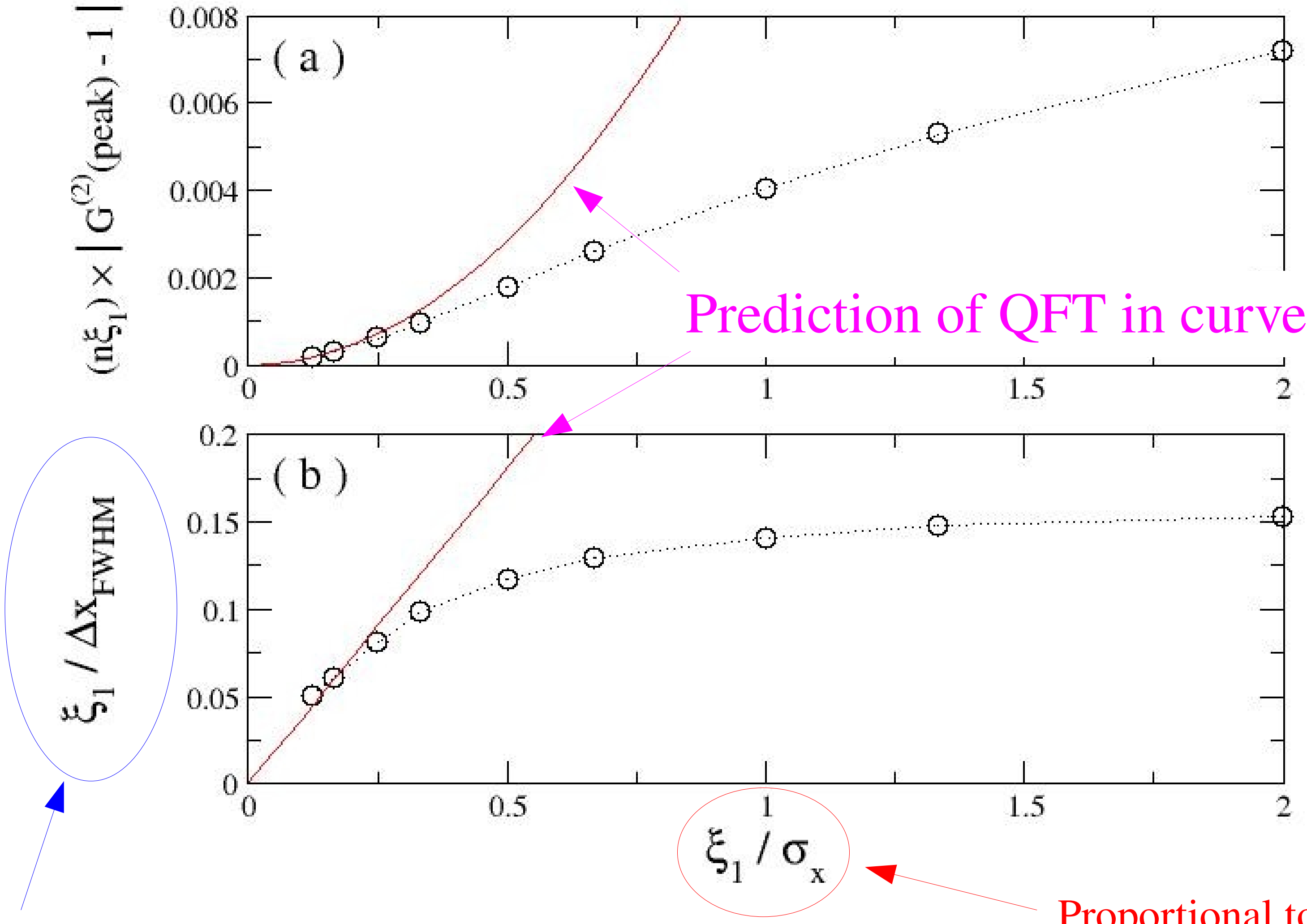
- long-range in/out density correlation which disappears if both $c_{1,2}<v_0$
- length grows linearly in t
- peak height, FWHM constant in t
- slope $\frac{v_0-c_2}{v_0-c_1}$ agrees with theory
  - pairs emitted at all t from horizon
  - propagate at sound speed



cuts of $G^{(2)}(x,x')$ - 1



Quantitative analysis



Prediction of QFT in curved space-time

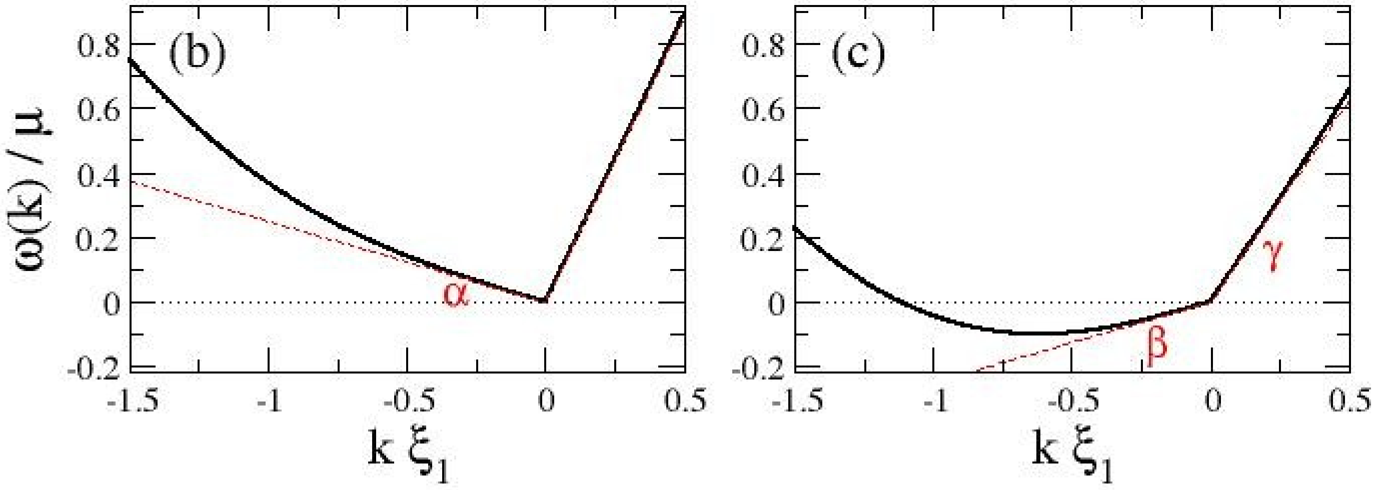
Proportional to surface gravity

Proportional to emission temperature

Analog model prediction quantitatively correct in hydrodynamic limit $\xi_1 / \sigma_x \ll 1$

Significant discrepancies for strong surface gravity

Features (iii, iv) : More on the Hawking signal



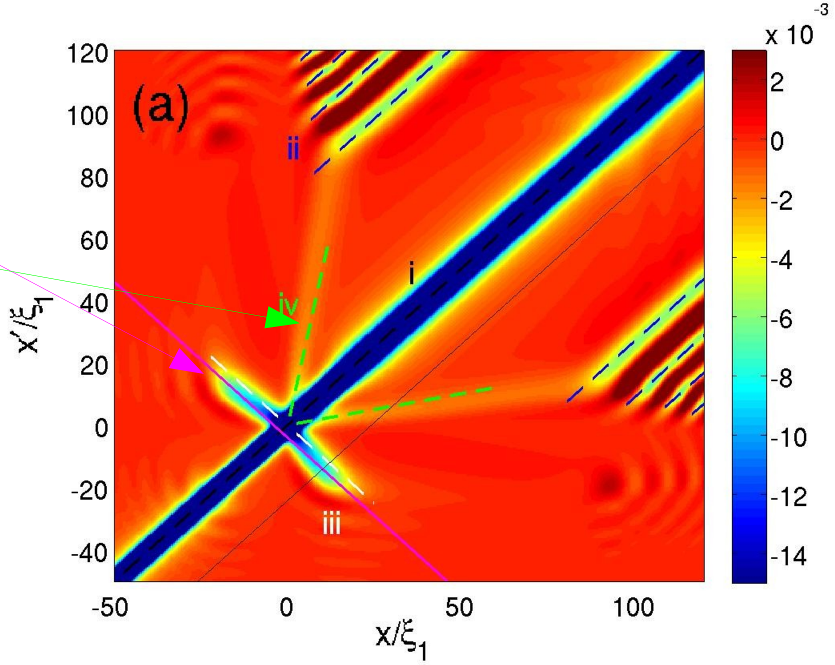
Two parametric “Hawking” processes:

- in/out: vacuum → α + β (feature iii)
- in/in: vacuum → β + γ (feature iv)

Energy conserved only if sub/super-sonic

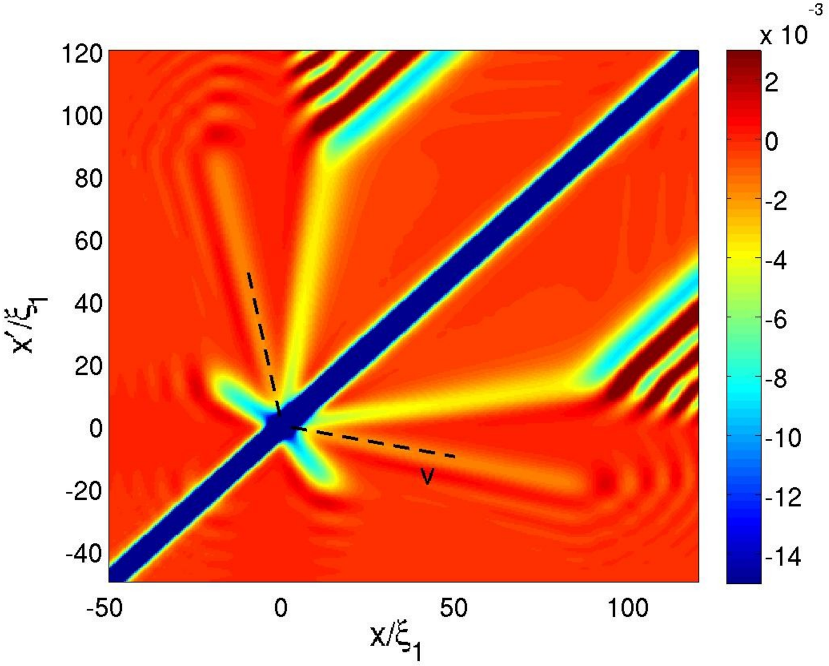
Momentum provided by horizon

Slope of tongues $\frac{v_0-c_2}{v_0-c_1}\simeq -1$, $\frac{v_0-c_2}{v_0+c_2}\simeq\frac{1}{5}$

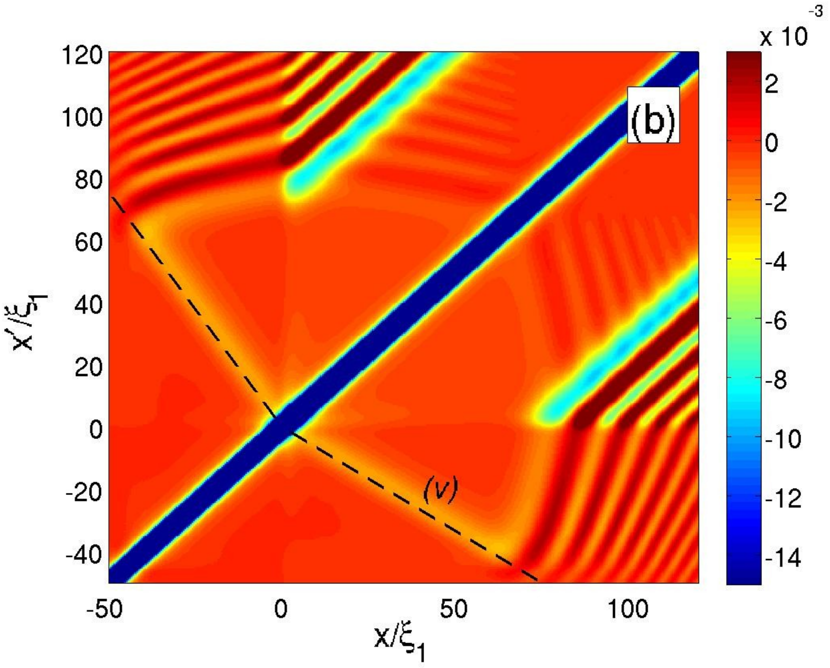


Effect of an initial T>0

- Hawking signal remains visible also for initial T comparable to T_H
- Stimulated Hawking emission
- Extra tongues (v) due to partial scattering of thermal phonons on horizon
- distinguishable from Hawking emission by different slope $\frac{(v_0-c_1)}{(v_0+c_2)}$

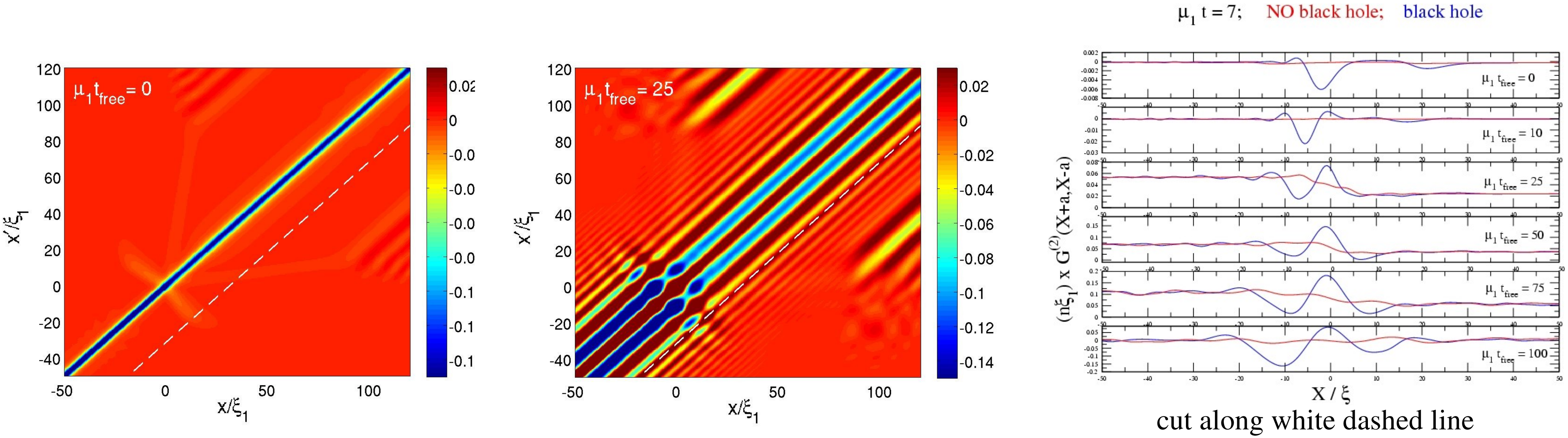


with Black Hole



without Black Hole

A trick to reinforce the Hawking signal



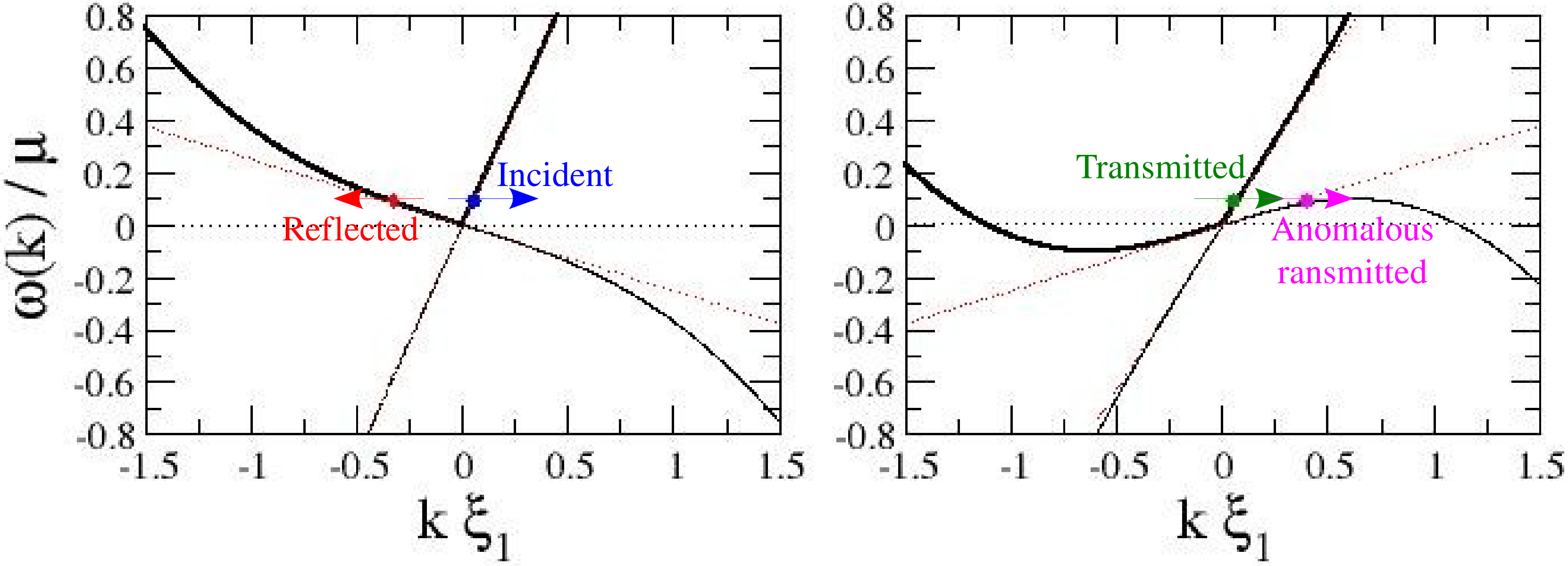
cut along white dashed line

- Hawking low-k phonons mostly phase waves with weak density modulation
- sudden switch off of interaction strength $g \to 0$
- time-of-flight expansion t_{free} : maps phase fluctuation into density fluctuation
- density correlation signal reinforced x10 (but also distorted)

Original idea: E. Cornell, EHR Workshop, Valencia 2009.
Similar trick to study 1D quasi-BECs: Dettmer et al., PRL **87**, 160406 (2001)

How does a quantum optician physically understand Hawking radiation ?

Scattering of Bogoliubov excitations on BH horizon



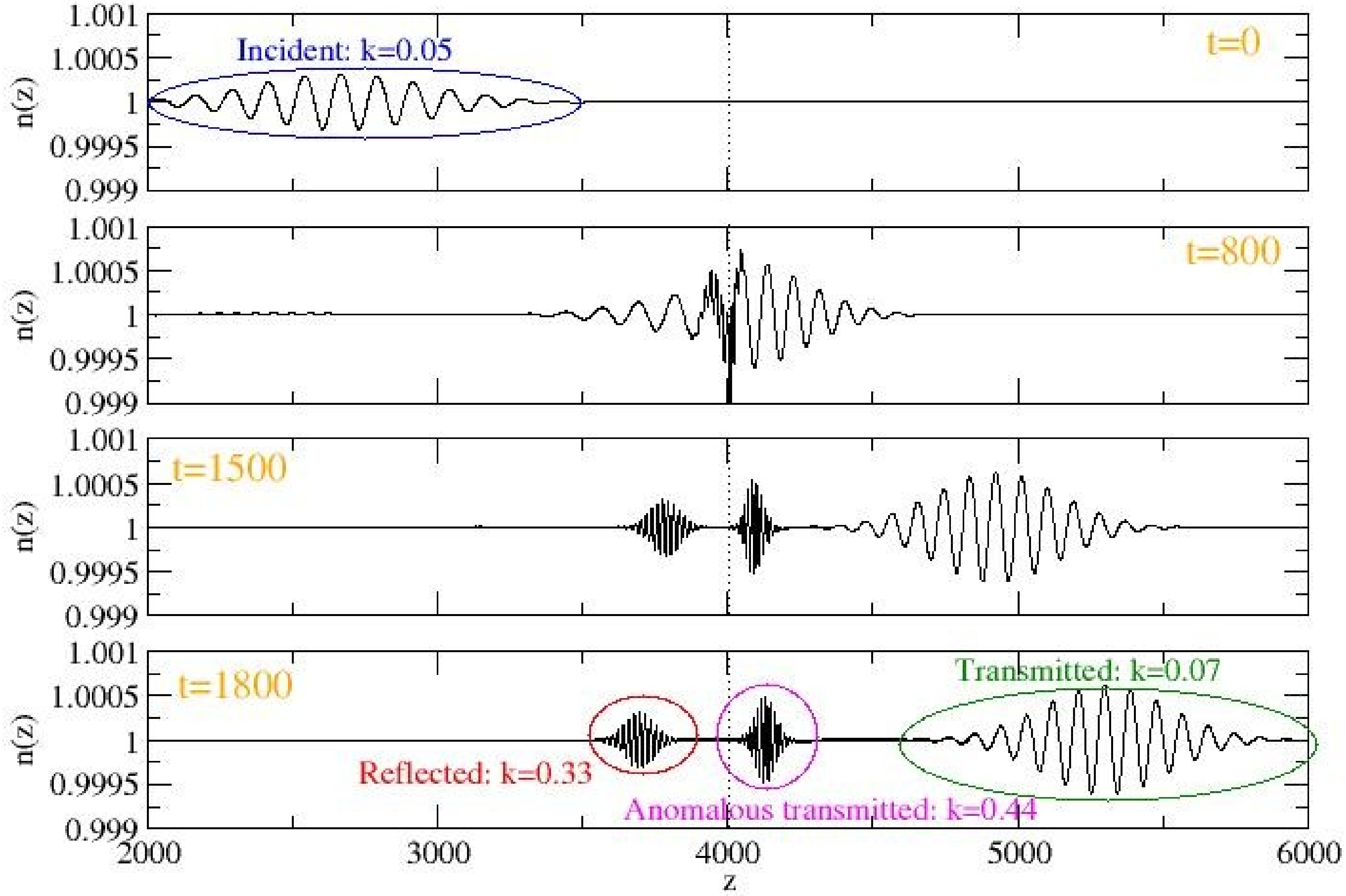
Energy conserved during scattering on stationary BH

Additional scattering channel available if black hole $c_1 > v_0 > c_2$

Incident plane wave → reflected, transmitted and anomalous transmitted

Anomalously transmitted mode is a Bogoliubov “ghost” mode

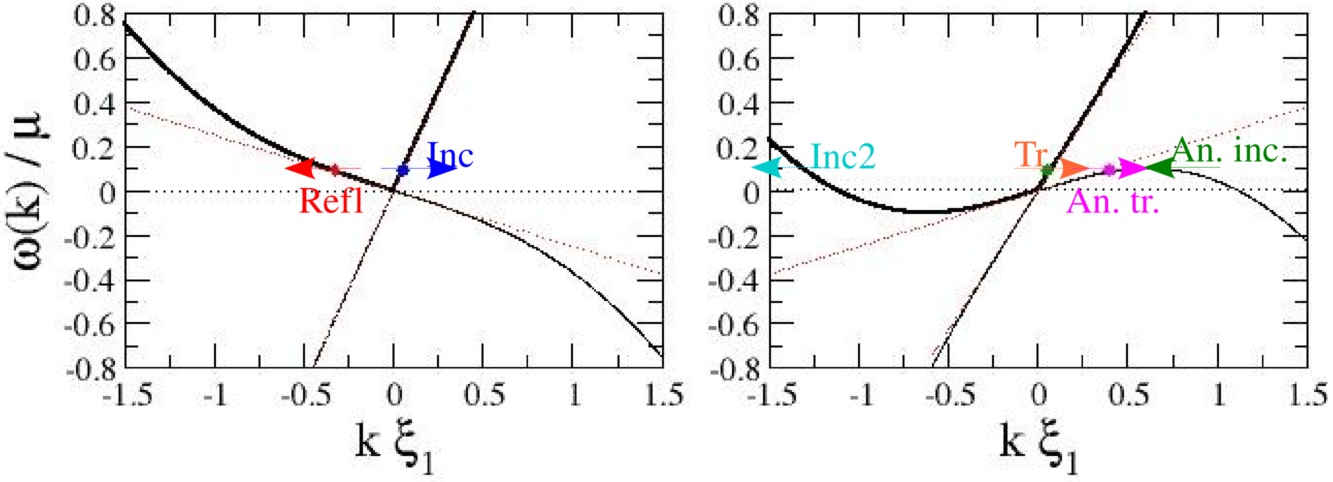
(Classical) wavepacket dynamics



Similar phenomenology observed in surface waves on moving fluids

G. Rousseaux et al, NJP 10, 053015 (2008)

Including quantum fluctuations: Hawking radiation



Input-output formalism of quantum optics

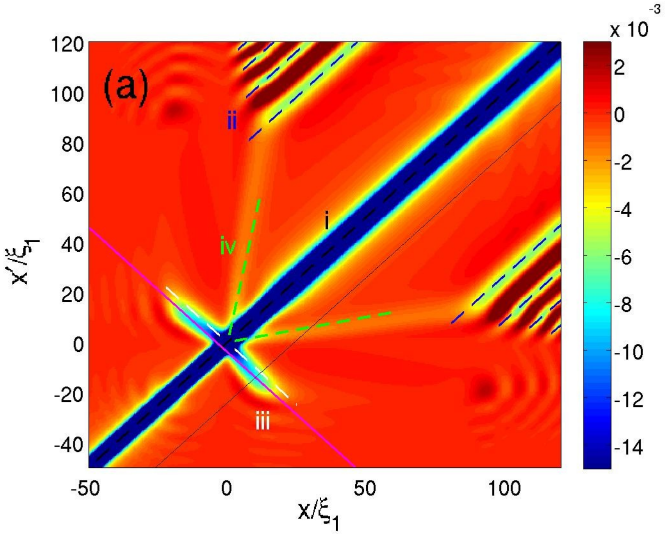
- $a_{an.inc}$, $a^{\dagger}_{an.tr}$ creation operators for Bogoliubov “ghost” branch
- zero-point fluctuations in incident beam becomes real transmitted particles

$$\langle a^{\dagger}_{an.tr.}\, a_{an.tr.} \rangle = |M_{3,3}|^2 \langle a_{an.inc.}\, a^{\dagger}_{an.inc.} \rangle + \ldots$$

- parametric emission of phonon pairs from vacuum
- energy conserved thanks to super-sonic flow; momentum provided by horizon

See also: Leonhardt & Philbin, arXiv:0803.0669; Macher & Parentani, arXiv:0903.2224

Why correlations?



Quantum correlations in emitted pairs:

- $\langle a_{refl}\, a_{an.tr.} \rangle = M_{1,3} M^{\star}_{3,3} \langle a_{an.inc.}\, a^{\dagger}_{an.inc.} \rangle$
- $\langle a_{tr}\, a_{an.tr.} \rangle = M_{2,3} M^{\star}_{3,3} \langle a_{an.inc.}\, a^{\dagger}_{an.inc.} \rangle$

Two-mode squeezing, thermal statistics when looking at one component

Simultaneous emission at all times t at horizon position

Propagate from the horizon with group velocity

Visible in density correlation function as signal peaked on lines $\frac{x}{v_{g1}} = \frac{x'}{v_{g2}}$

Slopes determined by c_1, v_0, c_2:

- in-out: $v_{g1}=v_0-c_1$, $v_{g2}=v_0-c_2$ → $\frac{v_0-c_2}{v_0-c_1} \simeq -1$
- in-in: $v_{g2}=v_0+c_2$, $v_{g2}=v_0-c_2$ → $\frac{v_0-c_2}{v_0+c_2} \simeq \frac{1}{5}$

A. Recati, N. Pavloff, IC, to appear (2009).

Conclusions

Analog Hawking radiation of phonons from acoustic black-hole
numerically observed via density correlation function

- microscopic simulations of condensate dynamics
  trans-Planckian, high-k modes in horizon region under control
- parametric Hawking emission of entangled phonon pairs from
  the horizon responsible for the observed correlated density fluctuations
- Hawking signal easily distinguished from other processes
  (e.g. Landau-Cerenkov, background thermal phonons)
- appreciable signal intensity for realistic parameters
- thermal spectrum at T_H in hydrodynamic limit
  significant deviations for stronger surface gravity

R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, IC, PRA 78, 021603 (2008)
IC, S. Fagnocchi, A. Recati, R. Balbinot, A. Fabbri, New J. Phys. 10, 103001 (2008)
A couple more works about to appear: stay tuned on the arXiv !!

New perspectives

- Dynamical Casimir density correlations sensitive to initial temperature
  - application to thermometry of ultracold atomic gases

- Back-action of quantum fluctuations onto horizon:
  - model for black-hole evaporation under Hawking radiation
  - in principle can be simulated by Wigner-MC method, numerically very demanding

- Analog HR in BECs of exciton-polaritons and/or nonlinear optics:
  - spatial modulation imprinted by lateral profile of pump beams
  - intensity correlations observable in emitted light
  - analogies/differences with standard optical parametric emission